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CONTENTS

Intestinal Parasites in Patients attending Medical City Teaching Hospital in Baghdad

SAMI Y. GUIRGES.

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ABSTRACT

Diarrhoea associated with intestinal parasitic infections is a common health problem in patients attending hospitals in Baghdad. The investigation was carried out on the incidence of intestinal parasites and fungi in patients attending Medical City Teaching Hospital in Baghdad.

The clinical material is based on 3564 faecal examinations by wet film preparation and culture for intestinal fungi. The infection rate with one or more of the intestinal parasites was found as high as 41.3% of the patients examined. The results showed that *Entamoeba histolytica* is the most prevalent protozoal parasite which was recovered in 26.5% of the patients. the intestinal flagellate *Giardia lamblia* showed an infection rate of 7.4%. *Dientamoeba fragilis* was recovered in 23 cases (0.6%). The infection with soil transmitted nematodes and cestodes ranged from 0.2 to 0.7%, the incidence of intestinal moniliasis and geotrichosis were 14.6 and 2.1% respectively. Two protozoal intestinal parasites *Dientamoeba fragilis*, *Embadomonas intestinalis* and one cestode *Hymenolepis diminuta* were recorded for the first time in Iraq.

الخلاصة

إن الإسهال المصحوب بالإصابة بالطفيليات المعوية هو من المشاكل الصحية للمرضى الوافدين إلى مستشفيات بغداد. أجري هذا البحث للتحري عن الإصابة بالطفيليات والفطريات المعوية للمرضى الوافدين إلى مدينة الطب في بغداد. المادة السريرية اعتمدت على فحص 3564 غائط بواسطة الشريحة الرطبة والزرع للفطريات المعوية.

إن الإصابة بواحد أو أكثر من الطفيليات المعوية بلغت 41.3 بالمائة من المرضى الذين تم فحصهم. وقد بينت النتائج إن طفيلي أميبا الزحار *Entamoeba histolytica* هو الأغلب إنتشارا

فقد وجدت نسبة الإصابة به 26.5 بالمائة، يأتي بعدها السوطيات المعوية *Giardia lamblia* حيث كانت نسبة الإصابة 7.4 بالمائة، وإن طفيلي الأميبا *Dientamoeba fragilis* تم تشخيصه في 23 حالة (0.6) بالمائة وقد سجل لأول مرة في القطر. وإن الإصابة بالديدان الخيطية والشريطية تراوحت نسبتها من 0.2 إلى 0.7 بالمائة. أما الإصابة بـ *Monilia* فكانت 14.6 بالمائة والـ *Geotrichum* 2.1 بالمائة. لقد تم تسجيل إثنان من الأوالي المعوية *Dientamoeba fragilis* ، *Embadomonas intestinalis* ودودة شريطية واحدة *Hymenolepis diminuta* لأول مرة في العراق.

INTRODUCTION

The importance of diseases due to intestinal parasites and their effect on the host is frequently overlooked especially in temperature climates where they are less common than in the tropics and subtropics (1). Surveys on the prevalence of intestinal parasites have been carried out in Iraq which involved mostly a healthy population of school children (2,3,4,5,6). Rare investigations were reported in patients attending hospitals (7). Accordingly, this study was carried out to report the prevalence of intestinal parasites of different age groups attending Medical City Teaching Hospital in Baghdad.

MATERIALS & METHODS

The clinical material on which this study is based consisted of 3564 faecal examination from patients attending the out-patient general laboratory of the Medical City Teaching Hospital in Baghdad for a 2-year period. Patients attending this hospital came from all areas of Baghdad province and other surrounding provinces. Also, this laboratory receives stool samples from in-patients of this hospital. In collecting fresh specimens of faeces, it was made certain that the person was not taking any preparation containing barium, bismuth or any medicine, which may have effect on the results of the examination. The stool was collected in clean, dry glass or plastic disposable stoppered container containing wooden applicator stick with a paper label provided for the name, age and sex. Male and female patients approximately equal in number were included and their ages ranged from one month to 70 years. Specimens were examined within one hour from the time passed. The stools were examined macroscopically for the presence of blood, mucus, tapeworm proglottides or adult worms which may be present in the stool and its consistency is recorded. Microscopic examination was made by preparing two direct wet smears, one in normal saline and the other in Lugol's iodine. Iron-alum haematoxylin stain was used occasionally for confirmation of the identification of the amoebae (3). Ova of hookworms were identified whether *Ancylostoma duodenale* or *Necator americanus* by culturing the stool specimen in Petri-dishes with charcoal according to the method of Headlee (8) and identified by the key given by the expert committee of WHO in helminthiasis (9). The budding cells of monilia were identified as *Candida albicans* from other species of *Candida* by culturing the specimen on Sabouraud's agar medium and subcultured on corn meal agar medium for the production of chlamydospores. Specimens showing barrel-shaped cells were identified as *Geotrichum candidum* by culturing the specimen on Sabouraud's medium and can be differentiated from other yeast like fungi by the flat rapidly spreading wrinkled colony (10).

RESULTS

The number of faecal specimens submitted for the study in the prevalence of intestinal parasites and found suitable for proper examination were 3564 specimens. We set a target for this high number of stool to be examined feeling that this high number was necessary if a more or less correct percentage of the prevalence of intestinal parasites was to be determined. Of all the patients examined 1470 (41.3%) were found infected. The infection rates with one or more parasites are shown in Table 1.

Table (1): Rates of infection with one or more of the intestinal parasites in 1470 patients.

No. of parasite	Total infected	Percent of infection
One parasite	1023	69.6%
Two parasites	290	19.7%
Three parasites	123	8.4%
Four parasites	30	2.0%
Five parasites	4	0.3%
Total	1470	100%

From the data above in table 1, it can be said that, about 70% of the patients proved to have single species infection and about 20% had double infections, 8.4% triple infections, 2% quadruple infections, and 0.3% had quintuple infections. In general stool examination, it was found that *Entamoeba coli* and *Endolimax nana* infections are in association with *Entamoeba histolytica* infections, adding to these *Giardia lamblia* and *Blastocystis hominis*.

The results of stool examinations of 3564 patients show that, the most common protozoal parasite is *E. histolytica* which was obtained in 938 (26.3%) of the patients. The majority of these patients were suffering either from abdominal pain or diarrhoea or both. It is worthwhile to mention that *E. histolytica* trophozoites were found in five patients less than one year of age and the youngest was six months. The commonesal parasite *E. coli* was found in 758 (21.3%) of the patients mostly in association with *E. histolytica* infection, *G. lamblia* was demonstrated in 265 (7.4%) which represent the next frequent pathogenic protozoal parasite encountered. It was more common in children below five years. The youngest age found infected with this parasite was 5-8 months where trophozoites of *G. lamblia* were found in 12 patients. Out of stool specimens examined 23 (0.6%) were diagnosed as trophozoites of *Dientamoeba fragilis*. The incidence of other protozoal parasites recovered in this study ranged from 0.5% to 6.5% (Table 2).

Table-2: Protozoal parasites in 3564 patients examined.

Parasites	Number infected	Percent of infection
1. *Entamoeba histolytica*	938	26.3%
2. *Entamoeba coli*	758	21.3%
3. *Endolimax nana*	87	2.4%
4. *Iodamoeba butschlii*	41	1.2%
5. *Dientamoeba fragilis*	23	0.6%
6. *Blastocystis hominis*	231	6.5%
7. *Giardia lamblia*	265	7.4%
8. *Trichomonas hominis*	61	1.7%
9. *Chilomastix mesnili*	122	3.4%
10. *Enteromonas hominis*	16	0.5%
11. *Embadomonas intestinalis*	45	1.3%

Results of the incidence of helminthic intestinal parasites in 3564 patients examined are listed in Table 3. Eight intestinal helminths were detected. Among these *Enterobius vermicularis* was the most frequently encountered which was recovered in 112 (3.2%) of the stool examined. The next common intestinal helminth is the cestode *Hymenolepis nana* which was obtained in 78 (2.2%) of the cases. *Hymenolepis diminuta* ova were recovered in 6 cases (0.2%). The infection with other cestodes and soil transmitted nematodes ranged from 0.2% to 0.7%. It is worthwhile to mention that, all the results of culturing the ova of hookworms *A. duodenale* larvae were obtained.

Table-3. Helminthic parasites in 3564 patients examined.

Parasites	Number infected	Percent of infection
1. Ascaris lumbricoides	25	0.7%
2. Hookworm	16	0.5%
3. Strongyloides stercoralis	10	0.3%
4. Trichuris trichiura	6	0.2%
5. Enterobius vermicularis	112	3.2%
6. Taenia Sp.	8	0.2%
7. Hymenolepis nana	78	2.2%
8. Hymenolepis diminuta	6	0.2%

On the other hand, in stools of 3564 patients examined budding cells of monilia were identified in 510 patients (14.6%), and the culture of the stool on corn meal agar revealed 312 (61.2%) to be *Candida albicans*. Moreover, it was also noted that this pathogenic monilia is mostly encountered in infants and children. The other fungus encountered in the stool examination is *Geotrichum candidum* which was found in 76 cases (2.1%) Table 4.

Table-4. Intestinal fungi in 3564 patients examined.

Fungus	No. positive	Percent positive
Monilia	510	14.6%
Geotrichum candidum	76	2.1%

DISCUSSION

The chronological record of prevalence of intestinal parasites in this country is far from complete. This work was carried out in the hope of throwing some light on the prevalence of intestinal parasites in patients attending a hospital in Baghdad. In addition investigation was carried out on other microorganisms including fungi which may infect the human intestine and can be recovered by stool examination. There are very few investigations which report all the intestinal parasites protozoal and helminths. In the present survey the prevalence rate was found to be as high as 41.3% in spite of the fact that the results are based on single stool specimen examination. This points to the fact that, the expected prevalence rate would be undoubtedly much higher if the three specimen technique was used.

Amoebiasis is known to occur in every part of the world and most people surveyed in tropical and subtropical countries with the exception of Esckimo and of poles with over 90.000 examinations with a nil report (11). It was estimated that *E. histolytica* infections are about 10% of the world's population, although its prevalence and severity may differ from one area to another (12). In the present study, *E. histolytica* infection was found in 26.3% of the patients examined which represents the highest incidence among all other intestinal protozoa. In 120 school children in Baghdad examined by other workers (3) the incidence of this parasite was found to be 23%. Recently in 3726 stool samples collected also from school children in Baghdad the incidence of this parasite was found to be 11.4% (6). In Sammarra city the incidence of infection with this parasite in patients attending the primary health care center was 36.7% (13). While in north of Iraq in Arbil City, the same workers found the incidence of this parasite to be 4.1% (14). Moreover, other workers (7) in Baghdad reported the incidence of *E. histolytica* infection to be 49.5% in 220 hospital patients with diarrhoea. The high incidence of *E. histolytica* reported by these workers because their cases were selected hospital patients suffering from acute diarrhoea. Another important protozoal parasite recovered in this study is *G. lamblia*, which was demonstrated in 7.4% of the patients examined. Giardiasis was particularly prevalent in children less than 10 years of age and most cases were associated with diarrhoea with various degrees of clinical features associated with intestinal malabsorption which have been demonstrated by standard technicques in giardiasis (15). A high incidence of giardiasis ranging from 31 to 33% was reported by other workers in pre-school and school children surveyed in

Baghdad (2,3). The high incidence of *E. histolytica, E. coli* and *G. lamblia* can be attributed to the fact that cysts of these parasites resist water. They are normally faeco-orally transmitted, but may become water transmitted when faecal material is discharged into the water supply system. Hence their high incidence at present, which is an indicator of drinking water pollution. In addition, the multiple infections can be attributed to the consumption of a highly contaminated food and water or to the impairment of the immunity. On the other hand, the incidence of other non-pathogenic intestinal amoebae like *E. nana* and *Iodoamoeba butschlii* was found to be less than 3% of our stool examination. Other workers did not mention these parasites in their work except that of Al-Dabagh et al. (2) which gave an incidence of 6 to 7%. Meanwhile *Dientamoeba fragilis* which was found in 23 cases (0.6%) of the stool examined. This parasite was not recovered previously by other workers and to the best of our knowledge this parasite is recorded for the first time in this country. The reason for not recording this parasite previously can be attributed to the unfamiliarity of the stool examiners with the amoebae especially the trophozoite stage which may be present in the stool and it requires a time and long search in the fresh stool preparation. It can be said that each fresh preparation requires a careful search for about half hour for watching the morphology and the movement of the trophozoite to give a correct diagnosis.

The intestinal flagellates recovered in our study in addition to *G. lamblia* mentioned above, are *Trichomonas hominis, Chilomastix mesnili, Enteromonas hominis and Embadomonas intestinalis* , their incidence were 1.7% , 3.4%, 0.5% and 1.3% respectively. The incidence of *T. hominis* reported by other workers ranged from 1% to 20% (2,3,6,7). On the other hand, the incidence of *C. mesnili* reported by other workers ranged from 3% to 9% (2,3). The other two flagellates reported in this study *E. hominis* and *E. intestinalis* have apparently not been previously recorded by other investigators except one worker (2) who reported a case of *E. hominis.*

On the other hand, the prevalence of the commoner helminth species encountered in our study are *E. vermicularis* and *H. nana* having an incidence of 3.2% and 2.2% respectively. These two parasites are faeco-orally transmitted. Their incidence reflects personal and family hygiene. All other helminth species had an incidence of less than 1%. The nematode *E. vermicularis* was probably much more common as our results is based on stool examination and not by Scotch tape techniques. A high incidence of 26%

of this nematode recovered by other workers (3,6) by using salt floatation method in the stool examination. This higher incidence obtained because their cases were selected elementary school children approximately 6 to 13 years old.

It was possible during the present study to identify the species of hookworm infecting patients, although it is well known that *A. duodenale* is the only species of hookworm present in this part of the world, but we wanted to make sure that this is true. It was found that *A. duodenale* is the only species of hookworm recovered. The percentage of infection with *Strongyloides stercoralis* obtained in this investigation corresponds to that obtained by other workers (3). The incidence of soil transmitted nematodes decreases with time in urban areas, particularly where sewage systems have been introduced.

The present investigation indicated the presence of the rat tapeworm *Hymenolepis diminuta* in 6 cases (0.2%). For the best of our knowledge this cestode is recorded for the first time. As it is known that both species of rats *Rattus rattus* and *Rattus norvegicus* are prevalent in Baghdad and it is possible to find them in every house and this increased population of rats may be attributed to the lack of control measures against them and these animals may cause contamination of food by their faeces and fleas and eaten or swallowed accidentally by man and cause infection with these cestodes.

In infection with monilia and specifically with *Candida* species, which may be present as a normal microflora in many parts of the human body, it may cause a number of infections under certain condition such as the use of broad spectrum antibiotics (16,17). The identification of the species of *Candida*, which cause gastroenteritis were fully studied previously (17) who showed that, both *C. albicans* and *Candida stellatoidea* are important in human infection in Iraqi patients., Meanwhile, in our study geotrichosis due to *G. candidum* was found in patients with diarrhoea resembling intestinal candidiasis. In this country there are no previous reports of such infections except that reported recently (18,19).

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Isolation of Fungi Strains from Sewage for production of Lemon Acid and Chemical Oxygen Demand Removal

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الخلاصة

استعملت المياه الثقيلة كمصدر للفطريات الدقيقة من اجل انتاج حامض الليمون (حامض الستريك) ، وهو احد المركبات الكيمياوية المهمة المستعملة في مختلف العمليات الصناعية. عزلت الفطريات الخيطية خصوصا الفطر *Aspergillus niger* من المياه الثقيلة المتكيفة فيها .

عزلت ثلاث سلالات فطرية من المياه الثقيلة وشخصت بأستعمال تقنية مزرعة الشريحة المتبوعة بفحص صور الفطريات النامية .

اختير في هذه الدراسة خمس سلالات من الفطر *A. niger* كونها أفضل سلالات متكيفة مع المياه الثقيلة ثلاثة منها (S1, S2 , S3) اختيرت من السلالات المعزولة من المياه الثقيلة نفسها . والسلالتان الاخريتان (L1&L2) من عزلات اصلية محفوظة في المختبر. جربت كل السلالات تحت ظروف تخمر مضبوطة مثل pH3، ودرجة الحرارة 32 م ، وعدد دورات الهز 200 دورة /دقيقة و 2% (وزن /وزن) من وحل المياه الثقيلة كمادة نمو اساس ، و3% (وزن/وزن) من الطحين كمادة نمو مساعدة ، وحجم اللقاح 2% (سبور/مللتر) وباستعمال طريقة المزرعة المغمورة في عملية التخمر للحصول على افضل انتاج من حامض الستريك .

قيمت امكانية الفطريات من خلال اعلى انتاج للحامض ، والنسبة المئوية للمواد الحيوية الصلبة العالقة الكلية (TSS%)، وازالة المتطلب الكيمياوي للأوكسجين (COD). اعطت السلالة S1 اعلى تركيز من حامض الستريك 0.139 غم/لتر ومواد حيوية صلبة عالقة كلية(TSS) 16.82 غم/لتر عند اليوم الرابع للنمو ، بينما كانت أعلى ازالة للـــ (COD)حصلت عند اليوم السادس للنمو.

ABSTRACT

Sewage Treatment Unit (STU) sludge was used as a source of micro fungi for production of Lemon acid (citric acid), one of the important chemicals used in various industrial processes. The isolation of filamentous fungi especially *Aspergillus niger* was done from STU sludge for better adaptability. Three strains of *A. niger* were isolated from STU sludge and identified using slide culture technique followed by image analysis.

Five strains of *A. niger* were selected as the best adapted strains in STU sludge. Three of them (S1, S2, and S3) were selected from isolated strains, and another two (L1 & L2)

from the lab stock. All strains were experimented under controlled fermentation conditions such as pH 3, temperature 32°C and agitation 200 rpm, using 2% (w/w) of substrate (STU sludge), 3% (w/w) co-substrate (wheat flour) with inoculum's size of 2% (spore /mL), using submerged culture fermentation process for the maximum production of citric acid.

Evaluation of fungal potentiality was done in terms of maximum citric acid production, biosolids production (TSS %) and chemical oxygen demand (COD) removal. Strain S1 produced high concentration of citric acid (0.139 g /L) and biosolids (16.82 g /L) on fourth day of fermentation, whereas COD removal (91.0 %) on sixth day.

INTRODUCTION

Some researches studied the utilization of Sewage Treatment Unit (STU) sludge that is an inexpensive and easily available raw material and a good source for growth of microorganisms because it has enough nutrients and trace elements. STU sludge is one of the final products of the treatment of sewage at a sewage (wastewater) treatment Unit. Treatment breaks down the organic matter and kills disease-causing organisms [1].

The main groups of the organic solids in the sludge are protein, carbohydrates, fats and oils [2], which vary with their origin, system and efficiency of the wastewater treatment Unit [3]. In Baghdad, STU sludge is the largest contributor of organic pollution to water resources and environment.

The sludge volume is expected to rise year by year. The management of the ever increasing organic wastes has been one of the important environmental issues in Baghdad, which requires a pragmatic and economic approach and study to utilize this sludge is vital to have a good waste management. STU sludge can be a very good source of carbon, nitrogen, phosphorus and other nutrients for many microbial processes that can add to the value of sludge by producing valuable metabolic product like citric acid.

Citric acid is one of the important chemicals used in various industrial processes. It is estimated that about 500,000 tons of citric acid is produced annually by fermentation of expensive raw materials like glucose and sucrose. In developed countries, 65% of citric acid consumption is in food and beverages and 20% in household detergents with the estimated global growth rate in demand between a 4 to 5% per year [4]. Various substrates like sugar cane molasses, inulin, kurma, date fruit syrup and carob pod [5-10] have been used for citric acid production by *Aspergillus niger*.

Some products, which were produced by using the microorganism *A. niger*, have been assessed as acceptable for daily intake by the World Health Organization [11].

Therefore, this study emphasized on utilization of a new substrate, sewage treatment unit sludge (STU sludge) for the production of citric acid as well as removal of COD. In order to achieve the target, isolation of filamentous fungi especially *A. niger* was done from STU sludge itself for better adaptability and its screening for effective bioconversion of the sludge into citric acid.

MATERIALS AND METHODS

Sample collection: STU sludge sample was obtained from Rustumya in Baghdad. The sample was collected in batch approximately every one week and, after maceration, stored in stainless in a cool room controlled at 4°C. To feed the lab experiments, a sample of sewage was drawn from the storage tanks after thorough mixing and then diluted with sterile distilled water for further use as a substrate [3].
Wheat flour is used as a co-substrate obtained from the market.

Isolation, purification and identification of microorganism: The media used for isolation of microorganism from sample (STU sludge) was a modified composition used by Ilias[12] (g per liter); KH_2PO_4 1.0, $MgSO_4.7H_2O$ 0.5, Peptone 5.0, Dextrose 10.0, Agar 10.0 and Streptomycin 2 mL (50 mg /L). All compositions were added prior to autoclaving at 121°C for 20 minutes, except Streptomycin, which was sterilized and added to the media after autoclaving. Dilution was done by mixing 5ml of sample with 100 ml sterile distilled water. Afterwards, 1 ml of diluted sample was put into Petri dish followed by 20ml of the media (3 replicates) and allowed to grow for 3-4 days in incubator at 32°C. Fungi were sub-cultured on Potato Dextrose Agar (PDA) medium to obtain pure strains. Identification was done visually and by micro morphological studies using Slide Culture Technique [16]. Olympus microscope was used to determine the morphology of the isolates.

Inoculum preparation: Inoculum preparation (spore suspension) was done according to the method suggested by Alam *et al*. [13]. Cultures grown on PDA medium in Petri dishes at 32°C for 7 days were transferred into Erlenmeyer flask (250 ml) containing 100 ml of sterile distilled water. It was shaken in a rotary shaker for 24 hours with 200 rpm and filtered. The filtrate was used as inoculum after measuring its concentration (spores / mL) by Haemocytometer.

Screening: Screening as done to get the best strain based on maximum citric acid production, treated biosolids and COD removal. Five strains of *Aspergillus niger* were selected for screening based on assessment of the best adapted strains in STU sludge. Three of them (S1, S2, and S3) were selected from isolated strains and another two (L1 & L2) from the lab stock. The screening experiment was done in a 500 ml of Erlenmeyer flask containing 100 ml of wastewater sludge with the fixed process conditions according to literature: substrate concentration of 1%(w/v), co-substrate concentration 2% (w/v), initial pH of 3, temperature of 30°C, agitation of 200 rpm and inoculum size of 2% (w/v). The sample was sterilized, inoculated and incubated in a rotary shaker for 2, 4 and 6 days of treatment. After treatment the sample was harvested to determine the parameters citric acid, chemical oxygen demand (COD) and biosolids content. Citric acid was determined according to the method of Marier and Boulet [15].COD and biosolids as the total suspended solids (TSS) were measured following the methods of APHA [15].

RESULTS AND DISCUSSION

Three strains of filamentous fungi – S12, S2 and S3 – were isolated from STU sludge and identified as *Aspergillus niger* by micro-morphological studies using slide culture technique [16] and by examining the size, shape arrangement and development of conidiophores and phialospores.

Two strains of *Aspergillus niger* (L1 & L2) were selected from lab stock. All five strains were screened using same process conditions and the best strain (S1) was selected on the basis of citric acid produced, COD removal, and treated biosolids. Citric acid concentration varies with fermentation time as shown in Fig. 1. High yield of citric acid was produced by all strains on 4th day of fermentation except for L2 and S3 and it reduced when the fermentation time increased. Although these two strains can produce highest citric acid in less time, the yield is insignificant (0.079 g /L and 0.075 g/ L for L2 and S3 respectively) compared to other strains. Overall, the highest citric acid produced (0.139 g/ L) was by strain S1 on 4th day and the lowest (0.005 g/ L) was by S2 on the 2nd day of fermentation. The difference of maximum citric acid yield for L1 and S1 is 0.008 g/ L for day 4 and 0.037 g/ L for day 2. For day 6, citric acid production of L1 is higher than S1 by only 0.001 g/ L. Since S1 obtained higher citric acid compared to L1 for most of days, strain S1 was selected for further study. The amount of citric acid produced during this study is very little, appearly due to presence of heavy metals and other components in STU sludge [13], which can decrease the production of this acid. The second parameter used to evaluate the fungal potential was COD. Removal of COD increased with fermentation time as shown in Fig. 2. This observation was expected as removal of COD is a percentage of the organic matter removed during the treatment because it is consumed by the fungi. The maximum COD removal (90%) was observed on day 6 for most of the strains. Since there is not much difference in terms of COD removal for all strains in terms of maximum removal and time of fermentations, S1 can be selected as the potential strain for maximum COD (91.0%) removal. When we observed the percentage of treated biosolids (Fig 3), strain S1 showed a good growth of biomass (1.682%) on day 2 and decreased as the time of fermentation increased. Other strains namely L1, L2, S2 and S3 reached their optimum values (1.452, 1.453, 1.590 and 1.494 % respectively) on 4th day of fermentation. Since none of the strains have significant difference in percentage of treated biosolids when compared to each other, S1 could be considered the best because it reached its maximum in day 2.

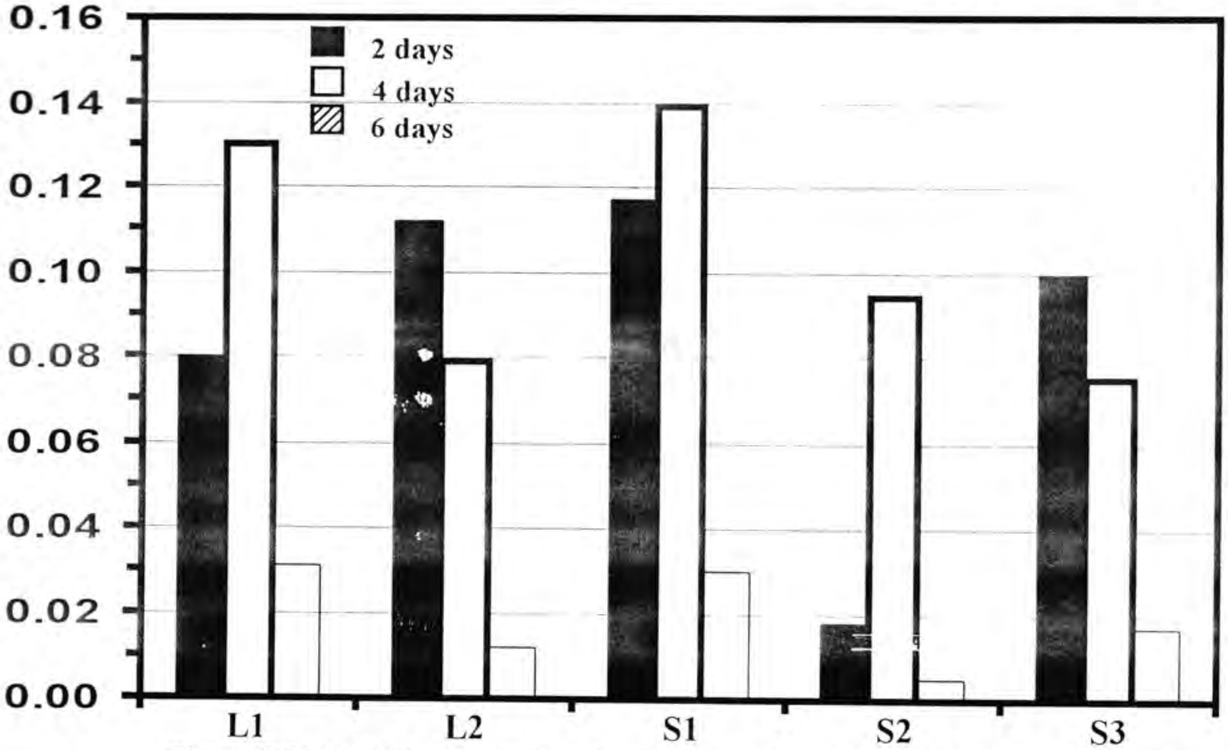

Fig 1: Effect of fermentation time (days) on citric acid concentration (g/L) by *Aspergillus niger*.

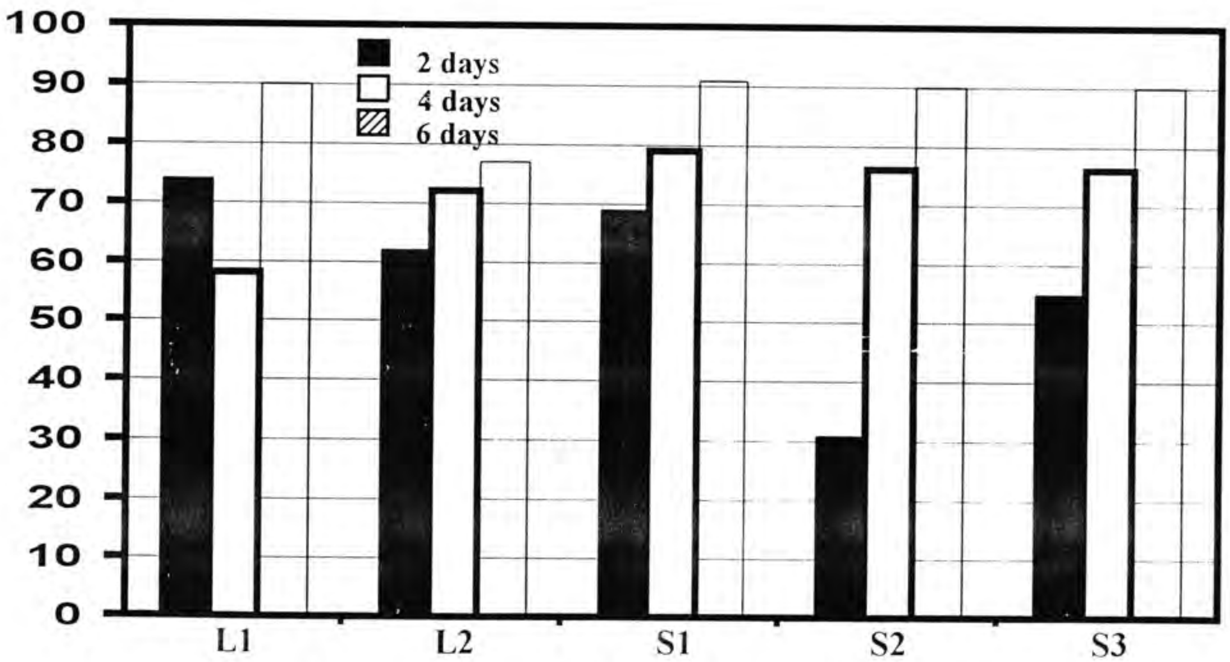

Fig 2: Effect of fermentation time (days) on COD % by *Aspergillus niger*.

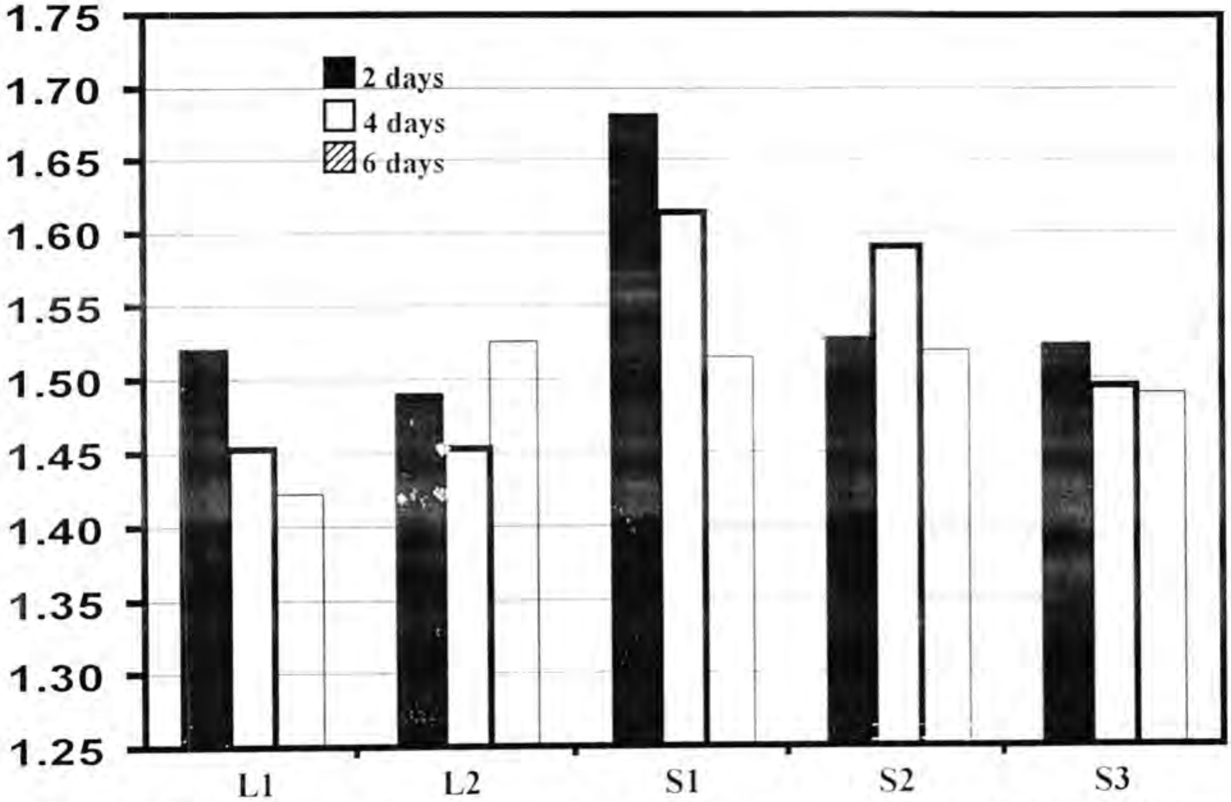

Fig 3: Effect of fermentation time (days) on treated biosolids % of *Aspergillus niger*.

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SYNTHESIS OF POLY [N-SUBSTITUTED ACRYLAMIDE] FROM POLYACRYLOYL CHLORIDE WITH DIFFERENT AMINES

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ABSTRACT

New nine polymers derivatives from poly acrylamide were prepared by reacti on of poly cryloyl chloride with different amines, primary, secondary, aliphatics and aromatics in absolute ethyl alcohol by using tri ethyl amine as catalyst. Polymers [N-substituted acryl amide] were prepared in good yields , 51-83%.

All the prepared polymers were studied by FT-IR spectroscopy. The softening points , melting points , and solubility were measured.

الخلاصة

حضرت تسع مشتقات لبوليمر الأكريل أمايد من تفاعل بولي كلوريد الأكريلويل polyacryloyl chloride مع امينات مختلفة تضمنت امينات اولية وثانوية اليفاتية واروماتية بمولات متساوية في الكحول الاثيلي المطلق وباستخدام ثلاثي اثيل امين كعامل مساعد. تراوحت نسبة التحول بين (51-83)% تم تشخيص هذه البوليمرات بتقنية الــ FT-IR، وقياس الخواص الفيزيائية من درجة التلين ودرجة الانصهار وكذلك الذوبانية.

INTRODUCTION

Polyacrylamides (PAMS) have many different applications because of their high viscosity. In agriculture, they make irrigation more efficient and prevent soil erosion. PAMS are also used as an additive in drilling muds , for photographic film and battery housing. In addition, PAMS are the major components used in gel electrophoresis for protines and nucleic acids[1-10]. For each of these applications and many more were behind the reasons of preparation of PAMS.

Polyamides (PAs), are made by reacting carboxylic acid with amines[11].

$$R-\overset{\overset{O}{\|}}{C}-OH+R'NH_2 \rightarrow R-\overset{\overset{O}{\|}}{C}-NHR'+H_2O$$

Acids may be converted to amides by treatment with thionyl chloride and then with ammonia[12].

$$R-\overset{\overset{O}{\|}}{C}-c\ell+2NH_3 \rightarrow RCONH_2+NH_4c\ell$$

Acrylamide $CH_2{=}CHCONH_2$, propenamide,exhibits, good thermal stability and long shelf life in the absence of light[13]. The acryl amide solution is stabilized by oxygen and small amounts (25-30) ppm based on acrylamide of cupric ion, several other types of stabilizers. such as ferric ion [14,15],and ethylene diamine tetraacetic acid (EDTA)[16-18]. For many years, acryl amide was made by reaction of acrylonitrile with H_2SO_4, H_2O followed by separation of acryl amide from its sulfate salt by using a base or an ion - exchange colum [11].

A series of novel polyacrylamides have been synthesized from polyacryloyl chloride ,palmitoyl chloride with different kinds of amines (primary ,secondary ,hetrocyclic amines) [19] .

Another main route have been used to prepare acryl amide from acryloyl halide - amine condensation [20].

$$CH2{=}CH\ \overset{\overset{O}{\|}}{C}-c\ell+RNH_2 \rightarrow CH_2=CH-\overset{\overset{O}{\|}}{C}NHR+Hc\ell$$

Experimental preparation of poly [N-substituted acrylamide]

Melting points and softening points were determined on Gallen Kamp melting point apparatus (MFB – 600) , and Reichert Thermovar , SP1 , 10 , 0.25 , 160 respectively, FT-IR absorption spectra were recorded using KBr discks on a FT-IR – 8400 S , FOURIER TRANSFORM INFRARED SPECTROPHOTOMETER ,SHIMADZU.

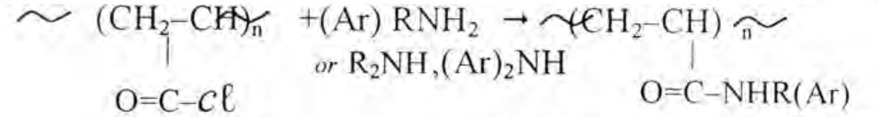
We get the Poly acryloyl chloride from the monomere acryloyl chloride which is polymerized by the light effect through the time. In a 100 ml round bottom flask provided with a reflux condenser placed (0.01

mole) of poly (acryloyl chloride) with (0.01 mole) of amine and 25 ml of absolute ethanol. The mixture was refluxed at 60°C for 30 minutes,then a (1 ml) triethylamine was added dropwise to the reaction mixture with contineous stirring. The mixture was refluxed for (2-7) hours depending on the type of amines. The reaction mixture was poured into a beaker containing chloroform and vigorosuly stirred ,the product was transffered to separatory funnel and the amide viscose layer was separated . The polyamide product was purified by dissolving in THF, DMF, or methanol with gentle heating and filtered. The clear filtered solution was precipitated by addition to suitable solvent such as acetone, diethyl ether, benzene and petrolium ether, and the formed precipitate was filtered, washed and dried.

Physical properties of the products are listed in table (1) , table (2), and table (3).

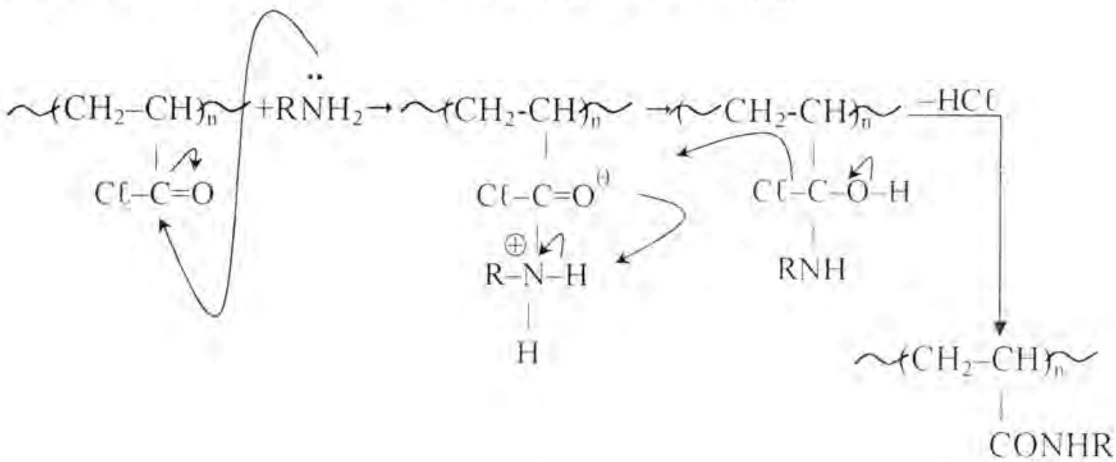
RESULTS AND DISCUSSION

Although there are several procedures for the preparation of poly [N-substituted acrylamide], one of methods is suitable for the preparation of poly [N-(isopropyl, sec.butyl, n-butyl,p-tolyl, o-anisidyl, o-chloro-o-tolyl, N, N-dimethyl, N, N-di-n-propyl, and morphoyl] acrylamide respectively from reaction of poly acryloyl chloride with different primary and secondary amine, aliphatic, aromatic and hetrocyclic using triethyl amine as catalyst to dehydrohalogenation of HCl. The condensation reaction between poly acryloyl chloride and different amines was yielded the poly [N-substituted acrylamide] with good yield (51-83)%.



acryloyl chloride **poly[N-substituted acrylamide]**

The mechanism of condensation reaction is as follows:



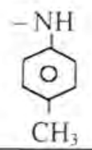
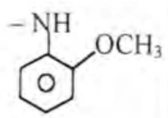
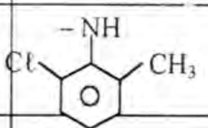
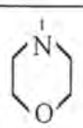
The melting point of poly acryloyl chloride is around (260-265) C.
All physical properties for prepared polymers are listed in tables (1,2,3).The measured softening point of produced amides were higher than those of softening point of the starting material (acryloyl chloride). This might be due to the hydrogen bonding arises from the −CONHR groups .
The FTIR spectra of all poly [N-substituted acrylamide] are shown in Figs. (2-4). The softening point of all prepared poly acrylamide indicated that these polymers are thermally stable , this due to hydrogen bonding of amide groups which enhanced the polar interaction between chains[15].

I.R. spectra of the resulted amides from the reaction acryloyl chloride ith primary amines ,showed an (−NH) absorption band at (3420-3456) cm^{-1} (compounds 1-6).This band (−NH) was disapeared ,as expected in the products resulted from the reaction of acryloyl chloride with secondary amines (compounds 7-9).

All prepared poly(N-substituted acrylamide) in this work ,showed strong absorption carbonyl amide band in the region (1647-1727) cm^{-1} .The stretching vibration (C-N, amides) absorption band appeared in (1313-1409) cm^{-1} .The compound (9) showed an additional absorption band at 1197 cm^{-1} for C-O-C ,due to symmetrical and unsymmetrical stretching vibration of cyclic ether group .

Table (1):- Physical properties of poly (N-substituted) acrylamides.

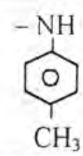
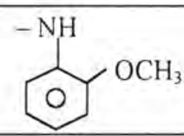
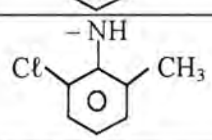
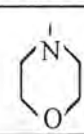
$$-\!\!(CH_2-CH)_n\!\!-$$
$$|$$
$$O=C-Am$$

No. Comp	Name of Poly (N-substituted) acrylamides	Am	Time of reaction (hrs.)	Conversion %	m.p °C	s.p °C	Color
1.	Poly [(N-isopropyl) acrylamide]	$-NH-CH(CH_3)_2$	(2-3)	65%	235-245	230-240	White
2.	Poly [(N-sec-butyl) acrylamide]	$-NH-CH(CH_3)-CH_2CH_3$	(3)	62%	302-307	300-305	Orange
3.	Poly [(N-(n-butyl) acrylamide]	$-NH(CH_2CH_2CH_2CH_3)$	(4)	83%	255-260	250-260	Gray
4.	poly[(N-(p-Tolyl) acrylamide]		(5)	71%	283-290	280-290	Dark-brown
5.	Poly [(N-(O-anisidyl) acrylamide		(5)	78%	311-320	308-315	Yellow
6.	Poly [(N(O-chloro-O-Tolyl) acrylamide]		(7)	51%	> 360	320-332	Brown
7.	Poly [(N,N-diethyl) acrylamide]	$-N(CH_2CH_3)_2$	(3)	80%	265-272	262-270	Yellow
8.	Poly [(N, N- di n-propyl) acryl amide]	$-N(CH_2CH_2CH_3)_2$	(3)	65%	330-340	325-335	Yellow
9.	Poly [(morpholinyl) acrylamide]		(4-5)	60%	> 360	310-325	Yellow

m.p : melting point, s.p : softening point . Am : Amine groups .

Table (2):- Solubility of the prepared poly (N-substituted) acrylamides.

$$\left(CH_2-CH \right)_n$$
$$|$$
$$O=C-Am$$

No. Comp	Am.	CHCl3	THF	DMF	DMSO
1.	$-NH-CH(CH_3)_2$	+	++	++	++
2.	$-NH-CH(CH_3)-CH_2CH_3$	+	++	++	++
3.	$-NH(CH_2CH_2CH_2CH_3)$	—	++	++	++
4.		—	+	+	+
5.		—	+	+	+
6.		—	+	+	+
7.	$-N(CH_2CH_3)_2$	+	++	++	++
8.	$-N(CH_2CH_2CH_3)_2$	+	++	++	++
9.		+	++	++	++

(++) = easy soluble.
(+) = Soluble.
(−) = Insoluble.

The prepared polymers are insoluble in the following solvents : H_2O,Et_2O,$(CH_3)_2C=O$,EtOH,PhH,CCl_4.

Table (3):- FT-IR Spectra of poly (acryl amide) and its derivatives .

$$\sim\!\!(CH_2-CH)_n\!\!\sim$$
$$|$$
$$O=C-Am$$

No. Comp	v(-NH) cm^{-1}	V(C=O) amide cm^{-1}	v(C-N) amide cm^{-1}	v(C-H) aliphatic cm^{-1}	v(C-H) aromatic cm^{-1}	Others
1.	3456	1764	1452	2875 2933	—	—
2.	3450	1647 1693	1423	2858 2925	—	—
3.	3355	1612 1676	1429	2812 2736 2522 2426	—	—
4.	3450	1739	1450 1496	2923	3150	v(C- Cl) (596) cm^{-1}
5.	3425	1728	1454	2677 2477	2939	v(C-O-C) Ether (1066) cm^{-1}
6.	3450	1739	1450	2923	3100	—
7.	—	1685	1494	2871 2925 2966	—	—
8.	—	1685	1495	2966	—	—
9.	—	1724	1409	2927	—	v(C-O-C) Ether (1197) cm^{-1}

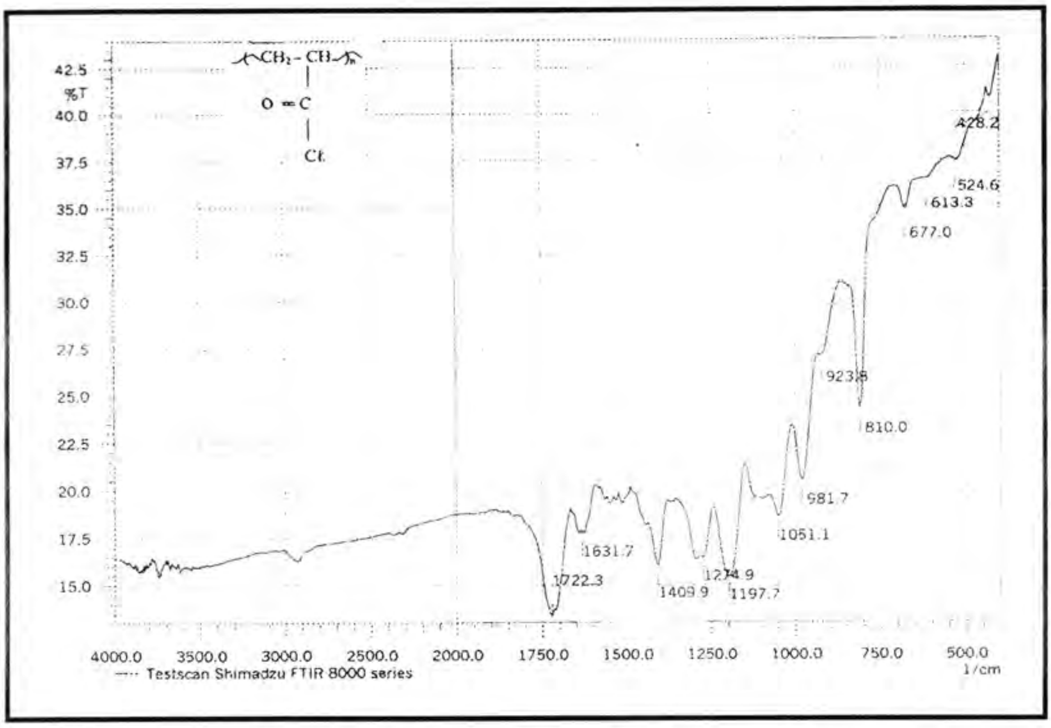


Fig. (1): Spectrum of poly acryloyl chloride

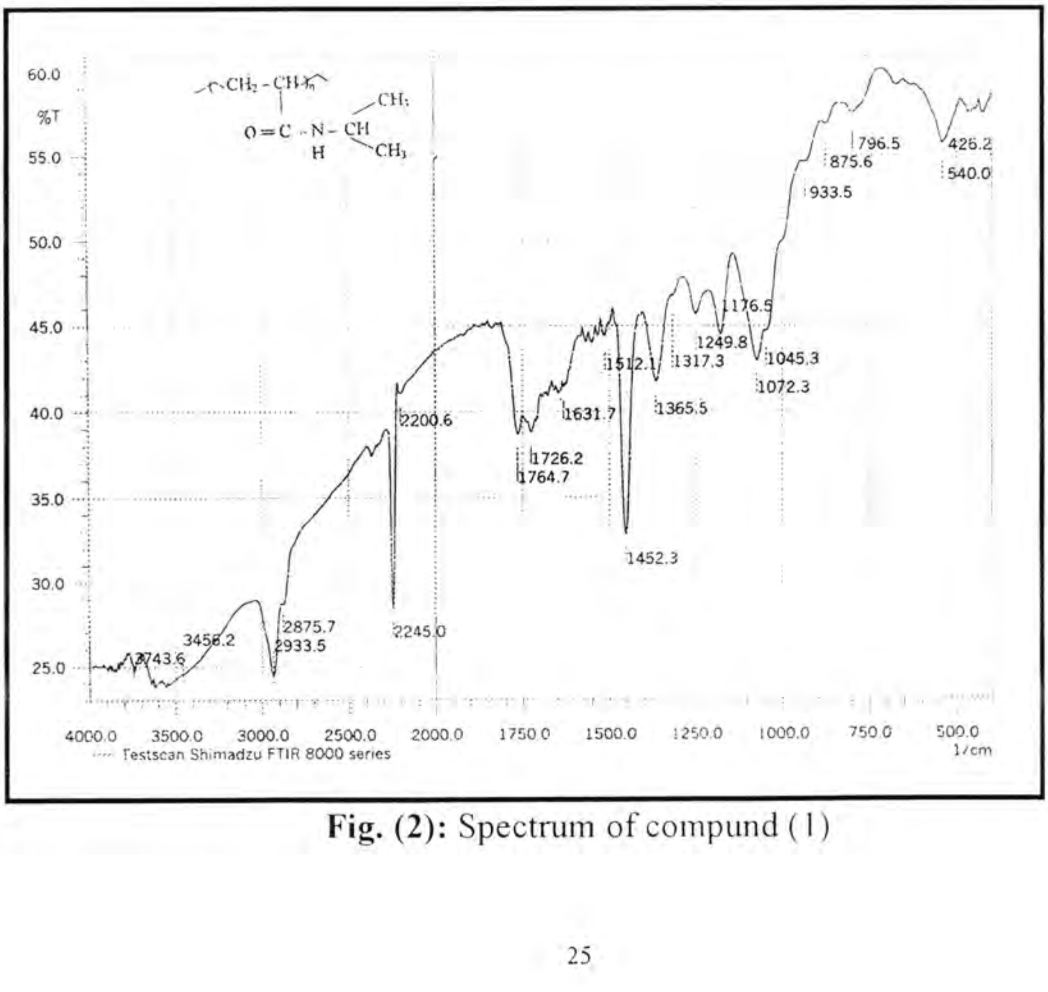


Fig. (2): Spectrum of compund (1)

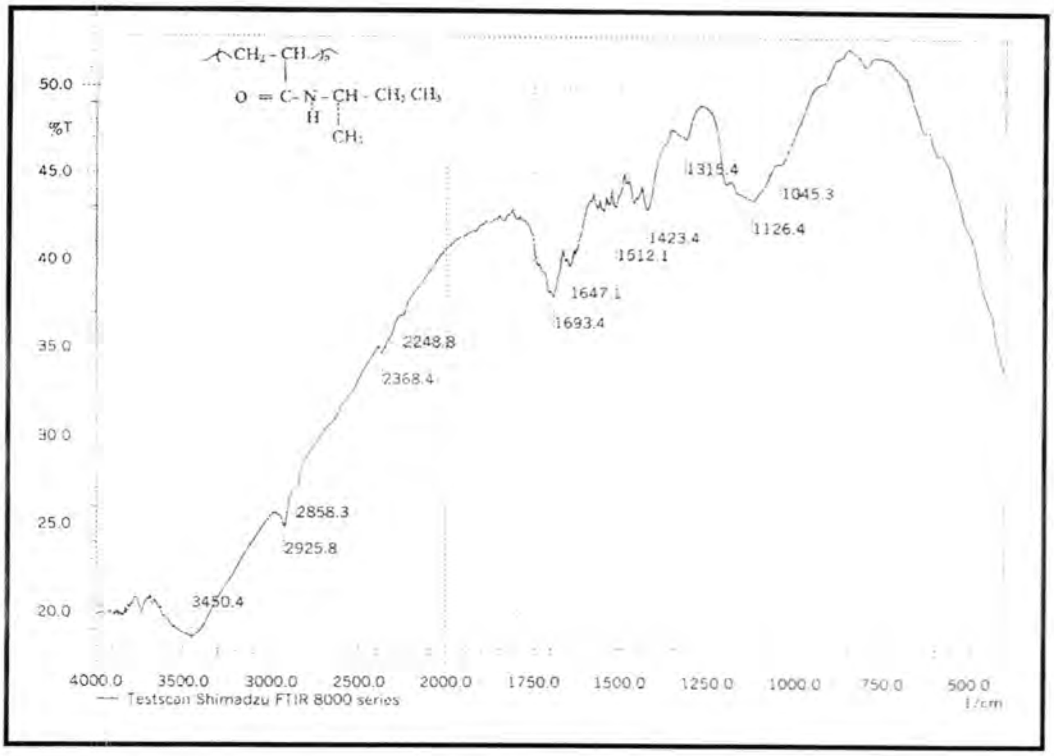


Fig. (3): spectrum of compund (2)

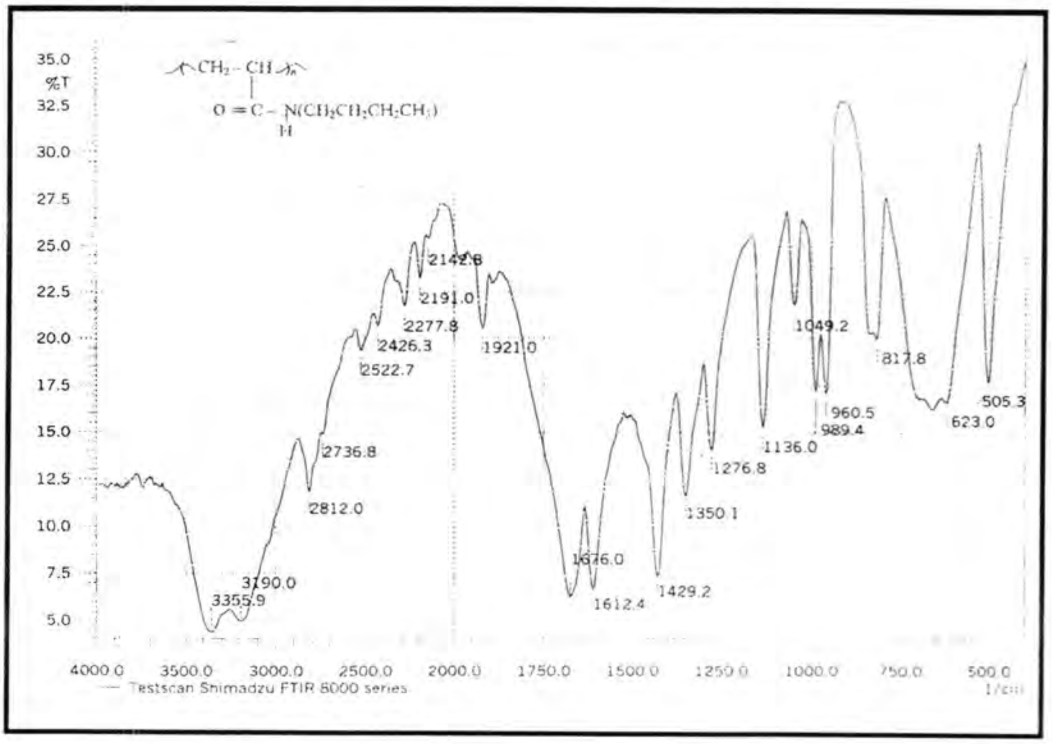


Fig. (4): spectrum of compund (3)

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Synthesis of New Pyrano[2, 3- C] Pyrazol- 6- ones

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تاريخ قبول البحث: 2006/6/25 تاريخ تقديم البحث: 2006/5/4

Abstract

Reaction of 5- chloro- 3, 4- dimethyl- 1- phenyl- 3- methyl- 1H, 6H pyrano [2, 3- C] pyrazol- 6- one (2) with acetylenic alcohols namely propargyl and 3- hexynyl alcohols gave the corresponding acetylenic ethers (3-4) which underwent Mannich reactions to give Mannich bases (5-8). On other hand, treatment of compound (2) with 2- mercaptoacetic acid gave the 5- (thioacetic acid)- 3, 4- dimethyl- 1- phenyl-3- methyl 1H, 6H pyrano [2, 3- C] pyrazol- 6- one (9), which upon refluxing with thionyl chloride yielded the corresponding 5- (thioacetyl chloride)- 3, 4- dimethyl- 1- phenyl- 3- methyl 1H, 6H pyrano [2, 3- C] pyrazol- 6- one (10). Heating of compound (10) with propyl, benzyl and ethyl alcohols in dry benzene for 7 hrs. affored the corresponding esters (11- 13) respectively.

The chemical structure of all synthesized compounds was confirmed on the basis of their some spectral data (IR, UV spectroscopy) and their elemental analysis.

الخلاصة

ان تفاعل 5- كلورو- 3، 4- ثنائي مثيل- 1- فنيل- 3- مثيل- 1H، H6 بـايرانو [2، 3- C] بايرازول- 6- أون (2) مع الكحولات الاستيلينية (بروباجيـل و 3- هكسـانايل) يعطي الايثرات الاستيلينية المقابلة (3-4) والتي تعاني تفاعل مانخ لينتج قواعد مـانخ (5-8). ومن جهة اخرى ان معاملة المركب (2) مع 2- مركبتو حامض الخليك يؤدي الـى تكـوين 5- (ثايو حامض الخليك)- 3، 4- ثنائي مثيل- 1- فنيل- 3- مثيل- 1H، H6 بـايرانو [2، 3- C] بايرازول- 6- أون (9) والتي عند تسخينها مع كلوريد الثايونيل يعطـي 5- (ثـايو كلوريد الاسيتيل)- 3، 4- ثنائي مثيل- 1- فنيل- 3- مثيل- 1H، H6 بـايرانو [2، 3- C] بايرازول- 6- أون (10). وان تسخين المركب (10) مع الكحولات التالية (بروبانول، بنزيـل الكحول والايثانول) في البنزين الجاف لمدة 7 ساعات تؤدي الى الاسترات المقابلـة (11-13) على التوالي.

تم اثبات التركيب الكيميائي للمركبات المحضرة بالاعتماد على بعض الخواص الطيفية (IR، UV) والتحليل الدقيق للعناصر (CHN).

Introduction

Condensed pyrazoles have been found to possess awide spectrum of considerable pharmacological, medical and biological activities such as analgesic, anti- inflammatory antimicrobial, vasodialators, hypotensive and hypoglycemic agents [1-4]. Also, the presence of basic Mannich side chains, acid, ester and acetylenic moieties are active as antibacterial insecticidal and antifungal [5-6].

This paper reports the synthesis of some new pyrano [2, 3- C] pyrazoles containing above moieties.

Experimental

Melting point were determined in open capillary tubes on a GALLENKAMP MELTING POINT APPARTUS and are uncorrected The IR spectra were recorded by KBr or film with SHIMADZU FTIR FOURIETRANSFORM INFRARED spectrophotometer- 8300. UV spectra were recorded with SHIMADZU UV- VISIBLE doublebeam scanning spectrophotometer 1650. Elemental analysis were done on a Carlo Erba Analyzer type 1106. starting chemical compounds were obtained from Fluka or Aldrich.

Synthesis of 3, 4- dimethyl- 1- phenyl- 1H, 6H pyrano [2, 3- C] pyrazol- 6- one (1)

This compound was prepared according to the method reported in the literature [7, 8].

Synthesis of 5- chloro- 3, 4- dimethyl- 1- phenyl- 1H, 6H pyrano [2, 3- C] pyrazol- 6- one (2)

To a solution of compound (1) (0.002 mole) in acetic acid a solution of sodium hypochloride (25 ml, 5%) was added drop wise (30 minutes). The mixture was poured onto crushed ice, the resulting oil collected and purified on a column of silica gel using chloroform as eluent.(Tables 1 and 4)

Synthesis of 5- alkynyloxy- 3, 4- dimethyl- 1- phenyl- 1H, 6H pyrano [2, 3- C] pyrazol- 6- ones (3-7)

General method:

To a mixture of compound (2) (0.005 mole) and appropriate acetylenic alcohol (0.005 mole) in dry benzene (25 ml), ten drops of pyridine was added. The mixture was refluxed for 4hrs. After cooling, water (15 ml) was added and the organic layer was separated and dried over $MgSO_4$. The filterate was evaporated under reduced pressure and the residue was collected and purified on a column of silica using chloroform as eluent.(Tables 1 and 4)

Synthesis of Mannich bases (5-8)

To a stirring solution of appropriate acetylenic alcohol (3, 4) (0.005 mole) in dry dioxane (15 ml) was added cuprous chloride (0.05 gm), and the mixture was heated for 10 minutes, then paraformaldehyde (0.005 mole) and the appropriate secondary amine (0.005 mole) was added. The mixture was heated at 90°C for 2 hrs. After cooling the mixture was filtered and poured onto ice water (80 ml). The precipitate was filtered and crystallized from suitable solvent. (Tables 2 and 4)

Synthesis of 5- (thioacetic acid)- 3, 4- dimethyl- 1- phenyl- 1H, 6H pyrano [2, 3- C] pyrazol- 6- one (9)

A stirred mixture of compound (2) (0.005 mole), mercaptoacetic acid (0.005 mole) and sodium carbonate (2 gm) in dimethylformamide (25 ml) was gently refluxed for 3 hrs. After cooling water (30 ml) was added and filtered. The filtrate was acidified with dilute HCl and the precipitate was collected and crystallized from appropriate solvent.(Tables 3 and 5)

Synthesis of 5-(thioacetyl chloride)-3,4- dimethyl- 1- phenyl- 1H, 6H pyrano [2, 3- C] pyrazol- 6- one (10)

A mixture of compound (9) (0.01 mole) and thionyl chloride (15 ml) was refluxed gently for 4hrs. Excess thionyl chloride was removed under vaccum to give brown oil of the acid chloride (10).(Tables 3 and 5)

Synthesis of 5- (thioacetic acid)- 3, 4- dimethyl- 1- phenyl- 1H, 6H pyrano [2, 3- C] pyrazol- 6- one alkylesters (11-13)

General method:

A mixture of acid chloride (10) (0.004 mole), appropriate alcohol (0.004 mole) in dry benzene (25 ml) was refluxed gently for 7hrs. The excess of the solvent was evaporated and the residue was purified on a column of silica using chloroform or benzene- chloroform mixture as eluent. (Tables 3 and 5)

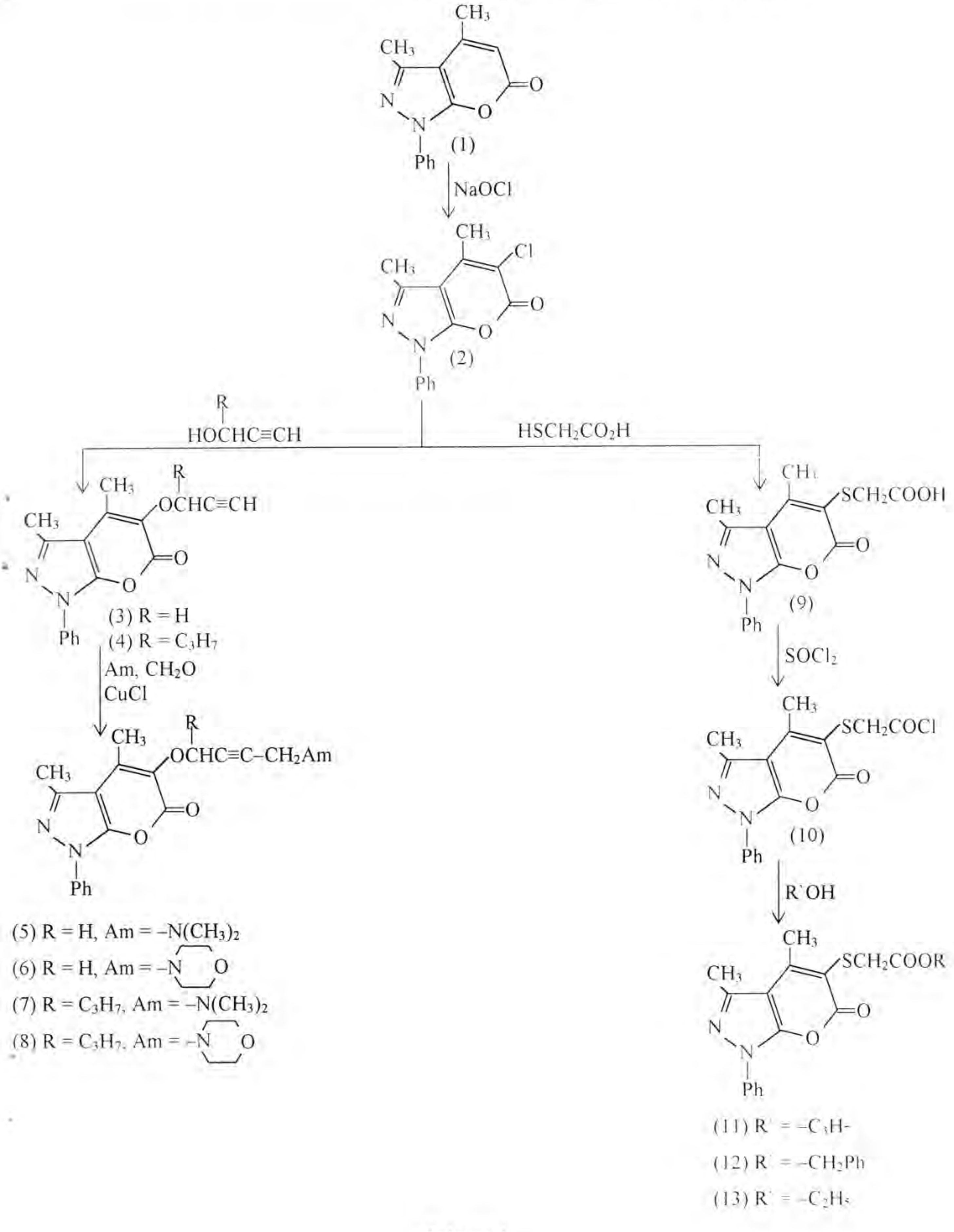
Results and Discussion

In the present work, 3, 4- dimethyl- 1- phenyl- 1H, 6H pyrano [2, 3- C] pyrazol- 6- one (1), which was previously prepared by us [7, 8] was used as the key intermediate for further synthesis. Thus compound(1) was treated with sodium hypochlorite in acetic acid, 5- chloro- 3, 4- dimethyl- 1- phenyl- 3- methyl- 1H, 6H pyrano [2, 3- C] pyrazol- 6- one (2) was obtained in 45% yield. The IR spectrum of it exhibited a C=O streching vibration near 1765 cm^{-1} and a new C–Cl absorption near 790 cm^{-1}.

Reaction of the compound (2) with acetylenic alcohols namely propargyl alcohol and 3- hexynyl alcohol in dry benzene afforded the corresponding acetylenic ethers (3-4) respectively, which displayed two bands at (3260-3300 cm^{-1}) and (2150- 2165 cm^{-1}) for the (≡CH) and (C≡C) stretching respectively, in addition to the bands at(1098-1105cm^{-1}) for C–O–C stretching. (Table 4). The formation of Mannich bases (5-8) were confirmed by the presence of a weak absorption near (2120-2140 cm^{-1}) and at (1250-1280 cm^{-1}) due to C–N stretching (Table 4).

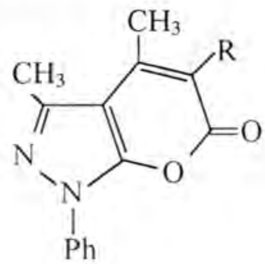
On the other hand, reaction of compound (2) with $HSCH_2CO_2H$ yields 5- (thioacetic acid) derivatives (9) which on treatment with thionyl chloride and alcohols namly, propyl, benzyl and ethyl alcohols afforded the corresponding 5- (thioaceticacid)- 3, 4- dimethyl- 1- phenyl- 1H, 6H pyrano [2, 3- C] pyrazol- 6- one alkylesters (11-13).

The structures of the derivatives (9-13) were proven on the basis of spectral data. The IR spectra of compounds (11-13) exhibited a C=O stretching vibrations near (1728-1740 cm^{-1}) and C–O–C absorption bands at (1150-1160 cm^{-1}).



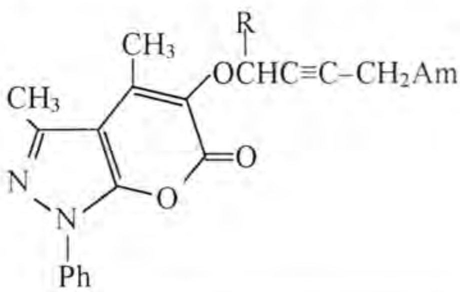
Scheme 1

Table (1) Some physical properties for compounds (1-4)



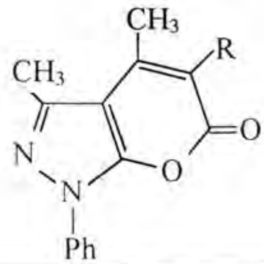
No. of Comp	R	Yield %	M.P. C°	Purification solvent	Elemental analysis Found% (Calc.%)		
					C	H	N
1	H	63	124	EtOH	70.00 (69.11)	5.00 (4.85)	11.66 (11.12)
2	Cl	45	Oily	$CHCl_3$	61.20 (61.05)	4.00 (3.56)	10.20 (10.14)
3	$-OCH_2C\equiv CH$	58	Oily	$CHCl_3$	69.38 (69.01)	4.76 (4.25)	9.52 (9.12)
4	$-OCHC\equiv CH$ C_3H_7	38	Oily	$CHCl_3$	71.42 (71.53)	5.95 (5.60)	8.33 (7.99)

Table (2) Some physical properties for compounds (5-8)



No. of Comp	R	Am	Yield %	M.P. C°	Purification solvent	Elemental analysis Found% (Calc.%)		
						C	H	N
5	H	$-N(CH_3)_2$	73	170-173	EtOH	68.37 (68.10)	5.98 (5.73)	11.96 (11.61)
6	H	–N O	55	197-200	Benzene	67.18 (66.86)	5.85 (5.62)	10.68 (10.22)
7	C_3H_7	$-N(CH_3)_2$	61	185-188	EtOH	70.23 (69.94)	6.87 (6.38)	10.68 (10.42)
8	C_3H_7	–N O	49	205-207	Benzene	71.59 (71.31)	6.92 (6.67)	10.02 (9.98)

Table (3) Some physical properties for compounds (9-13)



No. of Comp	R	Yield %	M.P. C°	Purification solvent	Elemental analysis Found% (Calc.%)		
					C	H	N
9	$-SCH_2CO_2H$	62	210-213	Benzene- EtOH	58.18 (57.89)	4.24 (3.98)	8.48 (8.19)
10	$-SCH_2COCl$	80	Oily	$CHCl_3$	–	–	–
11	$-SCH_2COOC_3H_7$	73	Oily	$CHCl_3$	–	–	–
12	$-SCH_2COOCH_2Ph$	78	Oily	Benzene-$CHCl_3$	65.71 (65.31)	4.76 (4.24)	6.66 (6.28)
13	$-SCH_2COOC_2H_5$	85	Oily	$CHCl_3$	–	–	–

Table (4) IR and UV spectral data for compounds (1-8)

No. of Comp	U.V λmax ($CHCl_3$)	Characteristic IR bands Cm^{-1}								
		C=O	C–H al.	C–H ar.	C–O	C=N	C–N	C≡C	C=C	Other
1	316 247 209	1750	2960 2890	3100	1120	1620	–	–	1560 1460	–
2	316 208	1765	2990 2885	3090	1105	1610	–	–	1550 1455	ν(C–Cl) (790)
3	315 248 209	1730	2925 2865	3059	1098	1595	–	2150	1545 1495	ν(≡CH) (3300)
4	245 209	1712	2925 2868	3070	1105	1595	–	2165	1548 1496	ν(≡CH) (3260)
5	269 209	1690	2985 2890	3030	1115	1598	1250	2120 (w)	1580 1475	–
6	304 265 209	1705	2910 2825	3060	1080	1620	1268	2135 (w)	1605 1520	–
7	367 286	1695	2970 2888	3090	1110	1630	1280	2125 (w)	1565 1489	–
8	346 273	1700	2927 2850	3030	1097	1620	1276	2140 (w)	1570 1490	–

Table (5) IR and UV spectral data for compounds (9-13)

No. of Comp	U.V λmax ($CHCl_3$)	Characteristic IR bands Cm^{-1}							
		C=O	C–H al.	C–H ar.	C–O	C=N	C–S	C=C	Other
9	354 244 213	1670	2923 2794	3066	1150	1598	755	1550 1494	ν(C=O) 1718 ν(OH) 3250-3280
10	310 230	1710	2965 2880	3100	1160	1610	770	1580 1500	ν(C=O) 1780 ν(C–Cl) (795)
11	320 264	1675	2985 2875	3090	1155	1590	762	1560 1490	ν(C=O) 1740 ester
12	346 214	1660	2905 2825	3080	1150	1630	765	1570 1495	ν(C=O) 1728 ester
13	367 235	1680	2990 2875	3090	1160	1620	760	1590 1485	ν(C=O) 1730 ester

al.= aliphatic, ar (arm)= aromatic

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Synthesis of Novel Cinnoline Derivatives

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Abstract:

Refluxing 5-methyl-4-chloro cinnoline with ethyl *p*-amino benzoate in boiling ethanol for 12 hrs lead to the formation of 5-methoxy-4(ethyl-*p*-amino benzoate) cinnoline [1] with good yield. On treatment of compound [1] with hydrazine (99%), hydrazide derivative [2] was obtained. This hydrazide was treated with phenyl isothiocyanate on refluxing in boiling ethanol for 4 hrs, gave thiosemicarbazide derivative [3], which have been treated with conc. KOH or H_2SO_4 afford thiadiazole [4] and triazole [5] respectively.

The synthesized compounds have been elucidated using some spectral data (IR and UV) and C.H.N. analyses.

We have been stimulated to test the antimicrobial activity of cinnoline derivatives on three species of pathogenic microorganisms (*Eschericha coli, Staphylococcus, Psendomonus*).

الملخص:

إن تسخين المركب 5-ميثوكسي-4-كلورو سينولين مع بارا امينو بنزوات الاثيل عند درجة (78 °C) لمدة 12 ساعة يؤدي إلى تكوين 5-ميثوكسي-4-(اثيل بارا امينو بنزوات) السينولين [1] وبمنتوج جيد وعند معاملة المركب [1] مع الهيدرازين المائي (99%) أعطى مشتق الهيدرازيد [2] وهذا يعامل مع فنيل آيزوثايوسيانات أدى إلى تكوين مشتق الثايوسيميكاربازايد [3] وعند معاملة الأخير مع قاعدة KOH وحامض H_2SO_4 أعطى مركبات الثايادايازول [4] والترايازول [5] على التوالي.

شخصت المركبات المحضرة باستخدام بعض الطرق الطيفية (IR, UV) والتحليل الدقيق للعناصر (C.H.N.).

ونظرا لأهميتها البايولوجية فقد تم في هذا البحث دراسة الفعالية البايولوجية لمشتقات السينولين المحضرة على ثلاثة أنواع من البكتريا هي *Eschericha coli, Staphylococcus, Psendomonus*.

Introduction:

Many cinnoline derivatives have been found to be considerable interest because of their bilogical activity (1-3). Some of these derivatives are useful antihypertensive agent (4), analgestic (5), antipyretic (6) and for their CNC depressant actions (7) and antitubercular activity (8).

Furthermore, several derivatives have found to possess antimicrobial (7), antifungal (8) and anticancer activities (9-13).

The present work involves synthesis of new cinnoline derivatives, which on step-wise reaction with various reagents give various derivatives of cinnoline.

Experimental:

Uncorrected melting points were determined on Gallen-kamp melting point apparatus. IR spectra were recorded on Pye Unicam SP3-100 spectrophotometer as KBr discs. The UV spectra were performed on a Hitachi / UV-2000 spectrophotometer. Elemental analysis of the compounds were carried out on C.H.N. analyzer type 1106-Carlo Erba in AL-Nahrain University.

❖ **Synthesis of 5-methoxy-4(ethyl-*p*-amino benzoate) cinnoline [1]:**

A mixture of the 4-chloro cinnoline (0.01 mole) and ethyl (*p*-amino benzoate) (0.01 mole) in ethanol (100 ml) was heated under reflux for (12 hrs). The solvent was removed in vacuum and the crude product was collected and recrystallized from ethanol to give product [1], (table 1).

❖ **Synthesis of 5-methoxy-4(*p*-anilino hydrazido) cinnoline [2]:**

❖ A mixture of compound [1] (0.01 mole) in ethanol (50 ml) and hydrazine hydrate (99%, 0.01 mole) was heated under reflux for (5 hrs). The product separated out, filtered off under vacuum and recrystallized from ethanol, (table 1).

❖ **Synthesis of 5-methoxy-4(*p*-anilino thiosemicarbazido) cinnoline [3]:**

❖ A mixture of compound [2] (0.01 mole) and phenyl isothiocyanate (0.01 mole) in absolute ethanol (100 ml) was heated under reflux for (4 hrs). The contents were poured into crushed-ice (100 gm), filtered and the product was recrystallized from ethanol to give [3], (table 1).

❖ **Synthesis of 5-methoxy-4[*p*-anilino-2'-(phenyl amino-1',3',4'-thiadiazolo)] cinnoline [4]:**

Compound [3] (0.005 mole) was dissolved in cold conc. H_2SO_4 (10 ml) and the contents were kept at room temperature for (2 hrs), stirred then poured into crushed-ice and the separated solid filtered off, washed with water and recrystallized from ethanol to give [4], (table 1).

Synthesis of 5-methoxy-4[*p*-anilino-(4'-aryl-3'-mercapto-1,2,4-triazolo)] cinnoline [5]:

Compound [3] (0.005 mole) was refluxed in KOH (20 ml, 2 mole/L) for (5 hrs), cooled, poured into excess of water, stirred and filtered. On acidification of the filtrate with HCl (2 N), the product was washed with cold water, dried and recrystallized from ethanol, (table 1).

❖ **Biological activity:**

The antimicrobial agents (1,2,3,4,5) were synthesized according to the reported procedures:

1. 5-methoxy-4-(ethyl-p-amino benzoate) cinnoline.
2. 5-methoxy-4-(p-anilino hydrazido) cinnoline.
3. 5-methoxy-4-(p-anilino thiosemicarbazido) cinnoline.

4. 5-methoxy-4-(p-anilino-2'-(phenyl amino-1',3',4'-thiadiazolo) cinnoline.
5. 5-methoxy-4-(p-anilino-(4'-aryl-3'-mercapto-1,2,4-triazolo) cinnoline.

Preliminary screening of the tested compounds (1-5) agents different strains of *Eschericha coli, Staphylococcus,Psendomonus* bacteria.

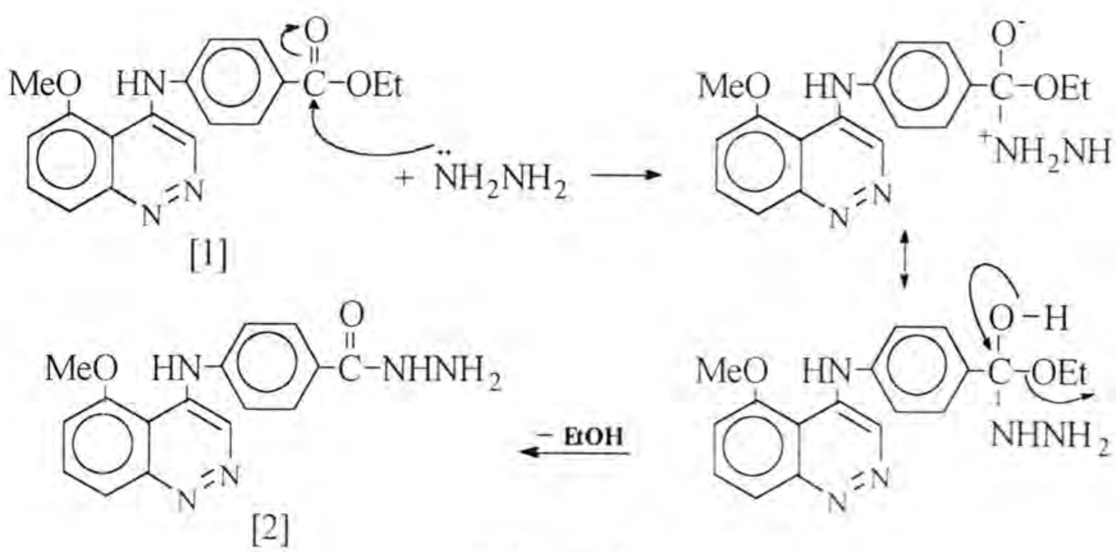
Determination of the minimum inhibitory concentration of the tested compounds against bacterias using the agar cup diffusion methods (14-17). The results are listed in table (3).

Results and Discussion:

The intermediate 5-methoxy-4-(ethyl-*p*-amino benzoate) cinnoline [1] was obtained in a good yield (60%)by refluxing of 5-methoxy-4-chloro cinnoline with ethyl(p-amino benzoate) in ethanol for (12 hrs). The reaction is followed by disappearance of C-Cl absorption band at 780 cm^{-1} and appearance of NH absorption band at 3200 cm^{-1}, for IR, UV, and C.H.N. analysis see tables (1 and 2).

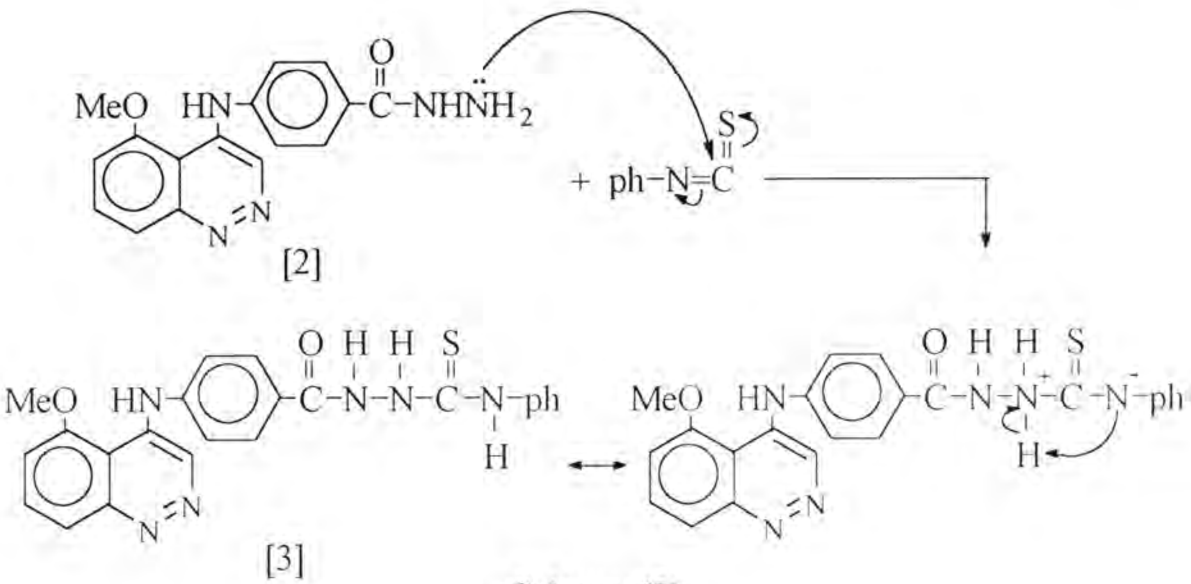
On treatment of compound [1] with hydrazine in boiling ethanol for (5 hrs) gave hydrazide derivative [2], the structure was elucidated by the appearance of new bands at 3200 cm^{-1}, 3300 cm^{-1} and 1665 cm^{-1}, which alternatively belonged to νNH and νC=O respectively and C.H.N. analysis.

The reaction may be following the mechanism shown in scheme (1) below:



Scheme (1)

The thiosemicarbazide derivative [3] was obtained by heating under reflux a mixture of the hydrazido derivative [2] and an equivalent quantity of phenylisothiocyanate in absolute ethanol for (4 hrs), the mechanism is shown in scheme (2) below:



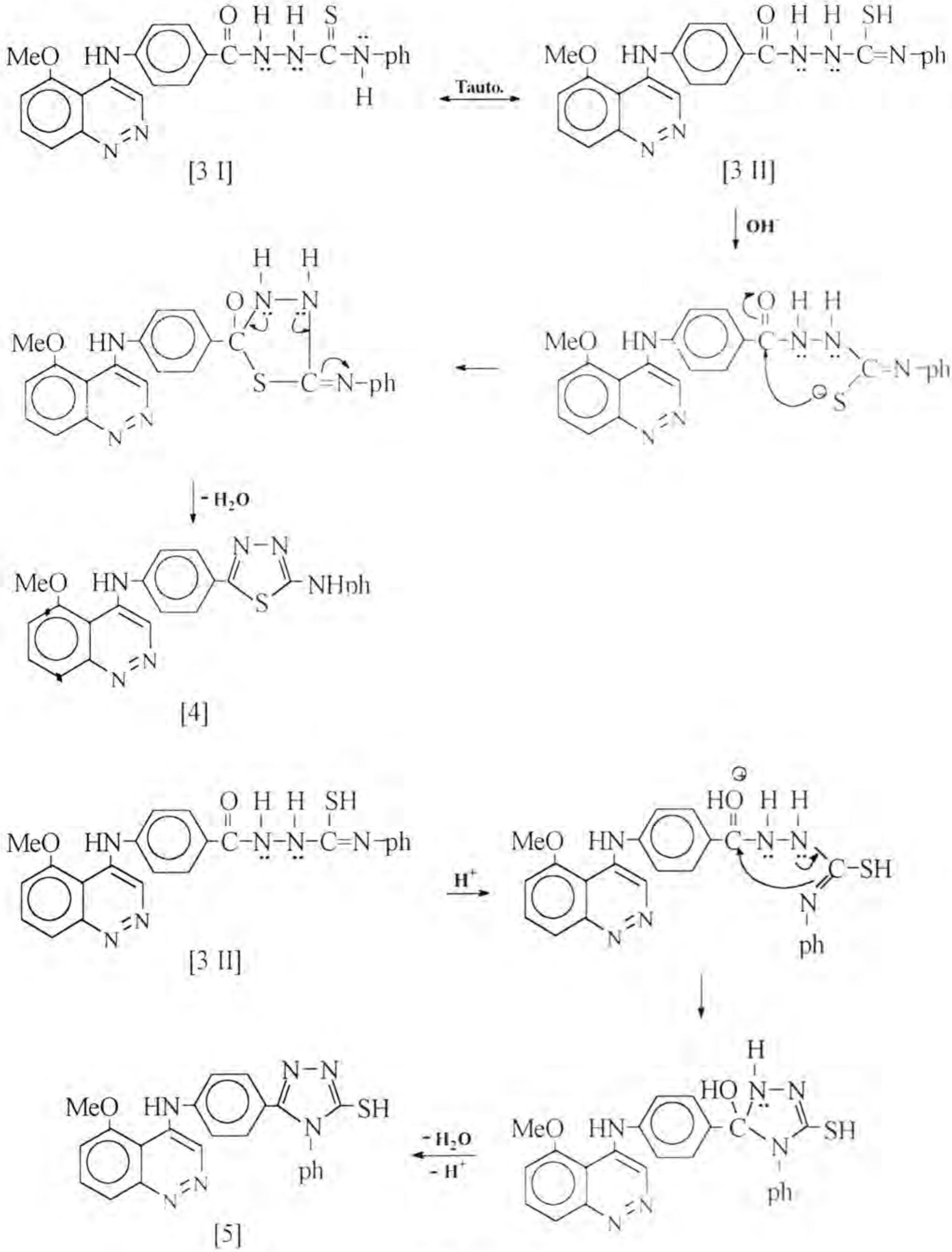
Scheme (2)

The structure of product [3] is confirmed by IR, UV (table 2) and C.H.N. (table 1), the IR spectrum of it showed a characteristic absorption band at 3115, 1220 and 2520 cm^{-1}, which attributed for νNH, νC=S and νSH groups respectively.

Compound [3] may be cyclized, on its treatment with base or acid. So, on reaction of compound [3] with KOH base led to the formation of thiadiazole [4].

The informed compound [4] was characterized using some spectral (IR, UV) and (C.H.N.) analysis. IR spectrum showed disappearance of νC=O band at 1660 cm^{-1}. And appearing of new bands at 1550 cm^{-1} and 1435 cm^{-1} for νN=C and νS-C-S respectively.

While the reaction of compound [3] with acid led to the formation of triazole [5]. Compound [5] show characteristic bands at 2350, 1530, 610 and 1575 cm^{-1} belonged to νSH, νN=C, νC-S and νNHph groups respectively. Furthermore, a band at 1660 was disappeared that attributed to νC=O. The mechanism is shown in scheme (3) below:



Scheme (3)

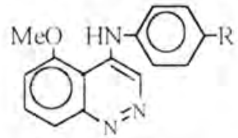
The antibacterial activities of the investigated series of 5-methoxy-(4-ethyl-p-amino benzoate) cinnoline against a representative species of bacteria namely *Eschericha coli, Staphylococcus,Psendomonus* are summarized in table (3).

These species of bacteria were chosen since they are known as pathogens for human beings. From the data obtained, it is evident that some of these compounds exhibited a good activity against the tested species of bacteria with the concentration used (0.1 ml / 0.02 gm), but specially significant are the compounds (4 and 5) which showed the highest activity among the compounds. As a conclusion, the preliminary *in vitro* studies of these compounds are promising since they exhibited activity against all species of bacteria tested without exception.

The main antibacterial activity of these compounds, may be attributed to the interaction between the functional group (i.e. NH, N+N, SH... etc.) in these compounds and binding sites on the bacterial cell envelop.

As a result of this interaction, the compounds may be metabolized to toxic products on degradation and thus affects the bacterial growth.

Table (1): Physical properties and C.H.N. analysis of the prepared compounds



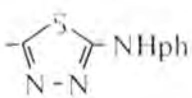
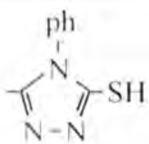
Comp No.	-R	M.p. °C	Colour	Yield %	Purification solvent	M. formula (M. Wt.)	C%		H%		N%	
							Cal.	Fou.	Cal.	Fou.	Cal.	Fou.
1	$-CO_2Et$	160	Pale yellow	60	Ethanol	$C_{18}H_{17}N_3O_3$ (323)	66.87	67.90	5.26	5.00	13.00	12.82
2	$-CONHNH_2$	206	Yellow	70	Ethanol	$C_{16}H_{15}N_5O_2$ (309)	62.13	61.98	4.85	4.81	22.65	21.86
3	-CONHNHCSNHph	219	Pale yellow	75	Ethanol	$C_{18}H_{20}N_6O_2S$ (444)	62.16	62.02	4.50	4.31	18.91	18.02
4		210	Yellow	61	Methanol	$C_{23}H_{18}N_6OS$ (426)	64.78	64.23	4.22	4.11	19.71	19.52
5		190	Yellow	60	Ethanol	$C_{23}H_{18}N_6OS$ (426)	64.78	64.21	4.22	4.00	19.71	19.21

Table (2): The characteristic IR and UV-Visible spectra of the prepared compounds

Comp. No.	ν_{N-H} cm^{-1}	$\nu_{C=O}$ cm^{-1}	ν_{C-O} cm^{-1}	ν_{C-H} cm^{-1}	$\nu_{N=N}$ cm^{-1}	ν_{Others} cm^{-1}	λmax EtOH (95%)×10^{-3}
1	3200	1730	1260	3000 2990 2880	1580	-	260, 280, 345, 390, 480, 230
2	3400-3100	1665	1230	3050 2990 2880	1600	ν_{C-N} (1510)	280, 360, 380, 450, 500, 232
3	3150	1660	1245	3000 2950 2860	1590	ν_{C-S} (1220) ν_{S-H} (2520)	260, 360, 370, 420, 510, 243
4	3290	-	1240	3000 2970 2855	1600	ν_{C-N} (1550) ν_{C-S-C} (1435) ν_{N-N} (1050)	255, 300, 390, 410, 490
5	3180	-	1250	3055 2960 2855	1610	ν_{S-H} (2530) ν_{C-N} (1530) ν_{C-S} (610) ν_{N-N} (1050) ν_{N-ph} (1575)	260, 280, 310, 350, 400

Table (3): Antibacterial activity of the prepared compounds on the bacteria

Compound No.	*Type of bacteria*		
	E. Coli	*Staph. aurous*	*Proteus mirabbilis*
1	+	+	+
2	++	+	++
3	+++	+++	+++
4	++++	++++	++++
5	++++	++++	++++

(0-3) mm = -

(6-9) mm = ±

(10-14) mm = +

(15-18) mm = ++

(19-21) mm = +++

(22-28) mm = ++++

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Multipol-Mixing Ratios of γ- Rays from the Heavy Ion of $^{106}Ag_{59}$ Levels Populated in the $^{96}Zr_{56}(^{14}N_7,4n)^{106}Ag_{59}$ Reaction Using Constant Statistical Tensor(CST) Method.

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Abstract

In the present work, the constant statistical tensor (CST) method has been successfully used to calculate the delta mixing ratios of gamma transitions from excited levels $^{106}Ag_{59}$ from the nuclear reaction $^{96}Zr_{56}(^{14}N_7,4n)^{106}Ag_{59}$.

The obtained results confirm the validity of this method in calculating the delta values and it capability in predicting any inaccuracies in the experimental data.

The comparisons of our calculations with the experimental data are in good agreement.

الخلاصة:

استخدمت في البحث الحالي بنجاح طريقة التتسور الإحصائي الثابت لحساب نسب الخلط لانتقالات كاما من مستويات متهيجة $^{106}Ag_{59}$ الناتج من التفاعل النووي $^{96}Zr_{56}(^{14}N_7,4n)^{106}Ag_{59}$.

بينت النتائج صحة هذه الطريقة وقابليتها على تحديد النتائج العملية الغير صحيحة . وقد كانت النتائج التي حصلنا عليها بتوافق مع القيم العملية.

Introduction:

The angular distribution measurements necessary for determining the character of the radiated multipole. This accurate knowledge of multipole character is very important in evaluating nuclear models and deducing the lifetimes. Thus angular distribution measurements have an extremely important role in nuclear spectroscopy [1].

The delta mixing ratios for gamma transitions can be calculated by many methods. One of these methods is (CST) method, it has been successfully applied by Youhana H. M. [2,3] was used this method to calculate δ-values for gamma transition in Zr (A = 90, 92, and 94) and Nd (A = 150) excited in the reactions of $(n, n'\gamma)$. Al-Shibany H.H[4] has used the $^{18}_{8}O_{10}(p,\gamma)^{19}_{9}F_{10}$,

$^{58}_{28}Ni_{30}(^{6}Li, pn\gamma)^{62}_{30}Zn_{32}$, $^{66}_{30}Zn_{36}(\alpha, n\gamma)^{69}_{32}Ge_{37}$ and $^{146}_{60}Nd_{86}(n, n'\gamma)^{146}_{60}Nd_{86}$ and $^{170}_{70}Yb_{100}(^{16}O,3n\gamma)^{183}_{78}Pt_{105}$ reactions, to calculate the δ-mixing ratios of γ-transitions from low and high spin states populated in $^{19}F_{10}$, $^{62}Zn_{32}$, $^{69}Ge_{37}$, $^{146}Nd_{86}$ and $^{183}Pt_{105}$ nuclei using CST method. Ibrahim K.S. et al. [5] were calculated the delta mixing ratios of gamma transitions from levels of nuclei populated in the $^{168}_{68}Er(n, n'\gamma)^{168}_{68}Er$ by using constant statistical tensor (CST) method.

Angular Distribution:

The angular distribution is defined as the distribution in angle, relative to an experimentally specified direction, of the intensity of photons or particles usually resulting from a nuclear reaction [6].

The total radiation from a radioactive sample is isotropic because, the nuclei are randomly oriented in space. An anisotropic radiation pattern can be observed only from an ensemble of nuclei that are not randomly oriented

We can arriving at such an ensemble consists in placing the radioactive sample at very low temperature in a strong magnetic field gradient and then measuring the angular distribution of the emitted radiation with respect to the direction of the applied field [1,6] or the subject of the present survey, consists in picking out only those nuclei whose spins happen to lie in a preferred direction. This case can be realized if the nuclei decay through successive emission of two radiation R1 and R2. The observation of R1 in a fixed direction k1 selects an ensemble of nuclei that has a non isotropic distribution of spin orientations [6]. However, nuclear orientation may also be achieved by nuclear reactions in which beams of polarized particles are produced or by atomic beam experiments [7].

In nuclear physics, orientation of atomic nuclei derives its main interest from the study of angular distributions (and polarizations) of the emitted radiation in the spontaneous decay of radioactive nuclei [6].

The subject of angular distribution can be divided into two parts viz, directional distribution and polarization distribution, depending on whether or not one ignores polarization effects [6, 8].

Selection Rules of Gamma Transitions:

The γ-transition for an initial state of spin and parity $J_i^{\pi_i}$ to a final state of spin and parity $J_f^{\pi_f}$ is possible if [9]:

$$\left|J_i - J_f\right| \le L \le \left(J_i + J_f\right) \qquad \text{...... (1)}$$

$$L \ne 0 \qquad \text{...... (2)}$$

Where L is the angular momentum which gives the multipolarity of γ-transition.

The parity change of electromagnetic radiation, EL, and ML, is as follows [9]:

$$\pi_i \,.\, \pi_f = (-1)^L \longrightarrow \text{Electric Transition} \qquad \text{...... (3)}$$

and

$$\pi_i \,.\, \pi_f = (-1)^{L+1} \longrightarrow \text{Magnetic Transition} \qquad \text{...... (4)}$$

When the parities of initial and final states are equal, the M1, E2, M3, E4 ...etc will conserved if not, then E1, M2, E3, M4 ... etc are possible

The Mechanisam of the $^{96}Zr_{56}(^{14}N_7,4n)^{106}Ag_{59}$ Reaction:

In the given work, we are calculating the mixing ratios, δ, for gamma ray transitions in the following reaction:

$$^{96}Zr_{56} + {}^{14}N_7 \longrightarrow [^{110}Ag_{63}]^* \longrightarrow [^{109}Ag_{62}]^* +_0^1n \longrightarrow [^{108}Ag_{61}]^* +_0^1n +_0^1n$$
$$[^{107}Ag_{60}]^* +_0^1n +2_0^1n \longrightarrow [^{106}Ag_{59}] +_0^1n +3_0^1n$$

This reaction is one of many types of heavy ions reactions and depending upon the velocity of the incident ions and the nature of the target, the compound nucleus formed may disintegrate by the loss of one or more

neutrons as shown in this reaction [10]. the decay scheme of this isotope with the spin sequence for each transition [fig.(1)].

Theorey:

1-Constant Statistical Tensor (CST) Method

This method depends on the fact that in a certain nucleus, the magnetic substates population parameters, $P(m_i)$, of levels with the same spin value neither depend upon the energy of the level nor upon its parity [11].

The statistical tensor coefficients $\rho_k(J_i, m_i)$, are also constant for the same J_i values[12].

Then according to the following equation

$$\rho_k(J_i) = \sum_{\substack{m_i=0 \\ \text{or } m_i=1/2}}^{J_i} \rho_k(J_i, m_i) P(m_i) \qquad \text{...... (5)}$$

the statistical tensor $\rho_k(J_i)$, would also be constant for levels with the same J_i values. Taking this fact into consideration, the experimental value of the angular distribution coefficients, a_2 obtained for certain and well known γ-transitions such as $|J_i - J_f| = 2$ with $\pi_i \cdot \pi_f = +1$ or $|J_i - J_f| = 0$ or 1 with $\pi_i \cdot \pi_f = -1$, can be used to calculate the statistical tensors $\rho_2(J_i)$ for initial levels and such transition[11] using this eq.

$$a_2(J_i - J_{f_2}) = \rho_2(Ji)\frac{F_2(J_{f_2}L_1L_1J_i) + 2\delta F_2(J_{f_2}L_1L_2J_i) + \delta^2 F_2(J_{f_2}L_2L_2J_i)}{1+\delta^2} \quad \text{......} (6)$$

Putting $\delta = 0$ since such transitions may be considered to be pure E2 or pure E1 transitions.

The $\rho_2(J_i)$ values thus obtained may then be used to calculate the δ-values for other transitions such as $(1^+ - 2^+)$, $(2^+ - 2^+)$, $(3^+ - 2^+)$, $(3^+ - 3^+)$, $(3^+ - 4^+)$, $(4^+ - 4^+)$... using equation (7).

2-Multipole Mixing Ratio

The multipole mixing ratio for the transition from an initial (J_i) to a final state (J_f) with angular momentum (L) as the ratio of the reduced matrix elements [6]

$$\delta(\gamma) \equiv \frac{\langle J_f \| L'\pi' \| J_i \rangle}{\langle J_f \| L\pi \| J_i \rangle} \quad \text{......} (7)$$

The ratio of the total intensity of the L'-pole to that of the L-pole is then equal to δ^2. The reduced matrix elements for γ-emission can always be chosen to be real thus the mixing ratio δ is real.

For a given intensity ratio δ^2, the mixing ratio δ can have either a positive sign or a negative sign, depending on the relative phase of the reduced matrix elements. However, the sign depends on the definition of the reduced matrix elements. This fact must be kept in mind when one compares the sign of δ as determined from a distribution experiment with that calculated on the basis of certain method [6,13,14].

Only two types of mixed γ-transitions have so far been observed experimentally, the rather frequent M1+E2 and the rare E1+M2 transition.

3-Results and Discussion

Table-1 shows that the most of δ-values, which are calculated by this method are in very good agreement with those of Ref. [15].The weighted average of $\rho_2(J_i)$ values are also presented in this table.

It should be noticed that, the sign of δ-values in the present work have been changed as long as the R_k-coefficients are related with the F_k-coefficients, as show in the following relation ship [16]:

$R_k(J_2\ L_1L_2J_1) = (-1)^{L1-L2+K}\ F_k(J_2\ L_1L_2J_1)$ ..... (8)

This indicates that for even values of k,

$R_k(J_2\ L_1L_2J_1) = F_k(J_2\ L_1L_2J_1)$ for $L_2 = L_1+1$ (9)

The discrepancies occur in the following cases which comparied with ref.[14]

1- The present values of δ-value for the transition gamma of 496.69(7) KeV (7^--7^+) is slightly different than the experiment data, because the measurements of a_2-coefficient is not accurate, due to some circumstance in the measurements.

2-The delta mixing ratio for gamma transition of 739.7(1)KeV(7^--6^+) is rather different but is still correct due to overlapping within the error of the present value and experimental data.

3- The δ-value of gamma transition of 169.75(5)KeV (9^--8^-) is not consistent with experimental data because the experimental a_2-coefficient is not accurate, due to statistics in the measurements, which depend on the angles of measurements mainly.

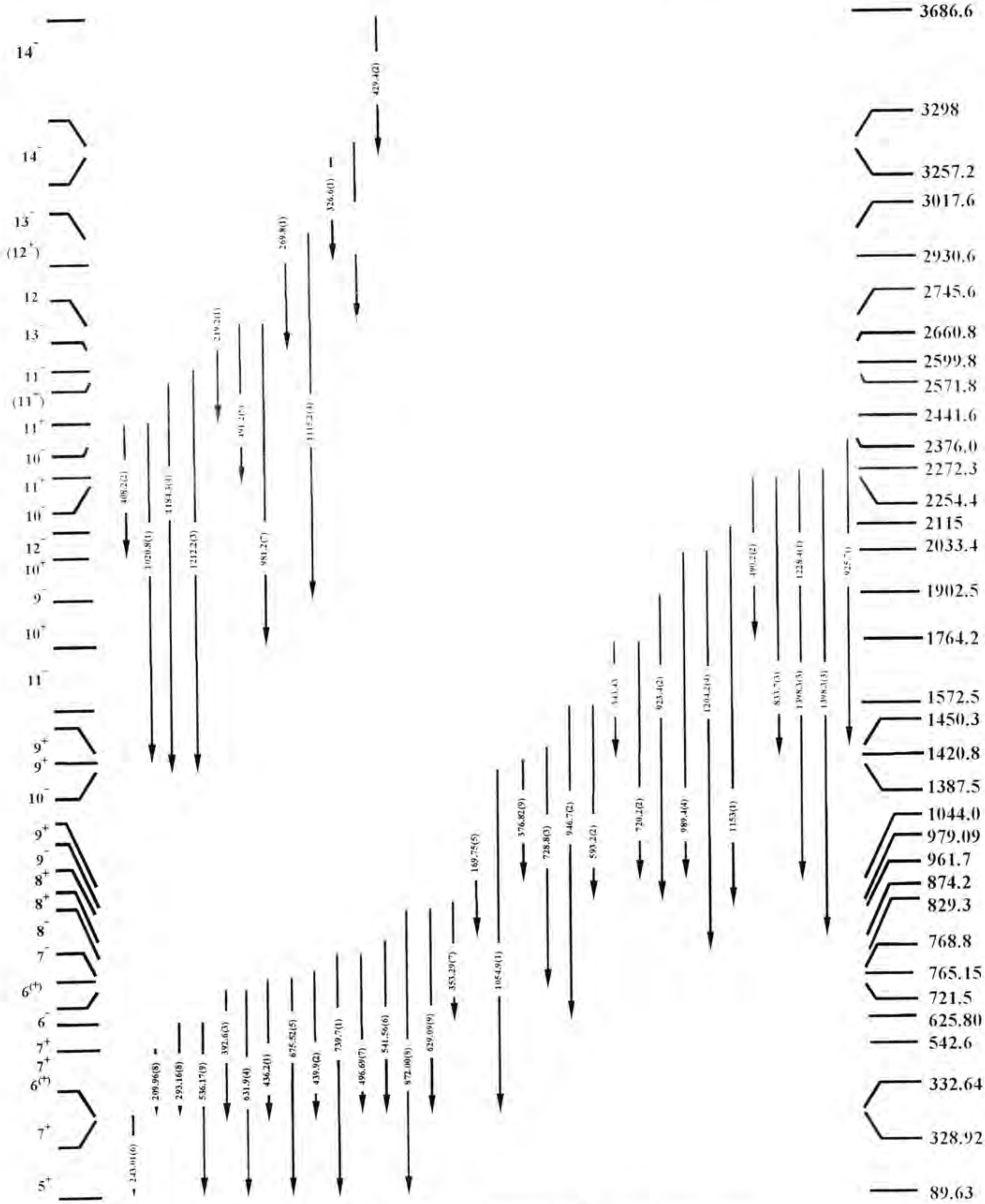

Fig.(1):Decay scheme of $^{106}_{47}Ag_{59}$ isotope.

Table-1: Multiple Mixing ratios of gamma transitions of $^{106}Ag_{59}$ calculated by using CST method

E γ (KeV)	E level (KeV)	$J_i^{\pi} - J_f^{\pi}$	$a_2(\Delta a_2)$	$\rho_2(J_1)$	W.a.	δ	
						Ref.[15]	CST(P.W.)
209.96(8)	542.6	6^+-7^+	-0.320(172)	-1.8030(9702)	-0.7572(224)	0.2(2)	0.16(15,-16)
436.2(1)	765.15	6^--5^+	-0.410(112)	-1.4536(3971)		—	-0.14(10)
439.9	768.5	$6^{(+)}-5^+$	-0.369(119)	-1.3083(4219)		0.15(5)	-0.12(10)
675.52(5)	765.15	6^--6^+	0.334(10)	-0.7536(225)		0.00(13)	0.00(3)
243.01(6)	332.64	7^+-6^+	-0.415(12)	-1.5101(478)	-0.8031(211)	-0.15(2)	-0.14(1)
293.16(8)	625.80	7^+-7^+	0.277(12)	-0.6235(270)		-0.17(8)	-0.18(3)

1/6 to be continued

E γ (KeV)	E level (KeV)	$J_i^\pi - J_f^\pi$	$a_2(\Delta a_2)$	$\rho_2(J_i)$	W.a.	δ	
						Ref.[15]	CST(P.W.)
392.6(3)	721.5	7^+-5^+	0.201(40)	-0.5147(1024)	-0.8031(211)	—	-0.12(4)
496.69(7)	829.3	7^--7^+	0.454(27)	-1.0220(607)		0.8(2)	040(32)
536.17(9)	625.80	7^+-6^+	0.372(39)	1.3608(1426)		0.5	0.5(5)
631.9(4)	721.5	7^+-6^+	1.099(392)	4.0203(14340)		—	1.07(105)
739.7(1)	829.3	7^--6^+	-0.468(70)	-1.7120(2560)		-0.06(14)	-0.20(5)
353.29	979.09	8^+-7^+	-0.129(19)	-0.4832(711)	-0.6683(460)	0.05(2)	0.04(2)
541.56(6)	874.2	8^--7^+	-0.217(17)	-0.8129(636)		0.00(2)	-0.03(2)
629.09(9)	961.7	8^+-7^+	0.172(36)	0.6443(1348)*		6(2)	5.4(8,-12)

2/6 to be continued

E γ (KeV)	E level (KeV)	J_i^{π} - J_f^{π}	$a_2(\Delta a_2)$	$\rho_2(J_i)$	W.a.	δ	
						Ref.[15]	CST(P.W.)
872.00(8)	961.7	8^+-6^+	0.267(73)	-0.7002(1914)	-0.6683(460)	—	0.02(6)
728.8(3)	1450.3	9^+-7^+	0.247(45)	-0.6600(1202)		—	0.03(6)
946.7(2)	1572.5	9^+-7^+	0.288(149)	-0.7695(3981)		—	0.10(4)
989.4(4)	2033.4	9^--9^-	0.233(83)	-0.5231(1863)		—	-0.15(?,-35)
1054.9(4)	1387.5	9^+-7^+	0.204(35)	-0.5451(935)	-0.6094(397)	0.00	-0.03(1,-5)
1204.2(4)	2033.4	9^--7^-	0.107(261)	-0.2859(6974)		—	-0.20(4)
169.75(5)	1044.0	9^--8^-	-0.162(13)	-0.6184(496)		0.04(2)	-0.002(10)
593.2(2)	1572.5	9^+-8^+	-0.286(119)	-1.0917(4542)		0.15(10)	-0.12(11,-12)

3/6 to be continued

E γ (KeV)	E level (KeV)	$J_i^\pi - J_f^\pi$	$a_2(\Delta a_2)$	$\rho_2(J_i)$	W.a.	δ	
						Ref.[15]	CST(P.W.)
376.82(9)	1420.8	$10^- - 9^-$	-0.168(8)	-0.6518(310)	-0.6507(300)	0.04(2)	0.00(0)
408.2(2)	2441.6	$10^- - 9^-$	-0.119(65)	-0.4611(2520)		0.07(5)	0.04(3,-5)
923.4(2)	1902.5	$10^+ - 8^+$	0.179(95)	-0.4856(2577)		——	-0.08(3,-13)
1020.8(1)	2441.6	$10^- - 10^-$	0.235(186)	-0.5272(4173)		——	-0.20(?,-50) 0.63(?,-80
1153(1)	2115	$10^+ - 8^+$	0.536(100)	-1.4541(2712)		——	0.56(27,?)`
1228.4(1)	2272.3	$10^- - 9^-$	-0.799(172)	-3.0965(6665)		-0.5(3)	0.92(1)
1398.3(3)	2272.3	$10^- - 8^-$	-0.044(96)	0.1193(2604)		——	-0.40(15,-2)
219.2(1)	2660.8	$11^- - 10^-$	-0.128(22)	-0.5023(863)		0.06(2)	0.05(2)

4/6 to be continued

E γ (KeV)	E level (KeV)	J_i^π - J_f^π	$a_2(\Delta a_2)$	$\rho_2(J_i)$	W.a.	δ	
						Ref.[15]	CST(P.W.)
343.43	1764.2	11^--10^-	-0.236(11)	-0.9261(431)		0.00(2)	-0.02(1)
720.2(2)	1764.2	11^--9^-	0.218(26)	-0.5988(714)		—	-0.07(3)
925.7(1)	2376.0	11^+-9^+	0.302(195)	-0.8296(5356)	-0.7865(334)	—	0.02(17)
1184.3(4)	2571.8	11^+-9^+	0.211(92)	-0.5796(2527)		—	-0.08(6,-10)
1212.2(3)	2599.8	(11^+)-9^+	0.593(193)	-1.6289(5301)		—	0.42(30,?)
269.8(1)	2930.6	12^--11^-	-0.058(30)	-0.2300(1189)		0.11(2)	0.10(3)
490.2(2)	2254.4	12^--11^-	-0.249(40)	-0.9874(1586)	-0.4948(689)	—	-0.14(6)
833.7(3)	2272.3	12^--10^-	0.281(46)	-0.7800(1276)		—	0.20(11,-14)

5/6 to be continued

E γ (KeV)	E level (KeV)	$J_i^\pi - J_f^\pi$	$a_2(\Delta a_2)$	$\rho_2(J_i)$	W.a.	δ	
						Ref.[15]	CST(P.W.)
1115.1(4)	3017.6	(12^+)-10^+	0.007(58)	-0.0194(1610)	-0.4948(689)	—	-0.30(10,-12)
326.6(1)	3257.2	13^--12^-	-0.013(34)	-0.0520(1360)	-0.0762(1347)	0.08(5)	0.04(24)
491.2(5)	2745.6	13^--12^-	0.083(124)	0.3321(4961)		—	1.12(23,?)
981.2(7)	2745.6	13^--11^-	0.487(354)	-1.3640(9915)		—	0.96(?,-3)
429.4(2)	3686.6	14^--13^-	-0.020(57)	-0.0806(2298)	0.3444(1829)	0.08(6)	0.17(9,-10)
552.2(3)	3298	$(14)^-$-13^-	0.268(75)	1.0806(3024)		0.34(8)	-0.34(30,?)

6/6

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Generalization Results in Redundancy

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ABSTRACT

This paper is concerned with the comparison between the two ways of providing redundant units for a system:

(1) Component redundancy.

(2) System redundancy.

The comparison between these ways is carried out by comparing the random variable representing the lifetime of the system resulted from applying component redundancy with the random variable representing the lifetime of that resulted from applying system redundancy using likelihood ratio ordering.

Keywords: Likelihood ratio ordering; Series system; Reliability function; Exponential distribution; Active redundancy; Cold-standby redundancy.

الخلاصة

يهتم هذا البحث بالمقارنة بين طريقتين لتجهيز وحدات اضافية لنظام معين:-

1- تجهيز الوحدات الاضافية على مستوى المركب(المجانبة على مستوى المركب).

2- تجهيز الوحدات الاضافية على مستوى المنظومة(المجانبة على مستوى المنظومة).

ان المقارنة بين هاتين الطريقتين تتم عن طريق مقارنة المتغير العشوائي الذي يمثل فترة الحياة للنظام الناتج عن تجهيز الوحدات الاضافية على مستوى المركب مع المتغير العشوائي الذي يمثل فترة الحياة لذلك النظام الناتج عن تجهيز الوحدات الاضافية على مستوى المنظومة باستخدام ترتيب نسبة الامكانية.

INTRODUCTION

One of the most important problems nowadays is" how to improve device's reliability?". In this work, the method of increasing reliability known redundancy is considered[1].

There are two basic types of redundancy, active redundancy and cold-standby redundancy [2]; also, there are two ways of providing each type of redundancy, component redundancy and system redundancy [3].

In this paper, we make a comparison between component redundancy and system redundancy with respect to each of the basic types of redundancy using likelihood ratio ordering (we say that the random variable T_1 is smaller than the random variable T_2 in the likelihood ratio ordering sense, written $T_1 \leq_{lr} T_2$, iff $\frac{f_2(t)}{f_1(t)}$ is increasing in t, where $f_i(\ )$ is the density function of T_i, i=1,2 [4]).Two results are presented herein, for the first result, we prove that active redundancy on component level is better than active redundancy on system level using likelihood ratio ordering, and in the second one, we prove that cold-standby redundancy on component level is better than that on system level using likelihood ratio ordering also.

The results of this paper are generalization results to two of those in [1]. The results in [1] concern the comparison of (active, cold-standby) redundancy on component and system levels by means of likelihood ratio ordering for the original system which is a series system of two units, while in the generalized results of this paper the original system is a series system of n units, defined in [5], instead of two.

In our results, we need to consider the following notation:

Notation

iid	independent, identically distributed.
T	$(T_1,T_2,\ldots\ldots,T_n)$: independent lifetimes of n components in system.
U	$(U_1,U_2,\ldots\ldots,U_n)$: independent lifetimes of an independent set of n spares for system.
$T_i \vee U_i$	$\max\{T_i, U_i\}$.
$\tau_{1-n:F}(T)$	$\min\{T_1,T_2,\ldots\ldots,T_n\}$.
$\tau_{1-n:F}(T \vee U)$	$\min\{T_1\vee U_1,T_2\vee U_2,\ldots\ldots,T_n\vee U_n\}$.
$\tau_{1-n:F}(T+U)$	$\min\{T_1+U_1,T_2+U_2,\ldots\ldots,T_n+U_n\}$.
$\tau_{1-n:F}(T)+\tau_{1-n:F}(U)$	$\min\{T_1,T_2,\ldots\ldots,T_n\}+\min\{U_1,U_2,\ldots\ldots,U_n\}$.
R(t)	reliability function, $0 \leq R(t) \leq 1$.

Also, in our results, we consider the following assumptions:

Assumptions

1.The lifetime random variables of all the units are independent.

2.The lifetime random variables of the systems in system level redundancy are independent.

3.The reliability of every unit is not affected by the type of redundancy.

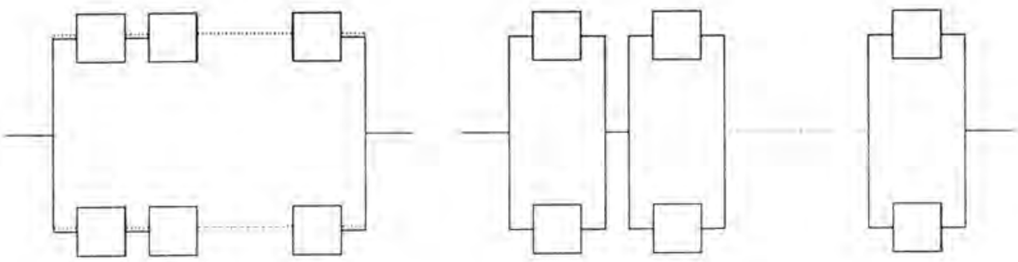
MAIN RESULTS

Result (1): Use assumptions (1-3)

Consider 1-out-of-n:F system. Let $T_1,T_2,\ldots\ldots,T_n,U_1,U_2,\ldots\ldots,U_n$ be iid lifetimes with common reliability function R, where $T_1,T_2,\ldots\ldots,T_n$ are the lifetimes of the n original units and $U_1,U_2,\ldots,U_n$ are the lifetimes of the n spare units. Then

$$\tau_{1-nF}(T)\vee\tau_{1-nF}(U)\leq_{lr}\tau_{1-nF}(T\vee U)$$

(Figure (1) below represents active redundancy on system and component levels).



(a) System redundancy

(b) Component redundancy

Figure (1)

Proof:-

By definition of likelihood ratio ordering, it is enough to show that $\frac{f_c(t)}{f_s(t)}$ is an increasing function of t.

Where

$$f_c(t)=-\frac{d}{dt}\{P(\tau_{1-n:F}(T\vee U)>t)\}$$

$$=-\frac{d}{dt}\{(2R(t)-R^2(t))^n\}$$

$$=n(2R(t)-R^2(t))^{n-1}.(2f(t)-2R(t).f(t)),\text{ where } f(t)=-\frac{d}{dt}R(t)$$

$$=2nf(t).(R(t))^{n-1}.(2-R(t))^{n-1}.(1-R(t))$$

and

$$f_s(t)=-\frac{d}{dt}\{P(\tau_{1-n:F}(T)\vee\tau_{1-n:F}(U)\phi\, t)\}$$

$$=-\frac{d}{dt}\{2R^n(t)-R^{2n}(t)\}$$

$$=2nf(t).(R(t))^{n-1}.(1-R^n(t))$$

Letting,

$$g_1(t)=\frac{f_c(t)}{f_s(t)}=\frac{2nf(t).(R(t))^{n-1}.(2-R(t))^{n-1}.(1-R(t))}{2nf(t).(R(t))^{n-1}.(1-R^n(t))}$$

$$=\frac{(2-R(t))^{n-1}(1-R(t))}{(1-R^n(t))}$$

To show that $g_1(t)$ is an increasing function of t, we must show that $\frac{d}{dt}g_1(t)\ \phi\ 0$

But,

$$\frac{d}{dt}g_1(t)=$$

$$\frac{\{1-R^n(t)\}.\{(n-1).(2-R(t))^{n-2}.f(t).(1-R(t))+f(t).(2-R(t))^{n-1}\}-\{(2-R(t))^{n-1}.(1-R(t))\}.\{nf(t)R^{n-1}(t)\}}{(1-R^n(t))^2}$$

i.e.,we must show that

$$nf(t).(1-R(t)).(1-R^n(t)).(2-R(t))^{n-2}-f(t).(1-R(t)).(1-R^n(t)).(2-R(t))^{n-2}$$
$$+f(t).(1-R^n(t)).(2-R(t))^{n-1}-nf(t).(R(t))^{n-1}.(1-R(t)).(2-R(t))^{n-1}\ \phi\ 0$$

Notice that,

$$f(t).(1-R^n(t)).(2-R(t))^{n-1}-f(t).(1-R^n(t)).(2-R(t))^{n-2}.(1-R(t))=f(t).(1-R^n(t)).(2-R(t))^{n-2}\ \phi\ 0$$

since $f(t)$,$(1-R^n(t))$,and $(2-R(t))^{n-2}>0$

and that,

$$nf(t).(1-R(t)).(1-R^n(t)).(2-R(t))^{n-2}-nf(t).R^{n-1}(t).(1-R(t)).(2-R(t))^{n-1}$$
$$=nf(t).(1-R(t)).(2-R(t))^{n-2}.(1-2R^{n-1}(t))\ \phi\ 0\quad \textit{under the condition that } (1-2R^{n-1}(t))\ \phi\ 0$$

i.e., under the condition that $R^{n-1}(t)\ \pi\ \frac{1}{2}$

This completes the proof. <

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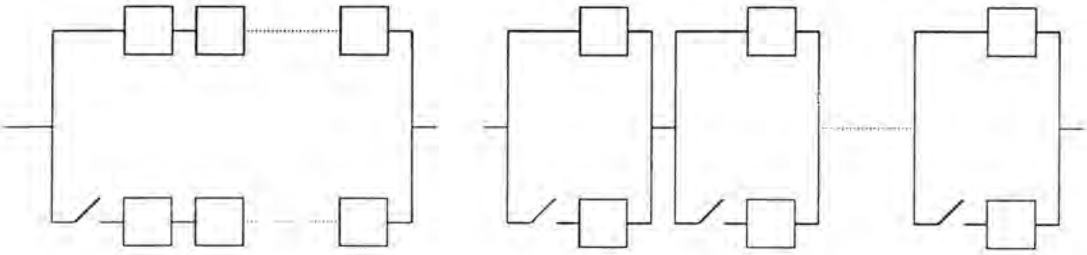
Result (2): Use assumptions (1-3)

Consider 1-out-of-n:F system. Let $T_1,T_2,\ldots,T_n$ be iid lifetimes of the original units. T_i is an exponential random variable with parameter α. Let $U_1,U_2,\ldots,U_n$ be iid lifetimes of the spare units. U_i is an exponential random variable with parameter 0.5α. Then

$$\tau_{1-n:F}(T)+\tau_{1-n:F}(U)\leq_{lr}\ \tau_{1-n:F}(T+U)$$

(Figure (2) below represents cold-standby redundancy on system and component levels).



(a) System redundancy (b) Component redundancy

Figure(2)

Proof:-

Define the distribution function of $\tau_{1-n:F}(T)$ by $(1-e^{-n\alpha t})$ and that of $\tau_{1-n:F}(U)$ by $(1-e^{-0.5n\alpha t})$ we have.

$$F_s(t) = P(\tau_{1-n:F}(T) + \tau_{1-n:F}(U) \le t)$$

$$= \int_0^t (1-e^{-0.5n\alpha(t-x)})\, d(1-e^{-n\alpha x})$$

$$= \int_0^t (1-e^{-0.5n\alpha(t-x)}).(n\alpha\, e^{-n\alpha x})\, dx$$

$$= \int_0^t n\alpha\, e^{-n\alpha x}\, dx - e^{-0.5n\alpha t}\int_0^t n\alpha\, e^{-0.5n\alpha x}\, dx$$

$$= (1-e^{-n\alpha t}) + 2e^{-0.5n\alpha t}(e^{-0.5n\alpha t}-1)$$

$$= 1 + e^{-n\alpha t} - 2e^{-0.5n\alpha t}$$

and

$$f_s(t) = \frac{d}{dt}F_s(t) = -n\alpha\, e^{-n\alpha t} + n\alpha\, e^{-0.5n\alpha t}$$

$$= n\alpha\, e^{-0.5n\alpha t}(1-e^{-0.5n\alpha t})$$

The distribution function of T_i+U_i is defined by:

$$P(T_i + U_i \le t) = \int_0^t (1-e^{-0.5\alpha(t-x)})\, d(1-e^{-\alpha x})$$

$$= \int_0^t (1-e^{-0.5\alpha(t-x)}).(\alpha\, e^{-\alpha x})\, dx$$

$$= \int_0^t \alpha\, e^{-\alpha x}\, dx - e^{-0.5\alpha t}\int_0^t \alpha\, e^{-0.5\alpha x}\, dx$$

$$= 1 - e^{-\alpha t} + 2e^{-0.5\alpha t}(e^{-0.5\alpha t} - 1)$$

$$= 1 + e^{-\alpha t} - 2e^{-0.5\alpha t}$$

Thus,

$$F_c(t) = P(\tau_{1-n:F}(T+U) \le t)$$
$$= 1-(1-P(T_i+U_i \le t))^n$$
$$= 1-(2e^{-0.5\alpha t} - e^{-\alpha t})^n$$

and

$$f_c(t) = \frac{d}{dt} F_c(t)$$
$$= n\alpha\,(2e^{-0.5\alpha t} - e^{-\alpha t})^{n-1}\,(e^{-0.5\alpha t} - e^{-\alpha t})$$
$$= n\alpha\, e^{-0.5n\alpha t}\,(2 - e^{-0.5\alpha t})^{n-1}.(1-e^{-0.5\alpha t})$$

Letting

$$g_2(t) = \frac{f_c(t)}{f_s(t)} = \frac{n\alpha\, e^{-0.5n\alpha t}(2-e^{-0.5\alpha t})^{n-1}.(1-e^{-0.5\alpha t})}{n\alpha\, e^{-0.5n\alpha t}(1-e^{-0.5n\alpha t})}$$
$$= (2-e^{-0.5\alpha t})^{n-1}\,\frac{(1-e^{-0.5\alpha t})}{(1-e^{-0.5n\alpha t})}$$

To prove the required result, we need to show that $g_2(t)$ is an increasing function of t.

Letting

$$L_1(t) = (2-e^{-0.5\alpha t})^{n-1}$$

and

$$L_2(t) = \frac{(1-e^{-0.5\alpha t})}{(1-e^{-0.5n\alpha t})}$$

$$g_2(t) = L_1(t).L_2(t)$$

We prove that $L_1(t)$ *and* $L_2(t)$ are increasing functions of t.

It is obvious that $L_1(t)$ is an increasing function of t.

Now, we prove that $L_2(t)$ is an increasing function of t.

We show that $\frac{d}{dt} L_2(t) \phi\, 0$

$$\frac{d}{dt} L_2(t) = \frac{0.5\alpha\, e^{-0.5\alpha t}(1-e^{-0.5n\alpha t}) - 0.5n\alpha\, e^{-0.5n\alpha t}(1-e^{-0.5\alpha t})}{(1-e^{-0.5n\alpha t})^2}$$

$\frac{d}{dt} L_2(t) \phi\, 0$ *if the numerator* $0.5\alpha\, e^{-0.5\alpha t} - 0.5\alpha\, e^{-0.5\alpha(n+1)t} - 0.5n\alpha\, e^{-0.5n\alpha t}$
$+ 0.5n\alpha\, e^{-0.5\alpha(n+1)t} > 0$

Note that,

$$(0.5n\alpha\, e^{-0.5\alpha(n+1)t} - 0.5\alpha\, e^{-0.5\alpha(n+1)t}) = 0.5\alpha\, e^{-0.5\alpha(n+1)t}\,(n-1)\ \phi\ 0$$

and that

$(0.5\alpha\, e^{0.5\alpha t} - 0.5n\alpha\, e^{0.5n\alpha t})\ \phi\ 0\ \textit{if}\ (1 - n\, e^{0.5(n-1)\alpha t})\phi\ 0$

i.e., if $t\ \phi\ \dfrac{2\ln(n)}{\alpha\ (n-1)}$

Thus,

$\dfrac{d}{dt} L_2(t)\ \phi\ 0$ *under the condition that* $t\ \phi\ \dfrac{2\ln(n)}{\alpha\ (n-1)}$

and $L_1(t)$ is an increasing function of t, for all t.

Thus,

$g_2(t)$ is an increasing function of t under the condition that $t\ \phi\ \dfrac{2\ln(n)}{\alpha\ (n-1)}$

and hence the result. <

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Directed Core graphs and their Up – down Pregroups

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Abstract

In this paper we wiil define the directed core graphs and will give a method to make any core graph a directed core graph (denoted by $\Gamma^{*}(H,T,T^{*},v^{*})$). Also we will show that the set of Schreier transversal is the set of all reduced paths in T with labeled $y_i \in X \cup X^{-1}$ that is in section 2. In section 3 we will show that the elements of the set of generators of the subgroup H of a free group F generated by $X=\{a,b\}$ are in form of up – down reduced words. In section 4 we will give an example to show the construction of the directed core graph of H. Finally in section 5 we will construct the up – down pregroup of the directed core graphs and then the universal group of the up – down pregroup of the directed core graph is isomorphic to H.

المستخلص

في بحثنا هذا أعطينا تعريفا لبيانات اللب الموجة $\Gamma^{*}(H,T,T^{*},v^{*})$ للزمر الجزئية H للزمر الحرة F المولدة بالمجموعة $X=\{a,b\}$ و بينا إن العناصر المولدة للزمـــرة الجزئيـــة Hللزمرة الحرة F هي على شكل أعلى – اسفل ومن ثم قمنا ببناء أعلى – اسفل ما قبل زمره Q للزمرة الجزئية H للزمرة الحرةF المولدة بالمجموعـــة $X=\{a,b\}$ مـــن بيانـــات اللـــب الموجه $\Gamma^{*}(H,T,T^{*},v^{*})$ للزمرة الجزئية H.

1.Introduction.

Let F be a free group generated by X. The **Cayley graph** of F on X (It is denoted by $\Gamma(F \times X)$) has vertex set F and set of edges $F \times X=\{(w,x); w \in F, x \in X\}$, such that the initial vertex of the edge (w,x) is w and the terminal vertex of the edge (w,x) is wx, for every edge (w,x) in $\Gamma(F \times X)$. For each edge (w,x) in $\Gamma(F \times X)$, there is an edge (wx,x^{-1}) is called the **inverse edge** of (w,x), such that the initial vertex of (wx,x^{-1}) is wx and the terminal vertex of (wx,x^{-1}) is w . The edges (w,x) of $\Gamma(F \times X)$ will be labeled by $x \in X$. The **Cayley coset** graph $\Gamma(F \times X)/H$ of a subgroup H of F on X (is denoted by $\Gamma(H)$) have vertex sets $\{Hw; w \in F\}$ and edge sets $\{(Hw,x); w \in F, x \in X\}$, such that the initial vertex of each edge (Hw,x) in $\Gamma(H)$ is Hw and the terminal vertex of each edge in $\Gamma(H)$ is Hwx. The Core graph of a Cayley coset graph of a subgroup H of a free group F on X is the smallest subgraph of $\Gamma(H)$ containing all cycles. It is denoted by $\Gamma^*(H)$. The number of cycles in $\Gamma^*(H)$ is called the cyclomatic number which is equal to the rank of the finitely generated subgroup H of a free group F on $X=\{a,b\}$. Let v be a vertex of $\Gamma^*(H)$, then the number of edges incident with the vertex v is called the **degree** of the vertex v. It is denoted by d(v). If $d(v)\geq 3$, then the vertex v is called a **branch point**. Let m be the rank of a finitely generated subgroup H of a free group F on X = $\{a, b\}$, such that $\Gamma^*(H)$ have vertices of degree 2 and 3 only, then $m=\frac{\#Br(\Gamma^*(H))}{2}+1$, where $\#Br(\Gamma^*(H))$ is the number of branch points in $\Gamma^*(H)$. Now if $\Gamma^*(H)$ has vertices of degree 4,3 and 2, then by isomorphic embed defined in [10] as below $\varphi: F\to F'$ by $\varphi(a)=uv^{-1}$ and $\varphi(b)=v^{-2}$, where F' is a free group generated by the set $\{u, v\}$ to have a new Core graph with the set of labeling $\{u, v\}$ and vertices of degree 2 and 3 only. Since the core graph $\Gamma^*(H)$ may have loops, so by isomorphic embed, defined by $\phi(a)=a^2$ and $\phi(b)=b^2$, then we have a new core graph $\Gamma^*(H)$ without loops. Henceforth we will assume that all core graphs $\Gamma^*(H)$ are core graphs without loops.

2.Directed Core graphs

***Definition* 2.1: A directed core graph** of a subgroup H of a free group F on $X=\{a,b\}$, can be made from $\Gamma^*(H)$ as below: i) Choose a base vertex v*; ii) Choose a maximal tree (spanning tree) T from $\Gamma^*(H)$; iii) Let the direction of all edges of T be away from the base vertex v*, that if the direction of an edge $e\in T$ down and labeled $x\in X=\{a,b\}$, then makes the direction of e be up and labeled $x^{-1}\in X^{-1}$; iv) The direction of all edges $e\in\Gamma^*(H)/T$ be as in $\Gamma^*(H)$, away from the base v*, which are called chords as in [3].

N.B. i) The directed core graph of H is denoted by $\Gamma^*(H,T,T^*,v^*)$. ii) The direction of all edges in T are up and away from the base vertex of T. iii) Let U be the set of all distinct reduced paths P in T with labeled y_e, where $y_{e_i}\in X\cup X^{-1}, e_i\in E(T)$ and $1\leq i\leq n$.

Definition 2.2: Let $u=y_{e_{i_1}}y_{e_{i_2}}\Lambda\, y_{e_{i_n}}$ be reduced path in T, then u is called a **maximal reduced path** in T if $t(u)=t(e_{i_n})$ is a maximal vertex in T and the set of all reduced paths in T is denoted by U'. Thus $U'\subset U$.

Definition 2.3: Let $u=y_{e_{i_1}}y_{e_{i_2}}\Lambda\, y_{e_{i_n}}$ be reduced path in T and let y_i be the labeled of the chord $e_{i_{n+1}}$ in $\Gamma^*(H)/T$, such that $i(e_{i_{n+1}})=t(e_{i_n})$, then define the set U^* to be the set $\{uy_{e_{i_{n+1}}} : u \in U, uy_{e_{i_{n+1}}} \in U$ if $e_{i_{n+1}} \in T, uy_{e_{i_{n+1}}} \notin U$ if $e_{i_{n+1}} \in \Gamma^*(H)/T\}$.

Henceforth we will denote the edge $e_{i_{n+1}}$ and the labeled $y_{e_{i_{n+1}}}$ of $e_{i_{n+1}}$ by e and y_e respectively.

Definition 2.4: Let u and v be any two elements in U^*, then we say that $u \le v$ if u is a subpath of v, $u < v$ if u is a subpath of v and $u \ne v$ and $u \sim v$ if $u \le v$ and $v \le u$.

Proposition 2.5: The relation $\sim$ defined above is an equivalence relation.□

Lemma 2.6: The set U^* has exactly one reduced path of each path in U^* under the relation $\sim$ defined above.

Proof: Let $x = uy_e$ and $y = vy_{e'}$ be reduced paths in U^* and suppose that $x \sim y$. Since u and v are unique reduced paths in T, so by definition of $\sim$, we have $x = y$. □

Lemma 2.7: The elements of U^* form a tree like, that is if x, y and z are elements in U^*, such that $x \le z$ and $y \le z$, then $x \le y$ or $y \le x$. Moreover the relation $\le$ is transitive.

Proof: Let $x = uy_e$, $y = vy_{e'}$ and $z = wy_{e''}$ be elements in U^*. Since $x \le z$ and $y \le z$, so either x and y are reduced paths in T and implies $x \le y$ or $y \le x$, or one of them is not in T and then we have $y \sim z$ or $x \sim z$. Therefore $x \le y$ or $y \le x$. It is clear that $\le$ is transitive. Therefore the elements of U^* form a tree T *.□

Since each reduced path in T * is unique so the relation $\sim$ (defined above) is an equivalence relation and each class is denoted by $[uy_e]$. Then the vertices of T^* are the classes $[uy_e]$.

Definition 2.8: For any two vertices $v_1 = [uy_e]$ and $v_2 = [vy_{e'}]$ in T^*, we say that $v_1 \equiv v_2$ if and only if $Huy_e = Hvy_{e'}$.

Lemma 2.9: If v_1 and v_2 are two vertices in T^* defined above, then $v_1 \equiv v_2$ if and only if $uy_e(vy_{e'})^{-1}$ forms a cycle in $\Gamma^*(H)$.

Proof: That $v_1 \equiv v_2$ if and only if $Huy_e = Hvy_{e'}$, if and only if $uy_e(vy_{e'})^{-1}$ is a cycle in $\Gamma^*(H)$.□

Proposition 2.10: The relation $\equiv$ defined above is an equivalence relation.

Proof: By the definition of $\equiv$ defined above the result follows.

Lemma 2.11: For any reduced path uy_e in T^*/T, there is a unique reduced path v in T such that $uy_e v^{-1}$ is a cycle in $\Gamma^*(H)$. i.e. $uy_e \equiv v$.

Proof: Since $uy_e \in T^*/T$ so y_e is a chord. Thus there is a reduced path v in T such that $uy_e v^{-1}$ is a cycle in $\Gamma^*(H)$ (by the definition of a chord).

Suppose now there is another reduced path z in T such that $uy_e z^{-1}$ is a cycle in $\Gamma^*(H)$. Therefore $t(v) = t(z)$ and vz^{-1} is a non – trivial cycle in T, a contradiction .Thus $v = z$. □

N.B. v (defined above) is denoted by $\overline{uy_e}$ in [4,5,6].

Definition 2.12: for any two edges e and e' in T * , we say that $e \approx e'$ if and only if (i) e and e' have the same labeled , (ii) $i(e) \equiv i(e')$ and $t(e) \equiv t(e')$.

Lemma 2.13: The relation $\approx$ (defined above) is an equivalence relation.

Proof: Directly by definition of $\approx$ the result follows. □

Lemma 2.14: T * has exactly one edge of each edge class under the relation $\approx$.

Proof: Let e and e' be any two edges in T * such that $e \approx e'$. Since T has exactly one vertex of each vertex class under the relation $\equiv$, so e and e' are not in T. Therefore either (i) e and e' are in T^*/T or (ii) $e \in T^*/T$ and $e' \in T$. If (i) holds, then $i(e)$ and $i(e')$ are in T, $i(e) = i(e')$, $t(e) = t(e')$ and then $y_e = y_{e'}$ otherwise we have a contradiction. Therefore we have an inconsistent graph T * a contradiction. If (ii) holds, we have $i(e) \equiv i(e')$ and $i(e), i(e') \in T$ a contradiction. □

Now by identifying the vertices of the same class in T * we have a directed core graph for H (defined above).

3. Nielsen – Seherier Theoerem

Lemma 3.1: If uy_e and $vy_{e'}$ are two reduced paths in T * such that $uy_e < vy_{e'}$ and $Huy_e = Hvy_{e'}$, then $vy_{e'} \notin$ T and $y_{e'}$ is a labeled of a chord e' and y_e is the labeled of an edge in T.

Proof: Suppose $vy_{e'} \in$ T. Since $Huy_e = Hvy_{e'}$ and $uy_e < vy_{e'}$, so $uy_e(vy_{e'})^{-1}$ is a cycle and $uy_e \in$ T respectively. Thus uy_e and $vy_{e'}$ are both reduced paths in T . Since T has no non – trivial cycles so $uy_e = vy_{e'}$ a contradiction. Hence $vy_{e'} \in T^*/T$ and e' is a chord and y_e is a labeled of an edge e in T. □

Therefore the reduced paths in T * form the minimum reduced paths in the class under equivalence relation ~.

Corollary 3.2: Let F be a free group generated by $X = \{a, b\}$. If x and y are two reduced words in F such that $x < y$ and $Hx = Hy$, then y is a labeled of a reduced path not in T.

Proof: By Lemma 3.1the result follows. □

Lemma 3.3: Let $x = y_{e_1} y_{e_2} \Lambda\, y_{e_{n-1}} y_{e_n}$ be a reduced word in F, $n \geq 1$. If x is a labeled of a reduced path in T,then the $y = y_{e_1} y_{e_2} \Lambda\, y_{e_{n-1}}$ is a labeled of a reduced path in T.

Proof: Since $y < x$ and $x \in T$, so y is a reduced path in T and then in U. □

Lemma 3.4: The set A = $\{uy_e(\overline{uy_e})^{-1}$; u is the labeled of a reduced path in T and $y_e \in X \cup X^{-1}$ is the labeled of an edge e in $T^*\}$ generates the subgroup H of a free group F on X = $\{a, b\}$.

Proof: Let $x = y_{e_1} y_{e_2} \Lambda\, y_{e_n}$ be a reduced word in H so $x = y_{e_1} y_{e_2} \Lambda\, y_{e_n}$ is a reduced closed path in $\Gamma^*(H,T,T^*,v^*)$ with labeled $y_{e_j} \in X \cup X^{-1}$ $1 \leq j \leq n$ starting and ending at the base vertex v^*. Therefore there is a sequence of maximal common reduced paths $u_1, u_2, \Lambda, u_{n-1}$ in T starting at v^* such that $u_{j+1} = \overline{u_j y_{e_j}}$, and $a_j = u_j y_{e_j} (\overline{u_j y_{e_j}})^{-1}$ in A for all j, $1 \leq j \leq n$, so $a_1 a_2 \Lambda\, a_n = u_1 y_{e_1} u_2^{-1} \Lambda\, u_n^{-1} u_n y_{e_n} u_{n+1}^{-1} = u_1 x u_{n+1}^{-1}$. Since $t(e_n) = v^* = t(\overline{u_n y_{e_n}})$ and $i(e_1) = v_0 = i(u_1 y_{e_1}) = i(u_1)$, so the maximal common reduced path between u_1 and e_1 is v^*, and also the maximal common reduced path between e_n and u_{n+1} is v^*. Thus u_1 = 1 and u_{n+1} = 1. Therefore $x = a_1 a_2 \Lambda\, a_n$ in H. □

Lemma 3.5: Let u be the labeled of a reduced path in T and $y_e \in X \cup X^{-1}$ is the labeled of the edge e in $\Gamma^*(H,T^*,T,v^*)$, then (i) $uy_e(\overline{uy_e})^{-1} = 1$ if and only if $uy_e \in T$. (ii) $u = \overline{\overline{uy_e} y_e^{-1}}$.

Proof: Since $\overline{uy_e}$ is the only reduced in T such that $uy_e(\overline{uy_e})^{-1}$ is a cycle in $\Gamma^*(H,T,T^*,v^*)$, so $uy_e(\overline{uy_e})^{-1} = 1$ if and only if $uy_e(\overline{uy_e})^{-1}$ is the trivial cycle in $\Gamma^*(H,T,T^*,v^*)$, if and only if $uy_e = \overline{uy_e}$ if and only if $uy_e \in T$. (ii) Since $uy_e \in T^*$ and $\overline{uy_e} \in T$, so $\overline{uy_e} y_e^{-1}$ is an up- down reduced subpath of $\overline{uy_e} y_e^{-1} u^{-1}$ in $\Gamma^*(H,T,T^*,v^*)$, such that $t(\overline{uy_e} y_e^{-1}) = t(u)$ and then u is the unique reduced in T such that $\overline{uy_e} y_e^{-1} u^{-1}$ is a cycle in $\Gamma^*(H,T,T^*,v^*)$,. Therefore $u = \overline{\overline{uy_e} y_e^{-1}}$. □

Lemma 3.6: If uy_e and $vy_{e'}$ are two reduced paths in T^*/T, then either (i) $y_e(\overline{uy_e})^{-1} \overline{vy_{e'}} = 1$, in which case $v = \overline{uy_e}$, $y_{e'} = y_e^{-1}$ and $u = \overline{vy_{e'}}$ or (ii) $y_e(\overline{uy_e})^{-1} \overline{vy_{e'}}$ is a reduced path of Length at least two such that $i(y_e(\overline{uy_e})^{-1} \overline{vy_{e'}}) = i(y_e)$ and $t(y_e(\overline{uy_e})^{-1} \overline{vy_{e'}}) = t(y_{e'})$.

Proof: Since uy_e and $vy_{e'}$ are in T^*/T and, y_e and $y_{e'}$ are the labeled of the chords e and e' respectively. Therefore there are unique reduced paths $\overline{uy_e}$ and $\overline{vy_{e'}}$ in T such that $uy_e(\overline{uy_e})^{-1}$ and $vy_{e'}(\overline{vy_{e'}})^{-1}$ are non trivial cycles in $\Gamma^*(H,T^*.T,v_0)$. Thus the maximal common reduced path between $\overline{uy_e}$ and v is k, implies that either (1) $(\overline{uy_e}=k=v)$, (2) $k=\overline{uy_e}, k<v)$, (3) $(k=v, k<\overline{uy_e})$ or (4) $(v<k, k, \overline{uy_e})$ holds. Now if (1) holds, then either $y_e y_{e'}=1$ implies $y_e=y_{e'}^{-1}$, $\overline{uy_e}=v$, $u=vy_{e'}$, and then $y_e(\overline{uy_e})^{-1}vy_{e'}=1$, or $y_e y_{e'}\neq 1$ implies $y_e y_{e'}$ is a reduced path of length 2 and then $y_e(\overline{uy_e})^{-1}vy_{e'}$ is a reduced path of length 2. If (2), (3) or (4) holds , then $(\overline{uy_e})^{-1}v\neq 1$, implies $y_e(\overline{uy_e})^{-1}vy_{e'}$ is a reduced path of length at least 2 such that $i(y_e(\overline{uy_e})^{-1}vy_{e'})=i(y_e)$ and $t(y_e(\overline{uy_e})^{-1}vy_{e'})=t(y_{e'})$.□

Lemma 3.7: If uy_e is a labeled of a reduced path in T^*/T, then all reduced paths $uy_e(\overline{uy_e})^{-1}$ are distinct and the set of them is equal to the disjoined union of the sets B and B^{-1}, where $B=\{uy_e(\overline{uy_e})^{-1}: u$ is a labeled of a reduced path in T and y_e is a labeled of an edge e in T^*/T, $y_e\in X\}$.

Proof: Since $uy_e\in T^*/T$, so y_e is a labeled of a chord . By Lemma 2.11 there is a unique reduced path $\overline{uy_e}$ in T such that $t(\overline{uy_e})=t(y_e)$ and $uy_e(\overline{uy_e})^{-1}$ is a non trivial cycle. Since all chord of $\Gamma^*(H,T,T^*,v^*)$ are distinct so all non trivial cycles $uy_e(\overline{uy_e})^{-1}$ are distinct of B . Now Let $B^{-1}=\{(uy_e(\overline{uy_e})^{-1})^{-1}: uy_e(\overline{uy_e})^{-1}\in B\}$. Since $(uy_e(\overline{uy_e})^{-1})^{-1}$ is the inverse of $uy_e(\overline{uy_e})^{-1}$, so $(uy_e(\overline{uy_e})^{-1})^{-1}=\overline{uy_e}y_e^{-1}u^{-1}$ is a non trivial cycle in $\Gamma^*(H,T,T^*,v^*)$. Since $y_e\neq y_e^{-1}$ so all elements of B^{-1} are distinct and then all elements of $B\cup B^{-1}$ are distinct .□

Theorem 3.8 (Nielsen – Schreier) : Let F be a free group generated by X of rank n, and H be a subgroup of F . If H is of finite index g , then H is finitely generated free subgroup of rank m such that $m = g(n-1)+1$.

Proof: Since H is of finite index g , so $\Gamma(H)=\Gamma^*(H)$ and then $\#V(\Gamma(H))=\#V(\Gamma^*(H)) = g$. By Lemmas 3.4 and 3.7 $A=B\cup B^{-1}\cup\{1\}$ generates H and then H is finitely generated by B. Since each non trivial cycle has only one edge of T^*/T, so the rank of $H = m = \#E(T^*/T)$. If $x_1x_2\Lambda\, x_t$ is a reduced word in H, where $x_i\in B, 1\leq i\leq t$, then by Lemma 3.6 $x_1x_2\Lambda\, x_t\neq 1$. Therefore H has no non- trivial relation and then H is free.

From graph theory we have $d(v)=2n$, $\sum_{i=1}^{g} d(v_i)=2\#E(\Gamma^*(H))$, $\#E(\Gamma^*(H))=\#E(T)+\#E(T^*/T)$ and $\#E(T)=\#V(T)-1$. Therefore $2ng=2(g-1+m)$. Thus $m = g(n-1)+1$. □

4. ***An Example.*** Let $\Gamma^*(H)$ of H of a free group F on $X = \{a, b\}$ be as below:

$T(\Gamma^*(H))$:

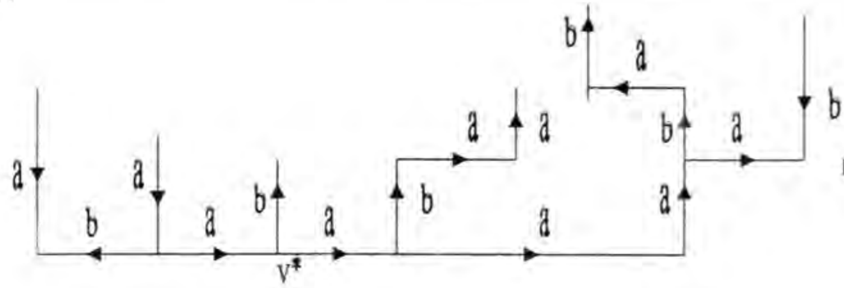

Figure 2

$T(\Gamma^*(H))$:

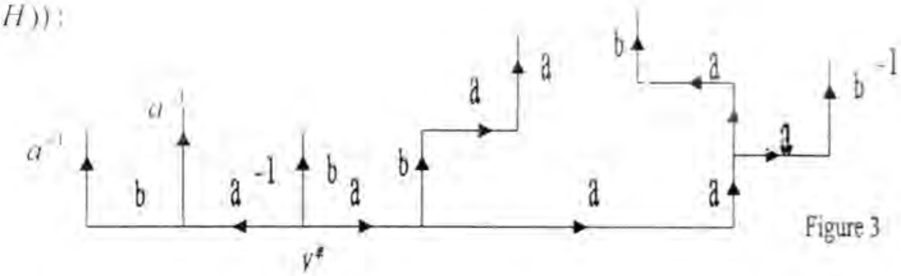

Figure 3

$T^*(\Gamma^*(H))$:

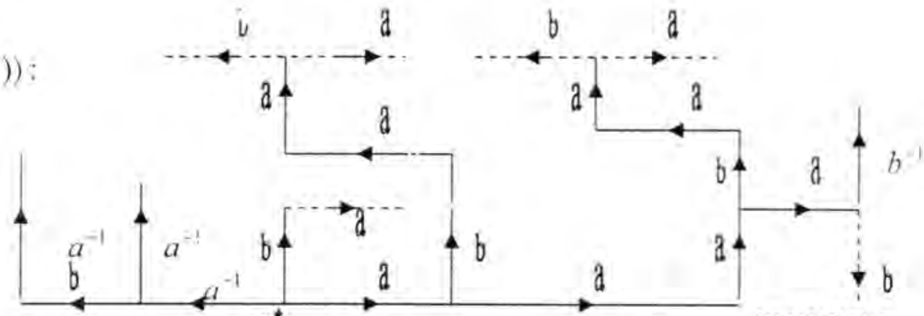

FIGURE 4

$\Gamma^*(H, T, T^*, v^*)$:

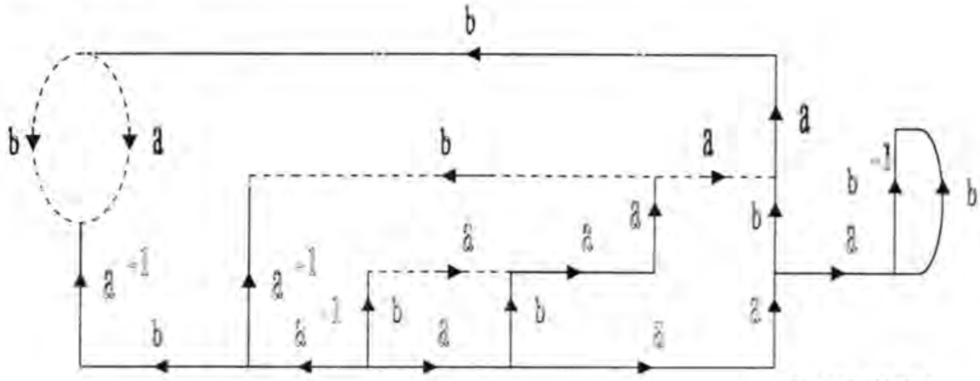

FIGURE 5

Therefore the set $B=\{x_1,x_2,x_3,x_4,x_5,x_6\}$, where $x_1=b.a.b^{-1}a^{-1}$, $x_2=abaa.a.b^{-1}a^{-1}a^{-1}a^{-1}$, $x_3=abaa.b.aa$, $x_4=aaabab.b.ab^{-1}a$, $x_5=aaabab.a.ab^{-1}a$, $x_6=aaaa.b.ba^{-1}a^{-1}a^{-1}a^{-1}$, where the letters between the dots refer to the labeled of the chords.

5. Directed Core graphs and their Up – down Pregroups

In this section we will construct an up-down Pregroup directly from the directed core graph $\Gamma^*(H,T,T^*,v^*)$ for subgroup H of a free group F on $X=\{a,b\}$.

Pregroups

The idea of Pregroups goes back to Baer [1] and independently the definition of Pregroup was given by Stallings [11] in 1971. The Theory of Pregroups were developed by Rimlinger [7], Stallings [11], Hoare [2] and Hoare – Jassim [3]. Now tern back to the origin definition of Pregroups [11].

Let P be a set with an element $1\in P$ and a mapping of a subset D of $P\times P$ into P, denoted by $(x,y)\alpha\ xy$. We shall say that xy is defined instead of $(x,y)\in D$. Suppose that there is an involution on P denoted by $x\,\alpha\ x^{-1}$, such that the following axioms hold:

P1: $x1=1x$ for all $x\in P$,

P2 : $xx^{-1}=1=x^{-1}x$ for all $x\in P$,

P3: If xy is defined , then $y^{-1}x^{-1}$ is defined and $(xy)^{-1}=y^{-1}x^{-1}$.

P4 : if xy and yz are defined then $(xy)z$ is defined if and only if $x(yz)$ is defined in which case the two are equal and we will say xyz is defined.

P5 : For any w, x, y and z in P , if wx, xy and yz are defined then either wxy or xyz is defined .

Hoare [2] showed that we could prove axiom P3 above by using the following proposition, P1, P2 and P4.

Proposition 5.1: If xy is defined, then $(xy)y^{-1}$ is defined and equal to x. □

Definition 5.2. [2] : For any $x\in P$, put $L(x)=\{a\in P: ax$ is defined $\}$. We write $x\le y$ if $L(y)\subseteq L(x), x<y$ if $L(y)\subset L(x)$ and $L(x)\ne L(y)$, and $x\sim y$ if $L(x)\ne L(y)$. It is clear that $\sim$ is an equivalence relation compatible with $\le$.

The following results are taken from Stallings [11] and Rimlinger[7].(See [2] for shorter proofs).

Proposition 5.3.

(i) If $x\le y$ or $y\le x$, then $x^{-1}y$ and $y^{-1}x$ are defined.

(ii) If xa and $a^{-1}y$ are defined , then $(xa)(a^{-1}y)$ is defined if and only if xy is defined in which case they are equal. □

By using axiom P5 above (will be denoted by P5(i)) Rimlinger [7] proved conditions P5(ii) and P5(iii) of Lemma 5.4 below.

Lemma 5.4 [2] . The following conditions on elements of P are equivalent :

P(i). If wx, xy and yz are defined , then either wxy or xyz is defined .

P(ii). If $x^{-1}a$ and $a^{-1}y$ are defined but $x^{-1}y$ is not , then $a < x$ and $a < y$.

P(iii). If $x^{-1}y$ is defined , then $x \le y$ or $y \le x$.

Therefore we will say P is a pregroup, if it satisfies axioms P1, P2. P4 and the conditions of Lemma 5.4 above. The universal group of a pregroup P [11] is denoted by $U(P)$ and has the following presentation $< P; x.y = xy$ whenever xy is defined, for $x, y, \in P >$. Now if P is a pregroup , then $(P, \le)$ is tree - like partial ordering ; that is P/~ has a minimum element and, for any x, y and z in P , $x \le z$ and $y \le z$ we have $x \le y$ or $y \le x$. Moreover Rimlinger in [7] showed that for any element x in P, we say that x has ***finite height*** $n \ge 0$, if there exists a maximal totally ordered subset $\{x_0, x_1, \Lambda, x_n\}$ of P such that $1 = x_0 < x_1 < \Lambda < x_n = x$. Also he showed that the elements of P form an order tree (denoted by $\boldsymbol{O}$), whose vertices , $[x]$, are the equivalence of elements of P under ~, and whose edges e, are formed by joining each vertex $[x]$ of height $n > 0$ to the unique vertex $[y]$ of height $n - 1$ satisfying $[y] < [x]$ and all edges e of $\boldsymbol{O}$ are directed away the base vertex $[x_0]$ of height 0. In [12] Stallings constructed an up – down pregroup for a free group F generated by $X = \{a, b\}$ of finite height and he showed that $U(P)$ the universal group of a pregroup P is isomorphic to F. In [3] we constructed a directed graph of groups of P directly from the order tree $\boldsymbol{O}$ of P and then we showed that the fundamental group of a graph of groups $\pi_1(G_v, G_e, Y, T, v^*, \varphi_e)$ is isomorphic to $U(P)$ and also we constructed an up – down pregroup Q directly from the directed graph of groups $(G_v, G_e, Y, T, v^*, \varphi_e)$ of a pregroup P and we showed that $U(Q)$ is isomorphic to $\pi_1(G_v, G_e, Y, T, v^*, \varphi_e)$ and then $U(Q) \cong U(P)$. In [8] Rimlinger constructed a pregroup structure Q for subgroups S of U(P) by using Cayley graph of $U(P)$ such that all elements in Q are E- reduced , that means if $x = x_1 x_2 \Lambda x_n$ is an E – reduced word in Q, then $x_1 x_2 \Lambda x_i \notin Q$ for all i such that $1 < i < n$.

Definition 5.5: Let $\overline{uy_e}$ and $vy_{e'}$ be reduced up – paths in T* such that $\overline{uy_e} = y_{e_1} y_{e_2} \Lambda y_{e_r}$ is a reduced path in T with labeled $y_{e_i} \in X \cup X^{-1}$, $1 \le i \le r$ and $vy_{e'} = y_{e'_1} y_{e'_2} \Lambda y_{e'_t} y_{e'_{t+1}}$ is a reduced path in T* with labeled $y_{e'_j} \in X \cup X^{-1}$, $1 \le j \le t$, and whether $y_{e'} = y_{e'_{t+1}}$ is a labeled of a chord or not. We say that $\overline{uy_e}$ is a subword of $vy_{e'}$ if and only if $e_j = e'_j$ and then $y_{e_j} = y_{e'_j}$ for all j, $1 \le j \le r \le t+1$. Thus we can write $vy_{e'} = \overline{uy_e} v' y_{e'} = y_{e_1} y_{e_2} \Lambda y_{e_r} y_{e'_{r+1}} \Lambda y_{e'_t} y_{e'_{t+1}}$ where $v' = y_{e'_{r+1}} \Lambda y_{e'_t}$.

Definition 5.5: For any x, y in B defined above , (where $x = uy_e\overline{uy_e}^{-1}$, $y = vy_{e'}\overline{vy_{e'}}^{-1}$), we say that $x.y = xy$ if and only if $\overline{uy_e}$ is a subword of $vy_{e'}$. More precisely if $\overline{uy_e}$ is a subword of v. It is denoted by $\overline{uy_e} \le vy_{e'}$.

Definition 5.6: Let B^* be the set of all reduced words $x_1x_2 \Lambda x_t$ with identity element 1, where $x_i \in B$ and x_ix_{i+1} is defined for all i, $1 \le i \le t-1$. Thus $B^* = \{x_1x_2 \Lambda x_t ; x_i \in B, x_ix_{i+1}$ is defined for all $i, 1 \le i \le t-1\}$.

N.B. (1) If $\alpha \in B^*$, then $\alpha = x_1x_2 \Lambda x_t$ and then $\alpha = u_1'y_{e_1}u_2'y_{e_2} \Lambda u_t'y_{e_t}\overline{u_ty_{e_t}}^{-1}$, where $u_1' = u_1$, u' is the terminal word of u_i of the element $x_i = u_iy_{e_i}\overline{u_iy_{e_i}}^{-1}$, $1 \le i \le t$ and y_{e_i} is the labeled of the chord e_i. (2) If $\alpha \in B^*$, then α is called an up – down word, where the up part of α is $u_1'y_{e_1}u_2'y_{e_2} \Lambda u_t'y_{e_t}$ and the down part of α is the down part of x_t which is equal to $\overline{u_ty_{e_t}}^{-1}$.

Definition5.7: For any two elements $\alpha_1, \alpha_2 \in B^*$, then we say that $\alpha_1 \le \alpha_2$ if and only if α_1 is a subword of α_2.

It is clear that the relation " is a subword of " is an equivalence relation.

Lemma 5.8: The relation " is a subword of " of elements of B^* is tree like , that if $\alpha_1 \le \alpha_3$ and $\alpha_2 \le \alpha_3$, then $\alpha_1 \le \alpha_2$ or $\alpha_2 \le \alpha_1$.

Proof: Directly by the definition of " a subword of " the result follows.

Remark: since $\Gamma^*(H,T,T^*,v^*)$ is a finite graph so B^* is a finite subset of H containing the identity element 1 of H.□

We now define the set K = $B^*.B^{*-1} = \{\alpha.\beta^{-1} ; \alpha\beta^{-1}$ is defined and $\alpha, \beta \in B^*\}$ is the set of all up – down elements of H.

Lemma 5.9: Let $k_1 = \alpha_1\beta_1^{-1}$, $k_2 = \alpha_2\beta_2^{-1}$ be two elements in K , then $k_1^{-1}k_2 = (\alpha_1\beta^{-1})^{-1}(\alpha_2\beta_2^{-1})$ is an element in K if and only if α_1 is a subword of α_2 or α_2 is a subword of α_1.

Proof : Let $\alpha_1 = u_1'y_{e_1}u_2'y_{e_2} \Lambda u_t'y_{e_t}\overline{u_ty_{e_t}}^{-1}$ and $\alpha_2 = v_1'y_{e_1'}v_2'y_{e_2'} \Lambda v_r'y_{e_r'}\overline{v_r'y_{e_r'}}^{-1}$. Then $k_1^{-1}k_2 = \beta_1\alpha_1^{-1}\alpha_2\beta_2^{-1}$ is defined if and only if $u_1'y_{e_1} = v_1'y_{e_1'}$ if and only if all edges e_i , e_i' in the reduced paths u_1', v_1' respectively are equal and then the labeled $y_{e_i}, y_{e_i'}$ of all edges e_i, e_i' in u_1', v_1' respectively are same, and the chord e_1 with the labeled y_{e_1} is equal to the chord e_1' with labeled $y_{e_1'}$, and more reduces happen only to up – down elements if and only if all edges e_i with labeled y_{e_i} in α_1 is equal to the edges e_i' with labeled $y_{e_i'}$ in α_2 in case $t \le r$ or all edges e_i' with labeled $y_{e_i'}$ in α_2 is equal to the edges e_i with labeled y_{e_i} in α_1 in case $r \le t$ and this happen if and only if $\alpha_1 \le \alpha_2$ or $\alpha_2 \le \alpha_1$ respectively.□

Lemma 5.10: Let $k_1=\alpha_1\beta_1^{-1}$ and $k_2=\alpha_2\beta_2^{-1}$ be reduced up – down elements in K. If α_1 is a subword of α_2, then $k_1=\alpha_1\beta_1^{-1} \le k_2=\alpha_2\beta_2^{-1}$.

Proof : Suppose that α_1 is a subword of α_2. Now if there exists $k_3=\alpha_3\beta_3^{-1}$ in K such that $k_3^{-1}k_2=(\alpha_3\beta_3^{-1})^{-1}(\alpha_2\beta_2^{-1})$ is defined that mean $k_3^{-1}k_2 \in K$ so by Lemma 5.9 α_2 is a subword of α_3 or α_3 is a subword of α_2. Since α_1 is a subword of α_2. so in both cases we have α_1 is a subword of α_3 or α_3 or α_3 is a subword of α_1. From both cases we have $k_3^{-1}k_1=(\alpha_3\beta_3^{-1})^{-1}(\alpha_1\beta_1^{-1})$ is defined and then an element in K. Therefore k_3^{-1} is an element of the set $L(k_1)$. implies that $L(k_2)$ is a subset of $L(k_1)$, and then $k_1=\alpha_1\beta_1^{-1} \le k_2=\alpha_2\beta_2^{-1}$.

Lemma 5.11: Let $k_1=\alpha_1\beta_1^{-1}$ and $k_2=\alpha_2\beta_2^{-1}$ be two elements in K . If $k_1^{-1}k_2$ is defined in K. Then $k_1 \le k_2$ or $k_2 \le k_1$.

Proof: By Lemma 5.9 above we have α_1 is a subword of α_2 or α_2 is a subword of α_1. Therefore by Lemma 5.10 we have $k_1 \le k_2$ or $k_2 \le k_1$ and then P(iii) holds.

Theorem 5.12: $K=B^*.B^{*-1}$ is a pregroup.

Proof: Since $K \subseteq H$, so P1, P2 and P4 hold and by Lemma 5.11 above P5(iii) holds. Therefore by the definition of P defined above we have $K=B^*.B^{*-1}$ is a pregroup . □

Example 2. In section 4, we constructed a directed core graph $\Gamma^*(H,T,T^*,v^*)$ for a subgroup H of a free group F on $X=\{a, b\}$, and from $\Gamma^*(H,T,T^*,v^*)$ we constructed the generating set B for H , such that $B=\{x_1,x_2,x_3,x_4,x_5,x_6\}$, where $x_1=b.a.b^{-1}a^{-1}$, $x_2=abaa.a.b^{-1}a^{-1}a^{-1}a^{-1}$, $x_3=abaa.b.aa$, $x_4=aaabab.b.ab^{-1}a$, $x_5=aaabab.a.ab^{-1}a$, $x_6=aaaa.b.ba^{-1}a^{-1}a^{-1}a^{-1}$, where the letters between the dots are referred to the labeled of the chords. Therefore the set B^* = $\{1,x_1,x_2,x_3,x_4,x_5,x_6,x_1x_2,x_1x_3,x_2x_4,x_2x_5,x_1x_2x_4,x_1x_2x_5\}$. Therefore the ordered tree of the up – down pregroup $K=B^*.B^{*-1}$ of the subgroup H of the free group F on $X=\{a, b\}$ is as below:

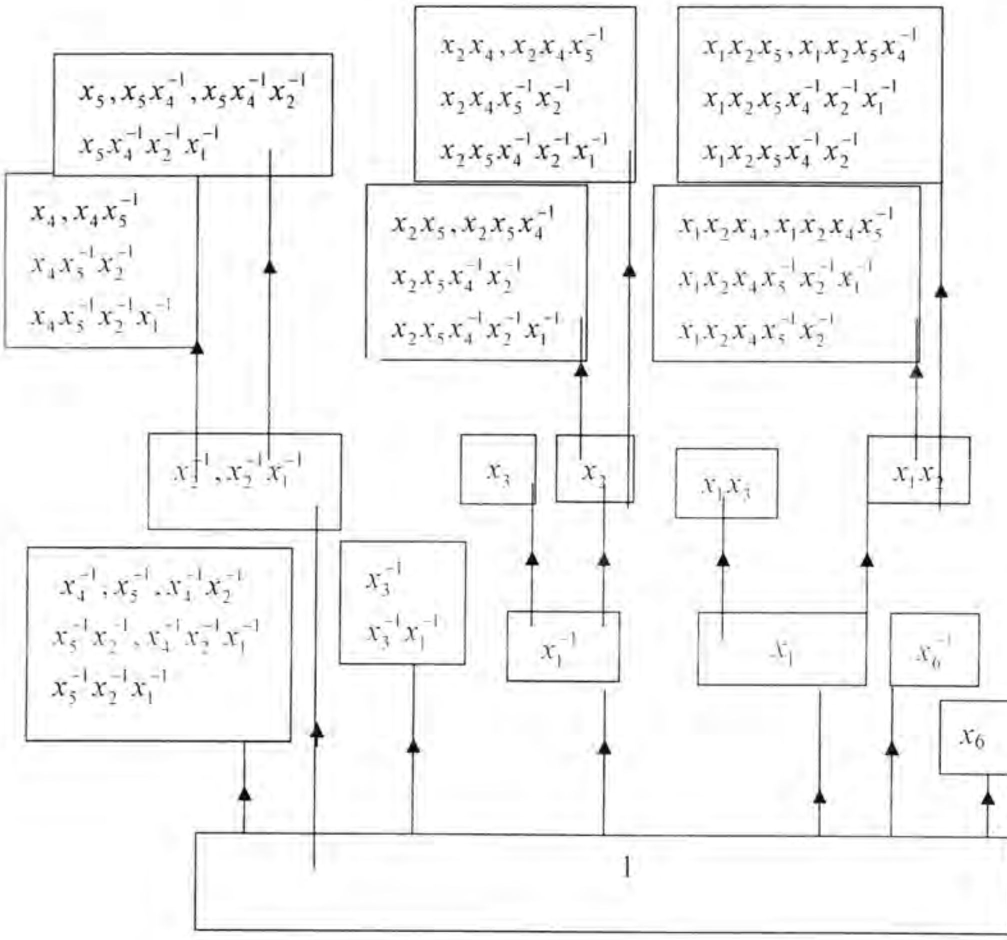

FIGURE 6

We now construct the directed graph of groups $(\{1\}, y_e, Y, T, v_0)$ for the up - down pregroup Q of the subgroup H of the free group F on $X = \{a, b\}$ by using the method which was given in [3] as below:

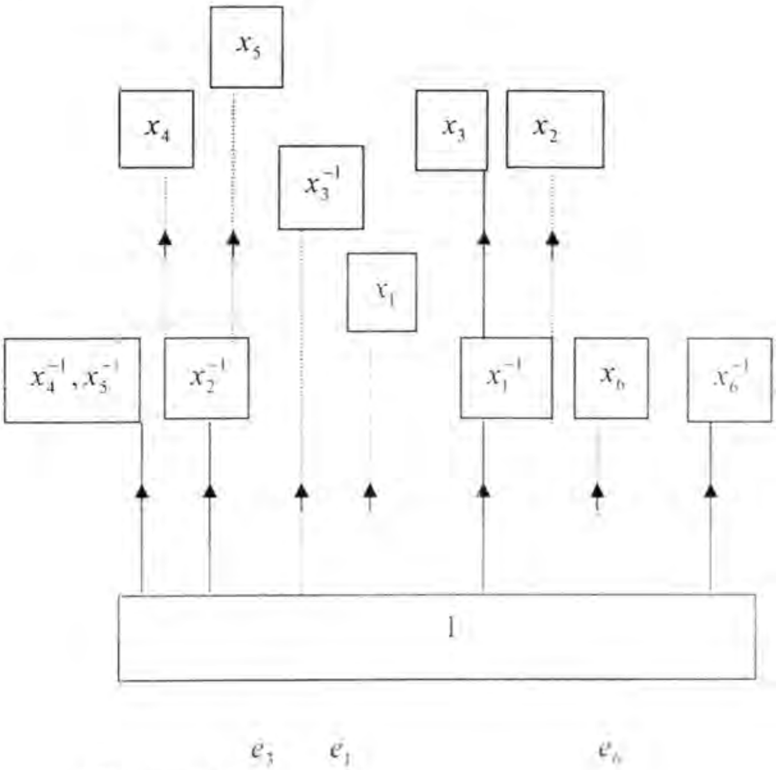

Figure 7

By identifying the vertices of the same classes in Figure 6 we have the directed graph of groups of the up – down pregroup Q which was constructed from the directed core graph $\Gamma^*(H,T,T^*,v^*)$ of the subgroup H of a free group F generated by $X = \{a, b\}$.

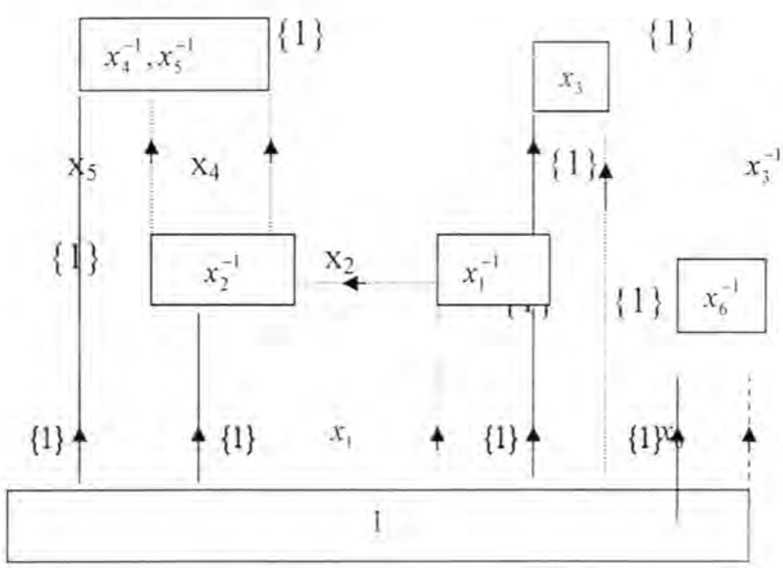

FIGURE 8

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On $g(g^{*})m$-closed sets

حول المجموعات المغلقة - $g(g^{*})m$

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Abstract

In this paper, we introduce two concepts, gm - closed sets and $g^{*}m$ - closed sets in the minimal structure space .Many properties of these concepts are studied comparable with the satisfying results relative to g - closed sets in the topological space and (g^{*}) closed sets in the Alexandroff space.

الخلاصة

في هذا البحث قدمنا مفهومي المجموعات المغلقة - gm والمجموعات المغلقة - $g^{*}m$ في فضاء البنية الاصغرية . العديد من الخواص لهذين المفهومين قد دُرست مقارنة بالنتائج المتحققة بالنسبة إلى المجموعات المغلقة - g في الفضاء التبولوجي و المجموعات المغلقة - g^{*} في فضاء الكسندروف.

Introduction

The concept of minimal structure space was introduced firstly in 1950 by H.Maki , J.Umehara and T.Noiri in they work "Every Topological space is pre$T_{1/2}$ ".Using these structure various mathematician turned their attention to investigate and studied these structure by considering m-open sets instead of open sets .While open sets are replaced by m-open sets, new results are obtained in some occasions and in other occasions substantial generalizations are exhibited.

In this directions, in 2000, Noiri and Popa [1] working on this structure and introduce the concepts of m-compact and m-connected, which various properties of these concepts are given and investigate.

Later in 2001 ,H.Jabar further the study of the concepts of m-compact and m-connected and introduce the concepts of m-separation axioms and m-almost continuous functions , later M.Aliohammad and M.Roohi [2] studied the fixed

* Keywords and phrases: Minimal structure space, Alexandroff space, g - closed sets, g^{*} - closed sets.

point property .Our aim in this paper is grew depending on work of Das and Rashid [3] ,that can be displayed as:

In 1970, Levine [4] generalized the concept of the closed sets to g – closed sets (A subset A of a topological space (X,τ) is said to be g -closed *iff* $cl(A) \subseteq O$, whenever $A \subseteq O$ and O is an open subset in X), and he shows that g -closed sets possess many of the familiar and important properties of closed sets.

In 2003, Das and Rashid [3], working on Levine research [4] present a new equivalent form of g -closed sets called g^* -closed sets by using another space called Alexandroff space where only countable unions of open sets are required to be open , various properties of these sets are investigate and they show that g^* -closed sets do not always behave like g -closed sets ,so in some of these cases they try to find out the conditions under which their behavior be the same. So, our main interest here is to introduce a new equivalent forms of g -closed sets, namely gm -closed sets and g^*m -closed sets, by using minimal structure space and investigate the results of Levine [4] and Das and Rashid [3] ,for which we show that some results in [3] are also satisfy in minimal structure space without needed to Alexandroff space assumption as in theorems 2.6, 3.9, 3.11 .Moreover , in section 4 ,we introduce the concepts of gm -open sets and g^*m -open sets and investigate their behavior with respect to g -open sets [4] and g^* -open sets [3] . Finally, I would to refer that the concepts of gm -closed (open) sets and g^*m - closed (open) sets, are novel to the best of our knowledge.

1. preliminaries

Definition 1.1[1][2].A subfamily m_X of the power set $P(X)$ of a none-empty set X is called a minimal structure (briefly, m -structure) on X if $\phi \in m_X$ and $X \in m_X$ .In this case (X,m_X) is called m -space.

Each member of m_X is said to be m_X -open and the complement of an m_X - open set is said to be m_X -closed set and $c(m_X)$ the collection of all m_X -closed sets.

Definition 1.2[1][2].Let (X,m_X) be an m -space, for a subset A of X , the m_X - closure of A and the m_X -interior of A are defined as follows:

(i) $m_X\text{-}cl(A) = \bigcap \{F : A \subseteq F, F \in c(m_X)\}$

(ii) $m_X\text{-}\mathrm{int}(A) = \bigcup\{U : U \subseteq A, U \in m_X\}$

Note that $m_X\text{-}cl(A)$ is not necessarily m_X -closed, also $m_X\text{-}\mathrm{int}(A)$ is not necessarily m_X -open.

Lemma 1.3[1][2].Let (X,m_X) be an m -space, for a subset A of X , the following hold :

$$m_X - \mathrm{int}(A^c) = [m_X - cl(A)]^c \text{ and } m_X - cl(A^c) = [m_X - \mathrm{int}(A)]^c \qquad (i)$$

(ii) If $A \in c(m_X)$, then $m_X - cl(A) = A$ and if $A \in m_X$, then $m_X - \mathrm{int}(A) = A$

(iii) $m_X - cl(\phi) = \phi$, $m_X - cl(X) = X$, $m_X - \mathrm{int}(\phi) = \phi$ and $m_X - \mathrm{int}(X) = X$

(iv) If $A \subseteq B$, then $m_X - cl(A) \subseteq m_X - cl(B)$ and $m_X - \mathrm{int}(A) \subseteq m_X - \mathrm{int}(B)$

(v) $A \subseteq m_X - cl(A)$ and $m_X - \mathrm{int}(A) \subseteq A$

(vi) $m_X - cl(m_X - cl(A)) = m_X - cl(A)$ and $m_X - \mathrm{int}(m_X - \mathrm{int}(A)) = m_X - \mathrm{int}(A)$.

Definition 1.4[1][2]. An m-structure m_X on an non-empty set X is said to have property (β) if the union of any family of subsets belonging to m_X belonging to m_X.

Lemma 1.5[1].For an m-structure m_X on a non-empty set X , the following are equivalent:
(i) m_X has property (β) .
(ii) If $m_X - \mathrm{int}(V) = V$, then $V \in m_X$.
(iii) If $m_X - cl(F) = F$, then $F \in c(m_X)$.

Lemma 1.6[1][2]. Let (X, m_X) be an m-space with property (β).For a subset A of X, the following properties hold:
(i) $A \in m_X$ *iff* $m_X - \mathrm{int}(A) = A$.
(ii) $A \in c(m_X)$ *iff* $m_X - cl(A) = A$.
(iii) $m_X - \mathrm{int}(A) \in m_X$,and $m_X - cl(A) \in c(m_X)$.

Definition 1.7[3]. Two sets A, B in an m-space (X, m_X) are said to be weakly separated if there are two m_X-open sets U, V such that $A \subseteq U, B \subseteq V$ and $A \cap V = B \cap U = \phi$.

2. $g(g^*)m$ closed sets

Definition 2.1. In an m-space(X, m_X), a subset A is gm_X – closed if $m_X - cl(A) \subseteq O$, whenever $A \subseteq O$ and O is m_X – open.

Definition 2.2. In a minimal space(X, m_X), a subset A is g^*m_X – closed if there is an m_X – closed set F containing A such that $F \subseteq O$, whenever $A \subseteq O$,and O is m_X – open.

Remark 2.3.Every m_X – closed set is g^*m_X – closed and every g^*m_X – closed is gm_X – closed i.e. m_X – closed$\Rightarrow$ g^*m_X – closed$\Rightarrow$ gm_X – closed.
The converse is not true in general as shown by the following examples:

Example 2.4.Let $X=\{a,b,c\}, m_X=\{\phi, X,\{a\},\{b\}\}$ and $A=\{a,b\}$. A is g^*m_X – closed since X is the only m_X – open and m_X – closed set which contain A , but A is not m_X – closed set.

Example 2.5.Let $X=\{a,b,c,d,e,f\}, m_X=\{\phi, X\{a,b,c,e\},\{b,c,d,f\},\{a,d,f\},\{a,d,e\}\}$ and $A=\{b,c\}$. $m_X-cl(A)=A$, so one can easily check that A is gm_X – closed, but A is not g^*m_X – closed since $\{b,c,d,f\},\{a,b,c,e\}$ are m_X – open containing A , but there is no m_X – closed set F , such that $A\subseteq F, F\subseteq\{a,b,c,e\}$ and $F\subseteq\{b,c,d,f\}$.

The proof of the following theorem is similar to the proof of theorem 3 [3], so is omitted.

Theorem 2.6. Let (X,m_X) be an m - space, and $A\subseteq X$. A is g^*m_X – closed ***iff*** there is an m_X – closed set F containing A such that $F-A$ dose not contain any non-empty m_X – closed set.

The following theorem and example show that theorem 2.6 does not hold if we replace g^*m_X – closed set by gm_X – closed set.

Theorem 2.7. Let (X,m_X) be an m - space, and $A\subseteq X$. If A is a gm_X – closed set, then $m_X-cl(A)-A$ does not contain any non-empty m_X – closed set.

Proof: Let F be a m_X – closed set and $F\subseteq m_X-cl(A)-A$, then $F\subseteq m_X-cl(A)$ and $F\cap A=\phi$ --------------→ (1)

Since F^c is m_X – open, $A\subseteq F^c$ and A is gm_X – closed, hence $m_X-cl(A)\subseteq F^c$, thus $F\subseteq[m_X-cl(A)]^c$ --------------→ (2)

From (1) and (2) we have that $F\subseteq m_X-cl(A)\cap[m_X-cl(A)]^c=\phi$, therefore $F=\phi$.

Example 2.8.Let $X=\{a,b,c,d,e\}$, $m_X=\{\phi, X,\{a\},\{b\}\}$ and $A=\{a\}$. $m_X-cl(A)=\{a,c,d,e\}$. Clearly A is not gm_X – closed, but ϕ is the only m_X – closed set contained in $m_X-cl(A)-A$.

The following theorem shows that the converse of theorem 2.7 holds under additional supposition.

Theorem 2.9. Let (X,m_X) be an m - space with property (β) and $A\subseteq X$. A is a gm_X – closed set ***iff*** $m_X-cl(A)-$ does not contain any non-empty m_X –closed set .

Proof: The necessity condition is theorem 2.7.

Sufficiency. Suppose that $A \subseteq U$ $(U \in m_X)$ and $m_X - cl(A) \not\subset U$. $m_X - cl(A) \cap U^c \neq \phi$, but $m_X - cl(A)$ by lemma (1.6, iii) is m_X −closed and U^c is also m_X −closed, thus by property (β) $m_X - cl(A) \cap U^c$ is m_X −closed, which is a contradiction. Thus $m_X - cl(A) \subseteq U$.

Corollary 2.10. Let (X, m_X) be an m - space with property (β) and A is a gm_X −closed set. $m_X - cl(A) - A$ is m_X −closed *iff* A is m_X −closed.
Proof : The result follows from lemma (1.6,ii , iii) and theorem 2.9 .

Remark 2.11. Clearly if $A = m_X - cl(A)$, then A is gm_X −closed, but the converse is not true as we shown by the following example.

Example 2.12.Let $X = \{a,b,c,d\}, m_X = \{\phi, X, \{a\}, \{d\}\}$ and $A = \{a,b\}$. Clearly $A \neq m_X - cl(A)$ and A is gm_X − closed.

We show in example 2.4 that $g^* m_X$ −closed set is not necessarily m_X −closed. In the following theorem we give equivalence between $g^* m_X$ −closed set and m_X −closed set under additional supposition.

Theorem 2.13.Let (X, m_X) be an m - space, and $A \in m_X$, then:
A is $g^* m_X$ −closed *iff* A is m_X −closed .
The following example shows that theorem 2.13 is not true if wereplace $g^* m_X$ −closed set by gm_X −closed set.

Example 2.14. Let $X = \{a,b,c,d\}, m_X = \{\phi, X, \{a,b\}, \{c\}, \{d\}\}$ and $A = \{a,b\} \in m_X$. Since $A = m_X - cl(A)$, thus A is gm_X −closed, but A is not m_X −closed.

The following theorem follows directly from definitions 2.1, 2.2.

Theorem 2.15. Let (X, m_X) be an m - space, and $A \subseteq X$.
(i) A is $g^* m_X$-closed *iff* there is an m_X-closed set F containing A such that $F \subseteq \ker(A)$, where $\ker(A) = \cap \{U : A \subseteq U, U \in m_X\}$.
(ii) A is gm_X-closed *iff* $m_X - cl(A) \subseteq \ker(A)$.

Remark 2.16. In a topological space the union of two g-closed sets is also g-closed [4][5] , this statement is also true in Alexandroff space [3]. But this is not true for gm_X-closed sets and also for $g^* m_X$-closed sets as shown by the following examples.

Example 2.17. Let $X = \{a,b,c,d,e,f\}$,
$m_X = \{\phi, X, \{a,b,c,e\}, \{b,c,d,f\}, \{a,d,f\}, \{a,d,e\}\}$ and
$A = \{b,c\}, B = \{d,f\}$. $m_X - cl(A) = A$ and $m_X - cl(B) = B$. One can easily check that A, B are both gm_X-closed sets. But $A \cup B = \{b,c,d,f\}$ is not gm_X-closed since $m_X - cl(A \cup B) = X$, and $A \cup B$ is m_X-open.

Example 2.18. Let $X = \{a,b,c,d,e,f\}$, $m_X = \{\phi, X, \{a,b,c\}, \{c,d,e,f\}, \{a,b,d,e,f\}\}$ and $A = \{a\}$, $B = \{c\}$. One can easily check that A, B are both $g^* m_X$-closed sets. But $A \cup B = \{a,c\}$ is not $g^* m_X$-closed set, since X is the only m_X-closed set which contains $A \cup B$ and $\{a,b,c\}$ is m_X-open containing $A \cup B$.

Remark 2.19. The intersection of two $g(g^*)m_X$ –closed sets is not necessarily $g(g^*)m_X$ –closed set as can be seen from example 2.5 [4].

3. Additional properties

Theorem 3.1. Let (X, m_X) be an m- space, for each $x \in X$, $\{x\}$ is m_X -closed set or $\{x\}^c$ is $g^*(g)$ m_X-closed.

Theorem 3.2. Let (X, m_X) be an m- space, and A, B be subsets of X, such that $A \subseteq B \subseteq m_X - cl(A)$, then:

(i) If A is $g^* m_X$- closed, then B is $g^* m_X$- closed.

(ii) If A is gm_X- closed, then B is gm_X- closed.

Proof:

(i) Let $O \in m_X$ and $B \subseteq O$, then $A \subseteq O$, thus there is m_X- closed set F such that $A \subseteq F \subseteq O$ (since A is $g^* m_X$- closed) but $m_X - cl(A) \subseteq F$, hence $B \subseteq m_X - cl(A) \subseteq F \subseteq O$, therefore B is $g^* m_X$- closed.

(ii) The proof is similar to part (i).

The following corollary is an immediately conclusion of theorem (3.1, ii).

Corollary 3.3. Let (X, m_X) be an m-space, and $A \subseteq X$. If A is gm_X- closed, then $m_X - cl(A)$ is g $- m_X$- closed.

Definition 3.4. Let (X, m_X) be an m-space and $A \subseteq X$. The collection $m_{XA} = \{O \cap A : O \in m_X\}$ is a m-structure for A called the relative m-structure for A. The pair (A, m_{XA}) is called m-subspace of (X, m_X) (briefly, m_{XA}-subspace).

Lemma 3.5[1]. Let (X, m_X) be an m-space and $A \subseteq X$, then $x \in m_X - cl(A)$ *iff* $U \cap A \neq \phi$, for each $U \in m_X$, $x \in U$.

Lemma 3.6. Let (X,m_X) be an m-space, (Y,m_{XY}) be an m-subspace of (X,m_X), and $A\subseteq Y\subseteq X$, then $\boldsymbol{m_{XY}-cl(A)=m_X-cl(A)\cap Y}$ (where $m_{XY}-cl(A)$ is the m-closure of A with respect to Y).
Proof: Clearly $\boldsymbol{m_X-cl(A)\cap Y\subseteq m_{XY}-cl(A)}$.
Suppose $x\in m_{XY}-cl(A)$. Now let $U\in m_X$, $x\in U$, then $U\cap Y\in m_{XY}$, thus $(U\cap Y)\cap A\neq\phi$ (lemma 3.5), which implies $U\cap A\neq\phi$, hence $x\in m_X-cl(A)\cap Y$.
This completes the proof.

Das and Rashid [3] proved that in Alexandroff space X if A is a subset of an open g^*-closed subset B of X, then A is g^*-closed relative to B *iff* A is g^*-closed in X.
The following theorem shows that the condition that B is open and g^*-closed subset of X are superfluous in the side when A is g^*-closed in X, also, another condition is given to make the other side hold.

Theorem 3.7. Let (X,m_X) be an m-space, and $A\subseteq B\subseteq X$, then :
(i) If A is g^*m_X- closed, then A is g^*m_{XB}- closed.
(ii) If A is gm_X- closed, then A is gm_{XB}- closed.
If (X,m_X) has property (β), then :
(iii) If A is g^*m_{XB}- closed and B is g^*m_X- closed, then A is g^*m_X- closed.
(iv) If A is gm_{XB}- closed and B is gm_X- closed, then A is gm_X- closed.
Proof: (i) Since A is g^*m_X- closed, then there is an m_X- closed set F, witnessing g^*m_X- closeness of A. Now let $U\in m_{XB}$ and $A\subseteq U$ (where $U=O\cap B, O\in m_X$), then $A\subseteq O$, hence $F\subseteq O$, that is $F\cap B\subseteq O\cap B(=U)$. Therefore A is g^*m_{XB}- closed.
(ii) Let $U\in m_{XB}$ and $A\subseteq U$ (where $U=O\cap B, O\in m_X$). Since A is gm_X- closed, and $A\subseteq O$, then $m_X-cl(A)\subseteq O$, thus $m_X-cl(A)\cap B\subseteq O\cap B(=U)$. Therefore by lemma 3.6 $m_{XB}-cl(A)\subseteq O$.
(iii) Since A is g^*m_{XB}- closed, then there is an m_{XB}- closed set $F'=F\cap B$, witnessing g^*m_{XB}- closeness of A (where $F\in c(m_X)$), also, since B g^*m_X- closed, then there is an m_X- closed set K, witnessing g^*m_X- closeness of B. Let $A\subseteq O$, $O\in m_X$, then $A\subseteq O\cap B(\in m_{XB})$, hence $F'\subseteq O\cap B$. Simple verification show that $B\subseteq O\cup F^c$ and by property (β) $O\cup F^c\in m_X$, hence $K\subseteq O\cup F^c$, thus $A=A\cap F\subseteq K\cap F\subseteq(O\cup F^c)\cap F$, then we have $A\subseteq K\cap F\subseteq O\cap F\subseteq O$, but $F\cap K$ is m_X- closed (by property (β)), therefore A is g^*m_X- closed.
(iv) Let $O\in m_X$ and $A\subseteq O$, then $\boldsymbol{A\subseteq O\cap B(\in m_{XB})}$, hence $m_{XB}-cl(A)\subseteq O\cap B$, then $\boldsymbol{m_X-cl(A)\cap B\subseteq O\cap B\subseteq O}$ (lemma 3.6), but $B\subseteq O\cup[m_X-cl(A)]^c$

and $m_X - cl(A)$ is m_X - closed (lemma 1.6,iii)and by property (β) $O \cup [m_X - cl(A)]^c \in m_X$, hence $m_X - cl(B) \subseteq O \cup [m_X - cl(A)]^c$, thus $m_X - cl(A) \subseteq O \cup [m_X - cl(A)]^c$, therefore $m_X - cl(A) \subseteq O$. This shows that A is gm_X - closed.

Corollary 3.8. Let (X, m_X) be an m -space with property (β). If A is $g(g^*)\, m_X$ - closed and B is m_X - closed, then $A \cap B$ is $g(g^*)\, m_X$ - closed.

Levine proved that in a topological space (X, τ) every subset of X is g - closed *iff* $\tau = c(\tau)$ (where $c(\tau)$ is the collection of all closed sets in (X, τ) (Theorem 2.10 [4]), and Das and Rashid proved that in Alexandroff space X , if every subset of X is g^*-closed , then $\tau = c(\tau)$, and shows by example 2.[3] that the converse is not true .
The following theorem illustrate that when we replace Alexandroff space by minimal space the theorem is also true.

Theorem 3.9. Let (X, m_X) be an m -space. If every subsets of X is $g^* m_X$ -closed, then $m_X = c(m_X)$.
Proof: Using theorem 2.13.
The following example shows that the converse of theorem 3.9 is not true (example 2. [3]).

Example 3.10.Let $X =$ R-Q and $m_X = \{\phi, X, G_i, A_i\}$, where Q is the set of all rational numbers , G_i and A_i runs over all countable and co-countable subsets of X , respectively . Clearly, $m_X = c(m_X)$. The set $B = X \cap (0, +\infty)$ is not an m_X - closed and since $\ker(B) = B$, hence B is not a $g^* m_X$ - closed.

However, Das and Rashid proved that in Alexandroff space (X, τ) , with $\tau = c(\tau)$, every subset of X is g^*-closed *iff* X is a topological space (theorem 9 [3]). The same argument is also satisfied in the minimal structure space as we show in the following theorem.

Theorem 3.11. Let (X, m_X) be an m -space with $m_X = c(m_X)$. Every subset of X is $g^* m_X$ - closed *iff* X is a topological space (Alexandroff space).
Proof: Necessity. Suppose every subset of X is $g^* m_X$ - closed.
(a) Let $\{F_\alpha\}_{\in \Lambda}$ be an arbitrary collection of m_X - closed sets and $F = \cap F_\alpha$. By assumption F is $g^* m_X$ - closed, then there is an m_X - closed set F' such that $F \subseteq F' \subseteq \ker(F)$ (Theorem 2.15, i). But $F_\alpha \in c(m_X) = m_X$ and, then $F \subseteq F_\alpha$, $F' \subseteq \ker(F) \subseteq \cap F_\alpha (= F)$, i.e. $F = F'$, thus F is m_X - closed .

(b) Let $\{K_i\}_{i=1}^{n}$ be an arbitrary finite collection of m_X - closed sets and $K = \bigcup_{i=1}^{n} K_i$.
By assumption K is g^*m_X - closed , then there is an m_X - closed K' such that $K \subseteq K' \subseteq \ker(K)$ (theorem 2.15)and then $K \subseteq K' \subseteq \ker(K) \subseteq \bigcup_{i=1}^{n} K_i (= K)$,thus $K = K'$, i.e. K is m_X - closed, therefore m_X is a topology (Alexandroff space)on X (theorem 3.4 [6]).

4. $g(g^*)m$- open

In this section we introduce the concept of gm_X - open and g^*m_X - open and we study some of their properties and characterizations of them are given.
Definition 4.1. In a minimal space (X, m_X), a subset A is called gm_X - open *iff* A^c is gm_X - closed.

Definition 4.2. In a minimal structure space (X, m_X), a subset A is called g^*m_X - open *iff* A^c is g^*m_X - closed.

Remark 4.3. Every m_X - open is g^*m_X - open and every g^*m_X - open is gm_X - open, i.e. m_X - open $\Rightarrow g^*m_X$ - open $\Rightarrow gm_X$ - open.
Using examples 2.4 and 2.5, we see that the converses is not true in general.

Theorem 4.4. Let (X, m_X) be an m -space and $A \subseteq X$, then:
(i) A is gm_X -open *iff* $F \subseteq m_X - \text{int}(A)$,whenever $F \in c(m_X)$ and $F \subseteq A$.
(ii) A is g^*m_X - open *iff* there is an m_X - open O , such that $F \subseteq O \subseteq A$,whenever $F \in c(m_X)$ and $F \subseteq A$.
Proof :(i) Necessity. Suppose A is gm_X -open .Let $F \in c(m_X)$ and $F \subseteq A$, then $A^c \subseteq F^c$ and $F^c \in m_X$, since A is gm_X -open , then A^c is gm_X -closed and $m_X - cl(A^c) \subseteq F^c$,thus $F \subseteq [m_X - cl(A^c)]^c$ but $m_X - cl(A^c) = [m_X - \text{int}(A)]^c$ (lemma 1.3,i),therefore $F \subseteq m_X - \text{int}(A)$.
Sufficiency. To prove A is gm_X -open it is enough to show that A^c is gm_X closed. Let $O \in m_X$ and $A^c \subseteq O$, then $O^c \in c(m_X)$ and $O^c \subseteq A$, then by assumption $O^c \subseteq m_X - \text{int}(A)$, thus $[m_X - \text{int}(A)]^c \subseteq O$, then by lemma (1.3,ii) $m_X - cl(A^c) \subseteq O$,which implies that A^c is gm_X - closed. therefore A is gm_X - open.
(ii) Necessity .Suppose A is g^*m_X -open, let $F \subseteq c(X)$ and $F \subseteq A$, then $F^c \in m_X$, A^c is gm_X -closed and $A^c \subseteq F^c$, hence there is gm_X -closed set K such that $A^c \subseteq K \subseteq F^c$, and then, $F \subseteq K^c \subseteq A$, $k^c \in m_X$.

Sufficiency .To prove that A is g^*m_X-open it is enough to show that A^c is g^*m_X closed .Let $O \subseteq m_X$,and $A^c \subseteq O$ then $O^c \in m_X$, $O^c \subseteq A$ and there is a m_X – open set, such that $O^c \subseteq U \subseteq A$, and then $A^c \subseteq U^c \subseteq O$, which implies that A^c is g^*m_X closed ,therefore A is g^*m_X-open.

Levine proved in a topological space a subset A is g – closed *iff* $A^c \cup \mathbf{int}(A) \subseteq U$ and U is open, implies $U = X$ (theorem 4.2. [4]) .Das and Rashid in Alexandroff space proved that A is g^* – closed *iff* there is an open set O , $O \subseteq A$ such that $A^c \cup O \subseteq U$ and U is open, implies $U = X$ .This equivalent is not true in general in minimal space as we shown by the following theorem and example.

Theorem 4.5. Let (X, m_X) be an m – space and $A \subseteq X$.

(i) If A is gm_X-open, then for every $V \in m_X$, such that $m_X - \mathbf{int}(A) \cup A^c \subseteq V$, implies $V = X$.

(ii) If A is g^*m_X - open, then there is an m_X -open set O , such that $O \subseteq A$, $O \cup A^c \subseteq U$ and $U \in m_X$, implies $U = X$.

Proof:

(i) Since $m_X - \mathbf{int}(A) \cup A^c \subseteq V$, then $A^c \subseteq V$, that is $V^c \subseteq A$, but A is gm_X - open and $V^c \in c(m_X)$, then $V^c \subseteq m_X - \mathbf{int}(A)$ (theorem 4.4, i), hence $[m_X - \mathbf{int}(A)]^c \subseteq V$ .Therefore $[m_X - \mathrm{int}(A)]^c \cup m_X - \mathrm{int}(A) \subseteq V$, which implies that $V = X$.

(ii) The proof is parallel to theorem 13 [4] and so is omitted.

Example 4.6. Let $X = \{a,b,c,d,e\}$, $m_X = \{\phi, X, \{a\}, \{b\}, \{b,d\}, \{a,c\}\}$ and $A = \{a,b,d,e\}$. $m_X - \mathbf{int}(A) = \{a,b,d\}$, simple verification show that A is not gm_X - closed, but X is the only m_X -open containing $m_X - \mathbf{int}(A) \cup A^c$. Using the same set A one can easily check that the converse of (4.5, ii) is also not true. However by adding the property (β) to the minimal space we have the following characterization of $g(g^*)\,m_X$ - open.

Theorem 4.7. Let (X, m_X) be an m – space with property (β) and $A \subseteq X$, then:

(i) A is gm_X-open *iff* if $m_X - \mathbf{int}(A) \cup A^c \subseteq V$, whenever $V \in m_X$, implies $V = X$.

(ii) A is g^*m_X - open *iff* there is an m_X -open set O , such that $O \subseteq A$, $O \cup A^c \subseteq U$ and $U \in m_X$, implies $U = X$.

The proof is omitted.

Levine proved that A is g-closed *iff* $cl(A)-A$ is g-open (theorem 14. [1]).Das and Rashid proved that in Alexandroff space, if $cl(A)$ is closed, then A is g^*-closed *iff* $cl(A)-A$ g^*-open. This equivalent is not true in general in minimal space as we show by the following theorem and example.

Theorem 4.8. Let (X,m_X) be an m-space and $A\subseteq X$, then:
(i) If A is gm_X-closed, then $m_X-cl(A)-A$ is gm_X-open.
(ii) If (X,m_X) with property (β), then the converse of part (i) is satisfy.
Proof: (i) the proof is immediate from theorems (2.7) and (4.4, i).
(ii) Let $O\in m_X$ and $A\subseteq O$, then by lemma (1.6 ,ii) $m_X-cl(A)$ is m_X-closed, and by property (β) $m_X-cl(A)\cap O^c$ is m_X-closed, but $m_X-cl(A)\cap O^c\subseteq m_X-cl(A)-A$, hence by theorem (4.4, i) $m_X-cl(A)\cap O^c\subseteq m_X-\mathrm{int}(m_X-cl(A)-A)$. Simple verification show that $m_X-\mathrm{int}(m_X-cl(A)-A)=\phi$, therefore $m_X-cl(A)\subseteq O$, which implies that A is gm_X-closed.

Example 4.9. Let $X=\{a,b,c,d,e,f,g\}$, $m_X=\{\phi,X,\{c,e\},\{f,g\},\{d,e,f\},\{e,f,g\}\}$ and $A=\{c\}$. One can easily check that $m_X-cl(A)=\{a,b,c\}$ and A is not gm_X-closed. $m_X-cl(A)-A=\{a,b\}$ is not m_X-open and ϕ is the only m_X-open and m_X-closed in $\{a,b\}$ which implies that $m_X-cl(A)$ is gm_X-open.
As a consequence of theorem 4.8, we have also the following corollary.

Corollary 4.10. Let (X,m_X) be an m-space and $A\subseteq X$, *if* A is g^*m_X-closed, then $m_X-cl(A)-A$ is gm_X-open.

Theorem.4.11.Let (X,m_X) be an m-space and $m_X-\mathrm{int}(A)\subseteq B\subseteq A\subseteq X$
(i) If A is g^*m_X-open, then B is g^*m_X-open.
(ii) If A is gm_X open, then B is gm_X open.
Proof :(i) let F be an m_X-closed set such that $F\subseteq B$, since A is gm_X-open and $F\subseteq A$, then there is an m_X-open set U such that $F\subseteq U\subseteq A$ (theorem 4.4, ii) B is g^*m_X-open.
(ii) Let K be an m_X-closed set such that $K\subseteq B$, since A is gm_X-open and $K\subseteq A$, then by theorem(4.4 , i) $K\subseteq m_X-\mathrm{int}(A)$,but $m_X-\mathrm{int}(A)\subseteq B$ and by lemma (1.3, iv, vi) $m_X-\mathrm{int}(A)\subseteq m_X-\mathrm{int}(B)$,thus $K\subseteq m_X-\mathrm{int}(B)$ which implies that B is gm_X- open.

Das and Rashid show that the union of two weakly separated g^*-open sets is g^*-open in Alexandroff space (theorem 15 [1]).

The following theorem shows that statement is also true in m-space with property (β) for this we introduce the following definition .

Definition 4.12.Two sets A,B in an m-space (X,m_X) are said to be weakly separated m-set if there are two m_X – open sets U,V such that $A\subseteq U, B\subseteq V$ and $A\cap V = B\cap U=\phi$.

Theorem 4.13. Let (X,m_X) be an m-space, with property (β) , then :
(i) The union of two weakly separated gm_X-open is gm_X-open.
(ii) The union of two weakly separated g^*m_X-open is g^*m_X-open.
Proof: (i) Let A_1 and A_2 be two weakly separated gm_X-open sets. Since A_1 and A_2 are two weakly separated, there are m_X – open sets U_1,U_2 such that $A_1\subseteq U_1, A_2\subseteq U_2$, $A_1\cap U_2 = A_2\cap U_1=\phi$. Let $F_i=U_i^c$, $(i=1,2)$, then $A_1\subseteq F_2$ and $A_2\subseteq F_1$ .Let F be an m_X – closed set such that $F\subseteq A_1\cup A_2$, simple verification shows that $F\subseteq (F\cap F_1)\cup(F\cap F_2)$, where $F\cap F_i$ is m_X – closed $(i=1,2)$ (by property (β)).Since A_i is $g-m_X$ – open $(i=1,2)$, then by theorem (4.4,i) $(F\cap F_1)\subseteq m_X-\text{int}(A_2)$ and
$(F\cap F_2)\subseteq m_X-\text{int}(A_1)$, therefore we have the following formula:
$F\subseteq (F\cap F_1)\cup(F\cap F_2)\subseteq m_X-\text{int}(A_1)\cup m_X-\text{int}(A_2)\subseteq m_X-\text{int}(A_1\cup A_2)$, which implies that $A_1\cup A_2$ is gm_X-open.
By same way one can proof part (ii), so is omitted.

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Minimizing a Quadratic Completion Time Problem with Set-up Times on a Single Machine

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Key words: scheduling, single machine, set-up, branch and bound.

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Abstract

In this paper, we study the weighted sum of square completion time's problem with set-up times on a single machine. The same problem with no set-up times was first solved by Townsend [1], and was solved by other references like [2], [3] and [4]. We generalize the problem described by the above references, and solve it with set-up times, in other word with families. A set-up time is required between two distinct families, and is required before the job that is scheduled first. We solve the problem by a branch and bound algorithm, a new improved lower bound and a new heuristic method have been used in the branching tree. The paper also considers some new results which can be used to reduce the size of the branching tree, and thus curtailing the enumeration efforts at the branching stage.

الخلاصة

تناولنا في هذا البحث دراسة مسألة المجموع الوزني لمربعات أوقات الاتمام بوجود اوقات اعداد لماكنة واحدة (the weighted sum of square completion times problem with set-up times). المسألة نفسها و لكن بعدم وجود اوقات اعداد (with no set-up times) قد تم حلها من قبل [1] Townsend و ايضا المصادر [2], [3] و [4] تطرقت لحل نفس المسأله المحلوله من قبل [1] Townsend، لقد قمنا بتعميم المسأله الموصوفه في المصادر اعلاه و قد تم حلها و لكن بوجود اوقات اعداد للماكنة (with set-ups) أى بوجود مجموعات(families)، وقت الاعداد ضروري بين مجموعتين مختلفتين و أيضا وقت الاعداد ضروري للعمل الذي يرتب أولا. قمنا بحل هذه المسأله بواسطة خوارزمية التفرع والتقيد (branch and bound)، و قد استعملنا قيد ادنى جديد و محسن (a new improved lower bound) و ايضا استعملنا طريقه تقريبيه جديده (a new heuristic method) في شجرة التفرع. البحث تطرق ايضا لبعض النتائج الجديده و التي استعملت لتقليل حجم شجرة التفرع و التي أدت لتقليل الجهود الحسابيه لمرحلة التفرع.

INTRODUCTION

This paper deals with a machine scheduling problem, which consists of n jobs and a single machine, the n jobs are divided into F families(f = 1, 2,..., F). A set- up time is necessary between two jobs of different families; also it is required before the job that is scheduled first. The objective is to find a processing order of the jobs, such that the weighted sum of square completion times $\sum w_i C_i^2$ is minimized. The same problem with no set-up times (with no families) was first solved by Townsend [1]. He used a branch and bound practice to solve it, Gupta and Sen [2] give some optimality conditions among the jobs of the same problem which described by [1], Szwarc et al. [3], Groce et al. [4] and Mondel and Sen [5] suggested additional precedence constraints among adjacent and non adjacent jobs of the problem. In this paper, we solve the quadratic completion time (QCT) problem with set up times, that is we generalize the problem described by the above references to a more complex one, we solve it by using a branch and bound algorithm, in section 2, we give some ordering rules on the jobs which can be used in the branching procedure. In section 3 we give some rules of composite jobs, in section 4, we describe a new modified and improved lower bound, section 5, describes our branch and bound algorithm, section 6 gives us some conclusions on the problem.

ORDERING CRITERIONS

In this section, we will give some rules to sort the jobs according to some criterions. Suppose that we have two jobs i and j, then for the QCT problem with no set-up times, Townsend [1] shows that if the following conditions are hold:

$$p_i / w_i \le p_j / w_j \quad and \quad ...(1)$$
$$w_i \ge w_j \quad ...(2)$$

Then i must precede j in an optimal schedule. We can generalize the above conditions to the QCT problem with set-up times as in the following theorem:

Theorem 1: For the QCT problem with set-up times if i and j are two distinct jobs, such that $p_i / w_i \le p_j / w_j$ and $w_i \ge w_j$, then i precedes j in an optimal schedule.

Proof: Consider the QCT problem with set-up times that is with families, then there are two possible states, the two jobs i and j are either from the same family or from separate families, if the two jobs are from the same family, then no set-up time will be required between them, thus the conditions (1) and (2) will remain the same as the two jobs are from distinct families, and then we observe that:

$$p_i / w_i \le p_j / w_j$$
$$\rightarrow \quad p_i w_j \le p_j w_i$$
$$\rightarrow \quad p_i w_j \le p_j w_i + s w_i$$
$$\rightarrow \quad p_i w_j \le (p_j + s) w_i$$
$$\rightarrow \quad p_i / w_i \le (p_j + s) / w_j \qquad (3)$$

Where s is the set-up time of the job j. This means that the conditions (3) and (2) are equivalent to that of the conditions (1) and (2), and for this reason i must precede j in an optimal schedule, and hence the theorem is proved. ■

Theorem 2: Let i and j be two jobs such that $(s_i + p_i)/w_i \le (s_j + p_j)/w_j$ and $w_i \ge w_j$, where s_i and s_j are the set-up times of the jobs i and j, which are either zero or positive values depending on the preceding jobs of i and j respectively, if they are from similar families or not, then i must precede j in an optimal schedule.

Proof: Consider the following two sequences $\sigma i \sigma' j$ and $\sigma j \sigma' i$. Now let T and T' be the two completion times of the two sequences σ and σ' respectively. Now the cost of the jobs i and j in the sequence $\sigma i \sigma' j$ is $w_i(T + s_i + p_i)^2 + w_j(T + s_i + p_i + T' + s_j + p_j)^2$, the cost of them in the sequence $\sigma j \sigma' i$ is $w_j(T + s_j + p_j)^2 + w_i(T + s_j + p_j + T' + s_i + p_i)^2$

The difference of the second cost from the first cost is:

$$\{-w_j(T' + s_j + p_j)^2 - 2w_j(T + s_i + p_i)(T' + s_j + p_j)\} +$$
$$\{w_i(T' + s_i + p_i)^2 + 2w_i(T + s_j + p_j)(T' + s_i + p_i)\}$$

Hence by using the conditions $(s_i + p_i)/w_i \le (s_j + p_j)/w_j$ and $w_i \ge w_j$, we deduce that the result of the difference will be a non positive value, which means that i must precede j in an optimal schedule, and as a result the theorem is proved. ■

The last theorem is true when the jobs are in adjacent or in nonadjacent positions, also if they are in adjacent positions and of the same family, then no set-up time will be required between them, so the conditions will be converted to $p_i / w_i \le p_j / w_j$ *and* $w_i \ge w_j$, which are identical to that of Townsend[1].

Now, for the QCT problem with no set-up times, Sen et al. [6] show that, job i will precede j if the following conditions are hold:

$$p_i \le p_j \quad \text{and} \ \ldots (4)$$
$$w_i \ge w_j \qquad \ldots (5)$$

We can generalize the conditions (4) and (5) for the QCT problem with set-up times as in the following theorem:

Theorem 3: Let i and j be two jobs such that $s_i + p_i \le s_j + p_j$ and $w_i \ge w_j$, where s_i and s_j are the set-up times of the jobs i and j, which are either zero or positive values depending on the preceding jobs of i and j respectively, if they are from similar families or not, then i must precede j in an optimal schedule.

Proof: This theorem can be proved straightly by using theorem 2 as follows: now since $s_i + p_i \le s_j + p_j$ and $w_i \ge w_j$, the last conditions can be converted to the following conditions $s_i + p_i \le s_j + p_j$ and $1/w_i \le 1/w_j$, now by multiplying the left hand side of the last conditions and the right hand side with each other we get that $(s_i + p_i)/w_i \le (s_j + p_j)/w_j$ and $w_i \ge w_j$, now by applying theorem 2 to the most recent conditions we deduce that i must precede j in an optimal schedule and hence the theorem. ■

Theorem 3 is true when the jobs are in adjacent or in nonadjacent positions, also if they are in adjacent positions and of the same family, then no set-up time will be required between them, so the conditions will be converted to $p_i \le p_j$ *and* $w_i \ge w_j$, which are identical to that of Sen et al.[6].

Also for the QCT problem Gupta and Sen [2] show that if $p_i w_i \ge p_j w_j$ and $p_i \le p_j$ then i should precede j in an optimal schedule, once more there result can be generalized to the QCT problem within set up times as in the following theorem:

Theorem 4: Suppose that i and j are two jobs such that $(s_i + p_i)w_i \ge (s_j + p_j)w_j$ and $s_i + p_i \le s_j + p_j$, where s_i and s_j are the set-up times of the jobs i and j, which are either zero or positive values depending on the preceding jobs of i and j respectively, if they are from distinct families or not, then i must precede j in an optimal schedule.

Proof: The proof can be deduced directly as follows: since $s_i + p_i \le s_j + p_j$, thus the second condition which is $(s_i + p_i)w_i \ge (s_j + p_j)w_j$ can not be reached unless that $w_i \ge w_j$, so we get the two conditions $s_i + p_i \le s_j + p_j$ and $w_i \ge w_j$ which are the same conditions described in theorem 3, so by using theorem 3 we deduce that if the conditions $(s_i + p_i)w_i \ge (s_j + p_j)w_j$ and $s_i + p_i \le s_j + p_j$, are satisfied, then i should precede j in an optimal schedule and hence the theorem is proved. ■

In theorem 4, if the two jobs i and j are in adjacent positions and from one family, then no set up will be required between them, and hence the conditions would be converted to $p_i w_i \ge p_j w_j$ and $p_i \le p_j$, which are the same of Gupta and Sen [2]

COMPOSITE JOBS

In the branching tree, if we have n jobs, then we will have n-1 levels. Now to form a composite job in any level of the n-1 levels, suppose that i is a sequenced job at the r^{th} level with a processing time p_i and weight w_i, suppose that j_k is a non sequenced job of the remaining n-r jobs $j_1, j_2 \dots j_{n-r}$. So we have two possibilities, either j_k from the same family of the job i or it is from a distinct family, now we can make use of theorem 1 as follows: first suppose that they are from the same family, now if the following conditions are hold: $p_{j_k}/w_{j_k} \le p_{j_l}/w_{j_l}$ and $w_{j_k} \ge w_{j_l}$ for all $l \in \{1,2,\dots,n-r\}/k$, then the job i and the job j_k will represent a composite job, secondly suppose that the two jobs i and j_k are from distinct families, so her a set-up time s_{j_k} of the job j_k will be required before the processing time p_{j_k}. Now if the following conditions are hold: $(s_{j_k}+p_{j_k})/w_{j_k} \le (s_{j_l}+p_{j_l})/w_{j_l}$ and $w_{j_k} \ge w_{j_l}$ for all $l \in \{1,2,\dots,n-r\}/k$ where s_{j_l} is either equal to zero or equal to a positive value depending on the job i, then i and j_k will represent a composite job.

LOWER BOUND

In this section we derive a new lower bound which based on a relaxation of set-ups. Before describing the lower bound, and according to theorem 1, we can rearrange the jobs of each family in non decreasing order of p_i/w_i. The computation of the lower bound consists of the following steps:

Step 1: Construct a sequence $\sigma = (\sigma(1), \sigma(2), \dots, \sigma(n))$, where the jobs of σ are arranged in non decreasing order of $(s_{\sigma(i)} + p_{\sigma(i)})/w_{\sigma(i)}$ where

$$s_{\sigma(i)} = \begin{cases} s_f & \text{if } \sigma(i) \text{ is the first job of family } f,\ f = 1,2,\dots,F \\ 0 & \text{otherwise.} \end{cases}$$

Step 2: Find the cost of the sequence $\sigma = (\sigma(1), \sigma(2), \dots, \sigma(n))$, in this sequence if we have two jobs i and j of the same family; such that $p_i/w_i > p_j/w_j$ then we notice that the maximum reduction in penalty that can occur through interchanging the order i j to j i is $\left(\frac{p_i}{w_i} - \frac{p_j}{w_j}\right) p_i p_j$. Use Townsend's method [1] to find the final value which represents a lower bound to the original problem.

In this lower bound for each family f, we use the set-up time s_f once and only once, which ensures to us that the lower bound is valid, Now to explain the lower bound we will give the following example: consider a problem with 6 jobs divided into 3 families $f_1 = \{1,2\}, f_2 = \{3,4\}$ and $f_3 = \{5,6\}$ the processing times and the weights of the jobs are: $p_1, p_2, \dots, p_6 = 2,3,4,2,3,2,$ $w_1, w_2, \dots, w_6 = 6,7,5,4,4,5.$ Also, the set-up times of the families f_1, f_2 and

f_3 are $s_{f_1}=3$, $s_{f_2}=2$ and $s_{f_3}=2$ respectively. Now before computing the lower bound we must rearrange the jobs of each family in non decreasing order of p_i/w_i, so the order of the jobs of each family will become $f_1=\{1,2\}$, $f_2=\{4,3\}$ and $f_3=\{6,5\}$. The lower bound can be computed by applying the above steps, from step 1 we get the sequence $\sigma=(\sigma(1),\sigma(2),...,\sigma(n))=(2,5,3,6,1,4)$ and finally by step 2, we get the cost 5875, which represents a lower bound to our problem.

The lower bound described above is a weak lower bound, since the contribution of the set-up times begins at the end of the sequence, while it must starts at the beginning of the sequence.

Now we can modify the above lower bound. The modified lower bound depends on some constructed sequences, first we construct a sequence $\beta=(\beta(1),\beta(2),...,\beta(F))$ where $\beta(i)$ is the first job of family f (f = 1, 2, ... ,F) and $(s_{\beta(1)}+p_{\beta(1)})/w_{\beta(1)} \le (s_{\beta(2)}+p_{\beta(2)})/w_{\beta(2)} \le ... \le (s_{\beta(F)}+p_{\beta(F)})/w_{\beta(F)}$. Secondly, construct a new sequence $\varphi=(\varphi(1),\varphi(2),...,\varphi(k))$, which consists of the remaining jobs (jobs which are not in the sequence β) where k = n - F and $p_{\varphi(1)}/w_{\varphi(1)} \le p_{\varphi(2)}/w_{\varphi(2)} \le ... \le p_{\varphi(k)}/w_{\varphi(k)}$.

Now to find the final value of the modified lower bound, we must apply the following steps:

Step 1: If $n_1=n_2=....=n_F$ where n_f is the number of jobs of family f (f = 1, 2, ..., F) then go to step 2, other wise arrange the n_f's in non increasing order, that is rearrange them as: $n_1 \ge n_2 \ge .... \ge n_F$.

Step 2: Let i = 1.

Step 3: Now choose $\beta(i)$ and the i^{th} n_i-1 job(s) of φ and arrange them in the i^{th} n_i^{th} positions.

Step 4: Set i=i+1,if i=F+1 then go to step 5, other wise go to step 3.

Step 5: A new sequence will now be constructed, in this sequence if we have two jobs i and j of the same family; such that $p_i/w_i > p_j/w_j$ then we notice that the maximum reduction in penalty that can occur through interchanging the order i j to j i is $\left(\frac{p_i}{w_i}-\frac{p_j}{w_j}\right)p_i p_j$ find its cost and apply Townsend's [1] method to find the final value which represents a lower bound to our problem.

In this lower bound the set-up time s_f is used at the beginning of each family f, which grantees that the new lower is a modification of the old one. Now to explain the modified lower bound, let us take the same example given above, now by applying the steps 1 through 5, we will get the sequence (6, 2, 1, 5, 4, 3), the last cost of that sequence is 6204 which represents a lower bound to our problem.

The modified lower bound can also be improved by using the composite jobs property. First for each family we can construct a batch, where a batch is a non empty set of jobs from the same family which can be sequenced contiguously. The batch of any family can be constructed by using the composite jobs property which described in the previous section. Let B_f denote the batch of the family f, and let m_f denote the number of the jobs in that batch B_f, where $m_f \le n_f$ (f = 1, 2 ,..., F). Construct a sequence β, where $\beta = (\beta(B_1), \beta(B_2),..., \beta(B_F))$, the elements of β are arranged in non decreasing order of the ratio P_{B_f} / W_{B_f} ,where $P_{B_f} = (s_f + \sum_{k=1}^{m_f} p_k)$ and $W_{B_f} = \sum_{k=1}^{m_f} w_k$, that is in the following form: $P_{\beta(B_1)} / W_{\beta(B_1)} \le P_{\beta(B_2)} / W_{\beta(B_2)} \le ... \le P_{\beta(B_F)} / W_{\beta(B_F)}$. For the remaining jobs (jobs which are not in the sequence β) rearrange them in non decreasing order of p_i / w_i, and hence we can construct a sequence φ, which contains the remaining jobs which are sequenced in non decreasing order of p_i / w_i. This lower bound can be found by using the following steps:

Step 1: If $n_1 = n_2 = ... = n_F$ where n_f is the number of jobs of family f (f = 1, 2, ..., F) then go to step 2, other wise arrange the n_f's in non increasing order, that is rearrange them as: $n_1 \ge n_2 \ge ... \ge n_F$.

Step 2: If the batches B_f (f = 1, 2, ..., F) consist of single jobs, then this lower bound will be equivalent to the lower bound described above, so we must terminate the improved lower bound, other wise go to the next step.

Step 3: i = 1.

Step 4: If $m_i = n_i$, then we will order the batch in the i^{th} position, other wise take a subsequence σ_i of the jobs of φ, the jobs of σ_i are lie in the positions $(\sum_{k=1}^{i-1} n_k)+1$ to the position $\sum_{k=1}^{i} n_k$, (where $\sum_{k=1}^{i-1} n_k = 0$; if i=1). Finally arrange the batch B_i and the subsequence σ_i in the i^{th} position respectively.

Step 5: i = i + 1, if i = F + 1, then go to step 6, other wise go to step 4.

Step 6: A new sequence will now be constructed, for this sequence; if we have two jobs i and j of the same family; such that $p_i / w_i > p_j / w_j$ then we notice that the maximum reduction in penalty that can occur through interchanging the order i j to j i is $\left(\frac{p_i}{w_i} - \frac{p_j}{w_j}\right) p_i p_j$. Find the cost of the last sequence and apply Townsend's[1] method to find the final value which represents a lower bound to our problem.

Now to explain the improved lower bound, let us take the same example given above, now by applying the steps 1 through 6, we will get the sequence (1, 2, 6, 5, 4, 3), the last cost of that sequence is 6279 which represents a lower bound to our problem, and it is a superior to the old one.

UPPER BOUND

Before describing the branch and bound procedure, we must use an upper bound at the root node of the branching tree, and this can be done by applying the following steps, first arrange the jobs of each family in non decreasing order of p_i / w_i, then rearrange the families according to non decreasing order of P_f / W_f, where $P_f = s_f + \sum_{i=1}^{n_f} p_i$ and $W_f = \sum_{i=1}^{n_f} w_i$, a new sequence now will be constructed, the cost of that sequence will represents an upper bound to our problem.

BRANCH AND BOUND ALGORITHM

The branch and bound method depends on the lower bound and the upper bound described in the preceding sections, some nodes of the branching tree can be eliminated by using theorem 1, 2 and 3. A node at r^{th} level will represent that there are r sequenced jobs and n-r un sequenced jobs, the cost of the r sequenced jobs is called an original cost while the cost of the n-r un sequenced jobs will represent the lower bound of the n-r jobs, hence the sum of the two costs will represent a lower bound to the n jobs. Branching continues from the node that has the least lower bound value if there is a tie we select arbitrarily.

We see that by using theorem 2, 3 or 4 on the above example, job 6 precedes the jobs 5, 4 and 3. Job 5 precedes job 4. And job 2 precedes the jobs 3, 4, 5 and 6, and hence we can reduce the number of nodes in the branching tree.

Figure 1 shows the complete branching tree for the example given in the previous section, by using the improved lower bound

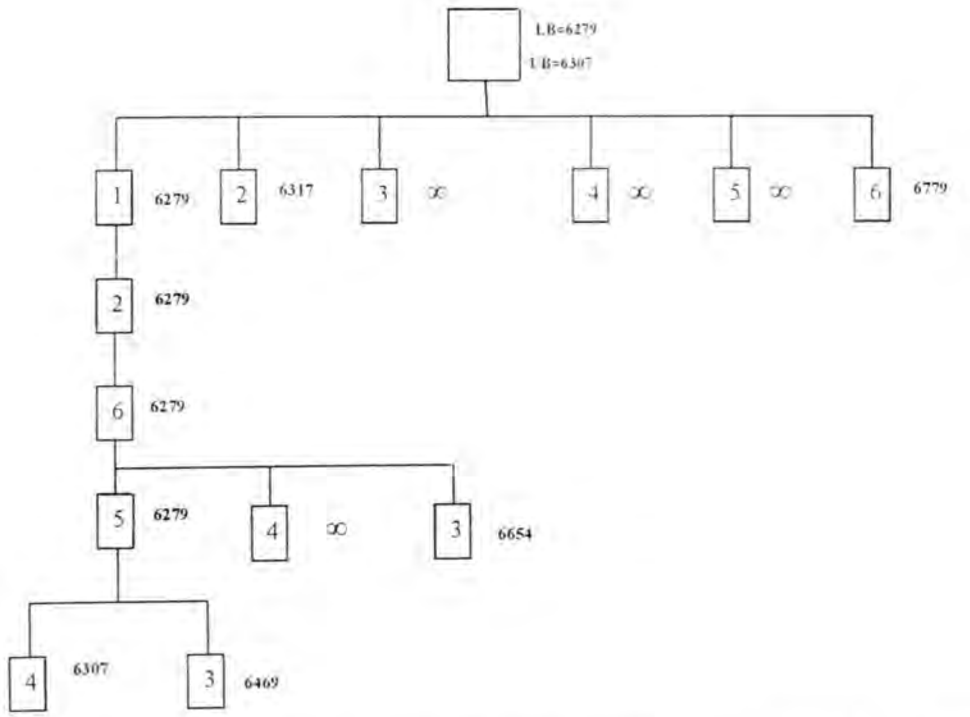

Figure 1: The branching tree by using the improved lower bound for the example.

The infinite nodes in Figure 1 denote according to theorem 2, 3 and 4; that these jobs have another jobs which preceding them. Now by using the composite jobs property the jobs 1, 2 and 6 will form a batch, and hence they must sequenced contiguously, that is the nodes 1, 2 and 6 of Figure 1, can form a compact node 1-2-6, and hence reducing the size of the branching tree and reducing our enumeration efforts for finding the optimal solution, thus Figure 1 can be graphed to a more compact one as shown in Figure 2.

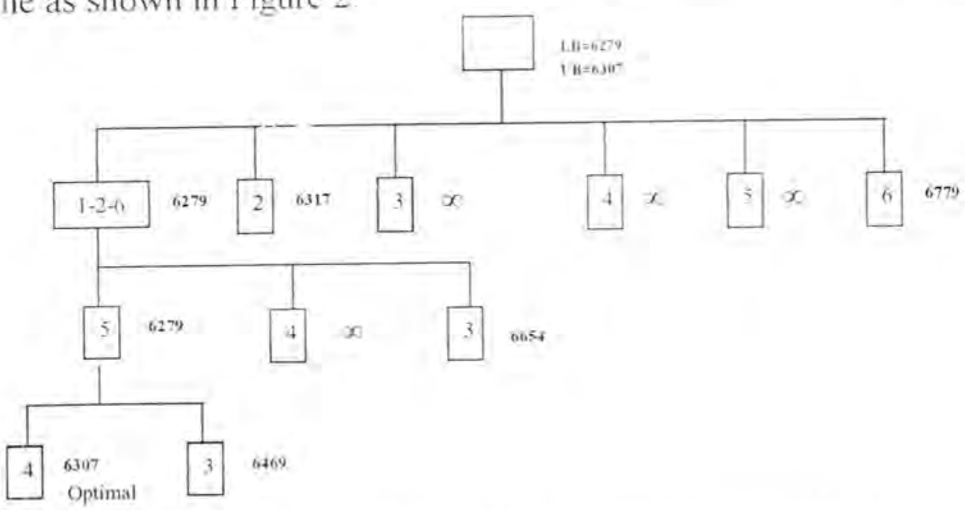

Figure 2: The branching tree in a more compact form.

CONCLUSIONS

In This paper we solve the quadratic completion time's problem, with set-up times, on a single machine, by using a branch and bound algorithm, this problem is a generalization to that of Townsend [1] and Croce et al. [4], and other references as mentioned above. A set-up time is required between two different job families and is required before the job that is scheduled first. The procedure depends on an improved lower bound and a new upper bound. Also we make use of some new results which provides links between adjacent and non adjacent jobs of the alike families or of distinct families, also we form batches between jobs from one family or from two distinct families and hence reducing the size of the branching tree, and hence reducing our efforts for finding the optimal solution. The next step is to find a solution to a more complex objective function which described in [1], [2] and in [6].

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Covert Channel in IP Packet Using DS Field

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KEYWORDS NDIS; TCP/IP; Covert Channel; OS field; Ethernet packet, Data Link Library (DLL); Datagram.

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List of Abbreviations

Abbreviation	The Idiom
DF	Don't Fragment
DLL	Data Link Library
DS	Differentiated Service
ID	Identification Number
IETF	Internet Engineering Task Force
IP	Internet Protocol
NDIS	Network Driver Interface Specification
NIC	Network Interface Card
OS	Operating System
OSI	Open System Interconnection
TCP/IP	Transmission Control Protocol/Internet Protocol
TOS	Type of Service

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ABSTRACT

Due to the wide spread use of computer networks in our life; the networks have used a universal protocol (TCP/IP) for communication, with increasing interests for security tools. In fact, little attention was paid to such traditional security aspect during the development of TCP/IP protocol. This work involves the manipulation of the Differentiated Service (DS) field in the IP packet header for implication of a covert channel.

The hidden data, which are carried by the DS field, are recognized at the receiver site using digital signature.

The work overcome the problems that faces the users of TCP/IP covert channels, these are mainly concentrated on the limitation of the message length and the recognizability of the hidden data by the intrusions.

The work is based on the development of the covert channel in the Microsoft Windows platform. Therefore, the work has deal with techniques that allow the interception of a subset of functions from the kernel mode. To do this a driver that hooks the Network Driver Interface Specification (NDIS) function is developed, thus manipulation of the IP header in the user mode becomes permitted.

Finally the extraction of the Ethernet packets is done by creation of Data Link Library (DLL).

الخلاصة

نظرا للانتشار الواسع في شبكات الحاسبات فقد استخدم بروتوكول عالمي (TCP/IP) في نظام الاتصالات الشبكي، وبذلك ازداد الاهتمام باستخدام الوسائل السرية لنقل المعلومات. في الحقيقة لم يعطى الاهتمام الكافي لنقل المعلومات بصوره سرية عبر شبكات الاتصال في أثناء إنشاء الـ TCP/ IP وبذلك أصبح ممكنا استغلال بعض الثغرات الموجودة في TCP / IP واستخدامها كقنوات إخفاء سريه لنقل المعلومات الحساسة بشكل لا يمكن ملاحظته. في هذا البحث تم معالجة حقل مفاضلة الخدمة بغية استخدامه كقناة سرية. البيانات المخفية في حقل مفاضلة الخدمة يتم تميزها عند جهة المستلم باستخدام علامة رقمية.

في هذا البحث تم تجاوز بعض المشاكل التي تواجه مستخدمي القنوات السرية في بروتوكول الـ TCP/IP والتي تتمثل بشكل رئيسي بتحديد طول البيانات السرية وكذلك إمكانية تمييز البيانات المخفية من قبل المعترضين.

كما و تم في هذا البحث الاعتماد على نظام تشغيل النوافذ في بناء القناة السرية، ولهذا تم التعامل مع التقنيات التي تسمح باعتراض جزء من مجموعة الدوال الموجودة في نواة نظام التشغيل (Kernel Mode) وبغية عمل ذلك تم إنشاء مشغل له القدرة على التحكم بدوال NDIS ، وبذلك أصبح بالإمكان معالجه ترويسة حزمة IP في جهة المستخدم لنظام التشغيل (User Mode).

وأخيرا فقد تم في هذا البحث تنفيذ قنوات الإخفاء عبر طريق برمجة مكتبة ارتباط البيانات المسؤولة عن دوال المستوى الأدنى مثل استخلاص حزمة IP ومعالجة حقل التشخيص وإعادة حساب فحص المجموع وإعادة الإرسال.

INTRODUCTION

Computer network are designed for communication, connectedness and collaboration; their specification is open for public, so it presents difficulties with respect to security. Therefore, it becomes necessary to address issues of protection, and in fact a flexible security for evolving network applications is required.

One of the sub disciplines of this broad concept is covert channels which is investigated and accordingly tied with security aspects of computer networks [1]. The concern for the presence of covert channels is common in high security systems such as military, banks, commerce, etc... where typically two observed users know that someone tries to listen to their conversations.

This work attempts to improve security in communication network employing IP header. It focuses on covert channels in computer networks for which data hiding takes place by making use of the network packet streams as the covert object. It first identifies covert channels in IP, and then suggest application scenario to enhance network security for current computer networks. It proposes a manipulation of the DS field in the IP packet header to implement the covert channel.

The network behavior for data hiding are taken into account in our work, since the packets traverse different network topologies and are shared at various network nodes before reaching their intended destination.

A. Covert Channels

A covert channel is a communication channel that allows two cooperating processes to transfer information in a manner that violates the system's security policy [2]. It also described as an information flow mechanism within a system resources not normally intended for communication between the users of the system [3]. It is thus a way of communication which is not part of the original design of the system, but can be used to transfer information to a process or user, who is not be authorized to access to that information [4].

Covert channels are in fact divided into two categories, storage channels (which are used in this work) and timing channels. In practice, when covert channel scenarios usage are constructed, a distinction between covert storage and timing channels is made, even though theoretically no fundamental distinction exists between them [5].

B. Datagram

Packets in the IP layer are called datagram's. It is a variable length that contains information which is essential to routing and delivery. Figure (1) shows the IP datagram header in 4-byte sections [6].

C. Differentiated Services (DS)

The Internet Engineering Task Force (IETF) has recently changed the interpretation and name of this 8-bit field. This field previously called Type of Service (TOS) and its interpretation was as follows: the first 3 bits are called precedence bits. The next 4 bits are called TOS bits and the last bit is not used. Now this field called DS and its interpretation is as follows: the first 6 bits make up the code point subfield and the last two bits are not used [7].

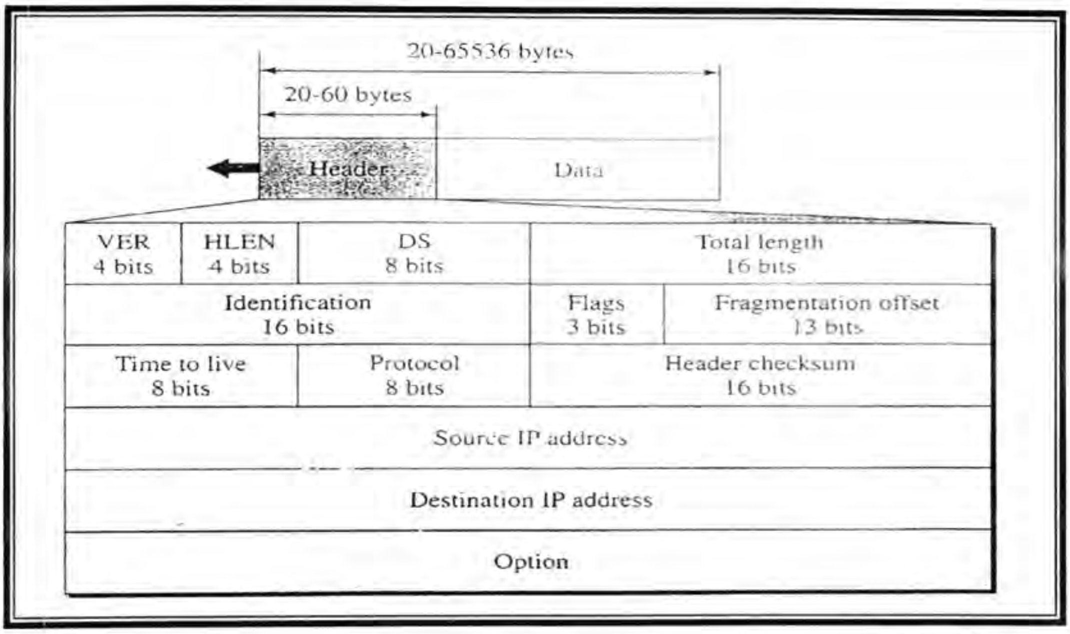

***Figure 1**: IP Datagram*

D. Network Driver Interface Specification (NDIS)

The (NDIS) is an interface between a protocol stack and network adapter card driver [8]. When a protocol driver intends to read or write messages formatted in its protocol's format from or to the network, the driver must do so using a network adapter [9]. Because the protocol drivers do not understand the nuances of every network adapter in the market (proprietary network adapters number in the thousands); the network adapter vendors then provide device drivers that can take network messages and transmit them via the vendors' proprietary hardware [10].

In 1989, Microsoft and 3Com jointly developed the (NDIS), which lets protocol drivers communicate with network adapter drivers in a device independent manner, so the NDIS device driver becomes related to the OSI model at the Data Link layer [11]. NDIS standardizes access to network card, so that the same software may be used to access any brand of network device [12]. In fact, it becomes a specification used when developing device driver for network card in all varieties of Windows [13].

FRAMEWORK

In all windows Operating Systems, the TCP/IP protocol is a proprietary, and its source code is not accessible, which means that the manipulation of the packet, in any of the TCP/IP protocol suite, is not possible from levels above the TCP/IP driver layer. To overcome this problem, a hooking technique can be used in order to control the packet at the point that links between the protocol driver and the Network Interface Card (NIC) card(s), which is represented by the NDIS. Therefore, NDIS hooking is developed during our work. From the operation point of view, the work in this paper is composed of two parts:

1) **Hooking the NDIS Technique:** which is composed of two parts as shown in Figure (2).

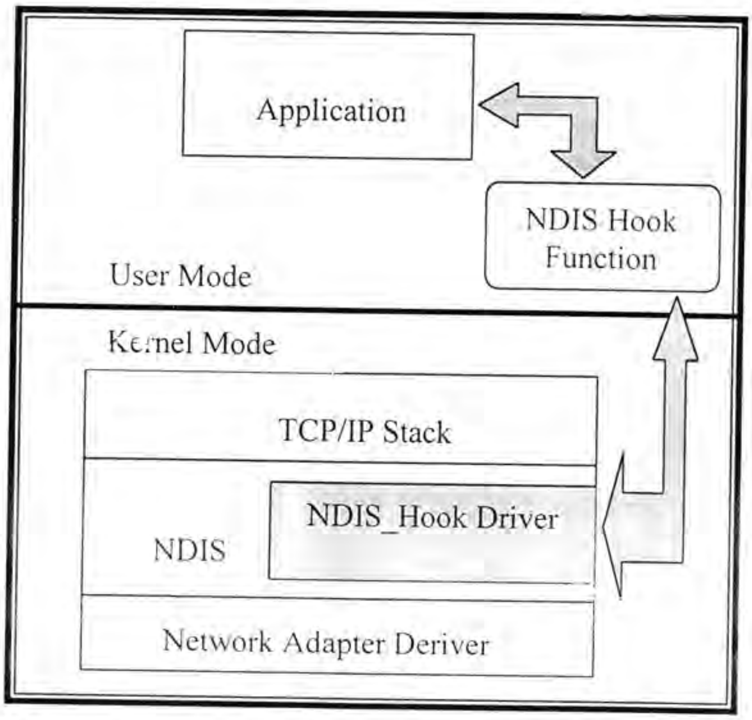

Figure 2: *NDIS-Hook Driver with Relation to User Mode.*

a) The NDIS-Hook Driver

The NDIS-Hook driver is logically similar to the NDIS Intermediate driver, but it is implemented differently. It inserts itself between TCP/IP and all of the adapters that bind with it. When TCP/IP sends a packet, the packet comes to the NDIS-Hook driver first. Likewise, packets that are to be indicated (received) on TCP/IP will go to the NDIS-Hook driver first.

The NDIS hook driver intercepts services that are exported by the windows NDIS wrapper at a point in the load sequence before NDIS protocols begin their binding process. These services are implemented in NDIS library. They deal with the TCP/IP stack and the NIC driver by sending/receiving a single or array of packets and other system events. Because the NDIS-Hook driver hooks exports services by the NDIS wrapper, it can intercept each protocol's call to NdisRegisterProtocol and replaces the protocol's services table with a table containing pointers to new functions within the NDIS-Hook driver. Later on, the NDIS-Hook driver intercepts each protocol's call to NdisOpenAdapter functions, so it can track the opened adapters.

At this step the new driver (hook driver) will be in full control of the original NDIS driver. Implementing the NDIS hook driver will indirectly enable the user mode application to control the original NDIS through I/O commands. Since the NDIS lies in the Data Link Layer, the new driver and thus the user mode hook application will deal with the packet as an Ethernet data form.

b) NDIS Hook Application

The second part of NDIS hook is a user mode application. It controls the hook driver, i.e. controls the packet traveling, through which the following functions are programmed in a Dynamic Link Library (DLL) file using visual C++.net language:

i. Loading the hook driver for the specified adapter.
ii. Controlling the original NDIS driver (by the hook driver), in order to be able to control the NDIS packet movement in the kernel mode as well as the packet movement from the kernel mode to user mode and return again to do the same procedure through a bobbin (reversible) process.
iii. Continuing the sending / receiving operation.

2) Implementing Data Hiding in IP Packet Header :

In this work we assume that two workstations A and B transfer information over a computer network, and employ data hiding involving the TCP/IP protocol suite to communicate covert supplementary information.

The most significant two bits in the DS field of the packet header are normally unused. These two bits represent random values that TCP/IP protocol do not use them. We consider, in this work, the use of these two bits as a covert channel that can transfer the proposed data across the network from the sender to receiver. When these two bits are used properly, the DS field will seem perfectly normal and the hidden data can not be detected by any network monitoring scheme.

For a better secured process, the paper suggest that two bits of data should be XOR with two bits of identification field before placing them in the DS field of the packet as an encryption.

DATA HIDING SCENARIO

A scenario is suggested between a sender A who tries to send a covert information to a receiver B as shown in Figure (3). The data hiding at the sender site is implemented through the algorithm that takes the followings as input:

1- The digital signature.
2- The length of covert information.
3- The desired covert information.
4- The network packet.

Host A will first send a digital signature to inform host B that it intends to send covert information. Next A sends the length of the covert information to B, followed the covert information it self.

The algorithm below shows the proposed scheme of the sender:

Sender Algorithm

Input: Reference sign, length of covert information, covert information and IP packet.

Output: IP packet containing two bits from the covert information.

Step 1: x = the first two least significant bits of the signature.
y = the first two bits of length of the covert information.
Signatureflag = lengthflag = lessthaninfolength = false.
i = j = k =0.

Step 2: Get the current sending packet
IF this packet is not destined to go to the specified receiver *THEN*
Go to step 7.

Step 3 : ***IF*** signatureflag = false ***THEN***
{
put x into the most significant bits of the DS field.
i = i +2
IF i < 16 ***THEN***
x = the next two bits of the signature.

```
        ELSE
          {
            signatureflag = true
            i = 0
          }
Step 4 : IF lengthflag = false and signatureflag = true THEN
        {
          put y into the most significant bits of the DS field.
          j = j +2
          IF j < 16 THEN
            y = the next two bits of the length.
          ELSE
            {
              lengthflag = true.
              J = 0.
            }
        }
Step 5: IF lessthaninfolength = false THEN
        {
          z = information [i].
          m = the first two bits of the z.
          lessthaninfolength = true.
          k = 0.
        }
        put z into the most significant bits of the DS field.
          k = k +2.
        IF k < 8 THEN
          m = the next two bits of z.
        ELSE
        If k < length then
          {
            k= k + 1.
            Lessthaninfolength = false.
          }
Step 6: Recalculate the IP checksum
Step 7: Go to step 2.
Step 8: End.
```

On the other hand, the algorithm of the receiver site will be run in the following sequence:

1- The receiver will always read the most significant bits of the DS field.
2- The receiver will read the length of the covert information if it receives a digital signature.

3- The information itself will be read after reading its length and the receiver will wait for another digital signature to arrive.

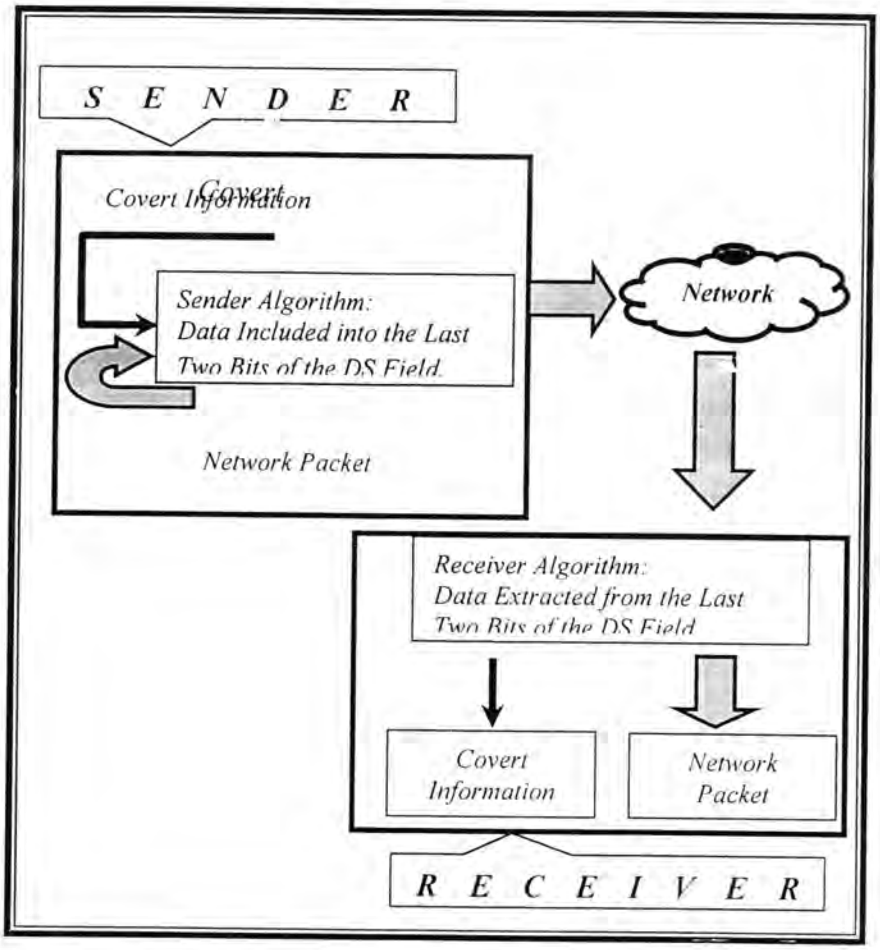

Figure 3: Schematic Diagram of the Data Hiding Scenario

Receiver Algorithm

Input: IP packet.
Output: The covert information extracting from the IP packet.

Step1: i= first two bits of the signature
M = 0
signitureflag , lengthflag , infoflag = false .

Step 2: Get packet.
Step 3: ***IF*** signitureflag = false ***THEN***
{
k = the two most significant bits of the DS field
IF i = k ***THEN***
i = the next two bits of the signature

```
        ELSE
            i = first two bits of the signature
        IF  i = the end of the signature THEN
            signature = true
        }

Step 4: IF lengthflag = false AND signature = true THEN
      {
        L = the two most significant bits of the DS field
        put l into buffer
        IF m = length THEN
          length = true
        ELSE
          m = m + L
      }
Step 5: IF infoflag = false AND lengthflag = true THEN
      {
        put the two most significant bits of the DS field into buffer
        j = j+2
        IF j = 8 THEN
          {
            put the buffer value into information[ y]
            y = y + 1
            j = 0
          }

      IF  y = length THEN
            infoflag = true
        }
Step 6: Go to step 2
Step 7: End
```

DISCUSSION

The data-hiding scenario in this work uses the most significant bits in DS field of the IP packet header which are not used before by the TCP/IP. To test the performance of this scenario it had been executed tens of times until successful implementation was reached.

Initially after executing the program and before sending any information, some improper text was appeared in a random way at the receiver site. This is because that the receiver site extracts the random text form the most significant bits of the DS field and considered it as a covert information as shown in Figure(4). This problem is solved by allowing the receiver to accept the hidden information when it receives a digital sign only.

Semi successful executions were early performed. The problem appeared again in one of the executions, when the value of the specific digital signature was coincidently equal to the improper text value. This is a very rare case, it is also overcame by sending a data which is different in value from the digital signature when no covert information is sent.

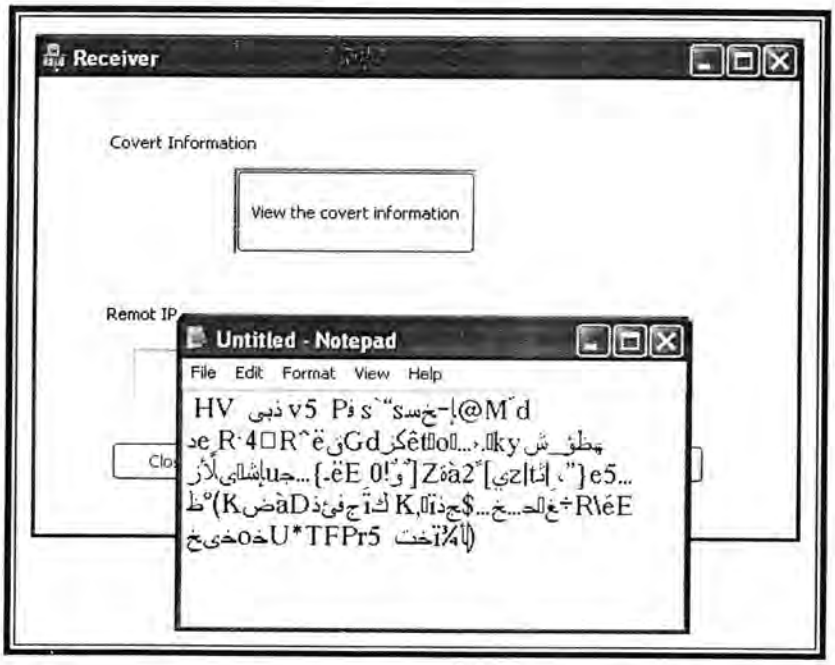

Figure 4: Improper Text Before The Use of a Reference Sign.

In some other executions, particularly during sending a large number of packets than the required ones for the covert information the problem of the improper text was also arisen, but after receiving the covert information.

The receiver received the proposed digital signature and starts accepting the covert information but with non stop procedure, it continued accepting non understandable data even after finishing all received covert information as shown in Figure (5). The problem was also solved by sending a length of information to the receiver soon after sending the digital signature, so that the receiver is to accept only the covert data and waits for a new digital signature. After treatment all of the above mentioned problems, no difficulties were experienced and all of the executions were performed confidently.

The present work uses reliably for the first time the most significant bits of the DS field for transmission of covert information. These two bits are unused and have no role in sending any information, besides that there uses do not interfere with any value of the other fields in the IP header.

Although there are no other workers that used these two bits to compare their results with that observed in the present work, nevertheless, the transmission of covert information can be compared with the results of kamran who used the Don't Fragment (DF) bit [14]. Unlike the present work, inspectors can notice peculiar value of the DF bit due to the difference of the identification field in packets stream.

Further more, the length of covert information is unlimited in the present work. The larger the length of covert information, the larger the number of packets that is required for transmission. Unlike other workers who used the ID field [15], they were only able to transmit only 256 characters per a message.

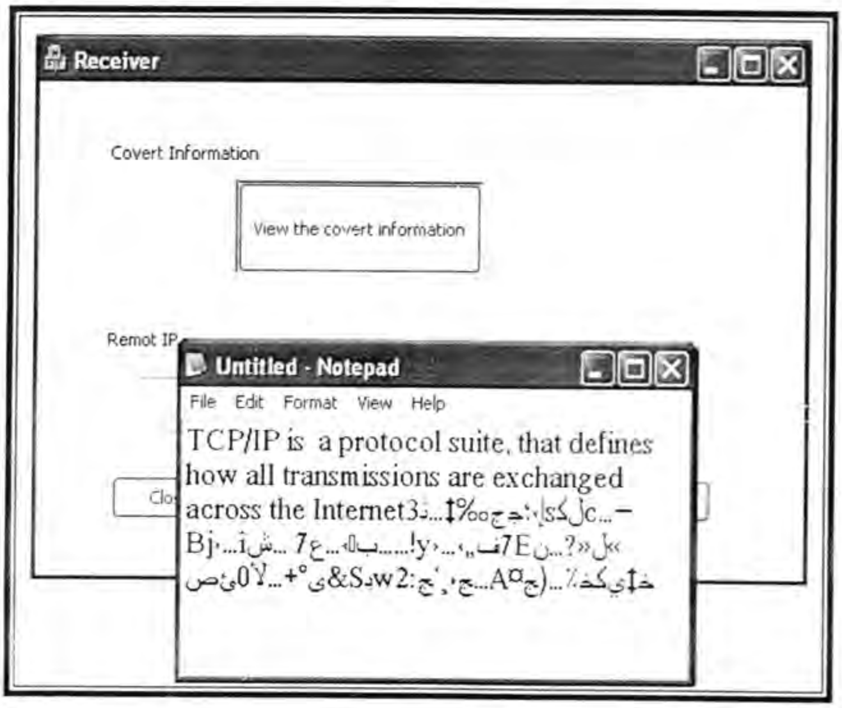

Figure 5: Output of The Receiver Site Before Limitation Length of Covert Information.

CONCLUSIONS

It is apparent that the use of computer network is very important in every aspect of communication and the secrecy of information transfer become extremely urgent in exchanging data. The uses of covert channel provide an effective mechanism in such a field. The following points are concluded throughout the work:

1- Unused bits are exploited during the manipulation of packet header of the TCP/IP protocol suite in order to identify the covert channel.

2- The arisen problem by windows XP architecture during covert channel development are solved by the use of NDIS hooking filter driver.

3- The NDIS can control and modify the packet at the application level through hooking process.

4- Adding additional information to the packet without affecting the packet size is possible.

FUTURE WORK

Covert channels find important applications in many fields, therefore, the future work may be directed in the following manner:

1. Using covert timing channel to hide the information in the packet.
2. Analyzing the IPv6 header for a new covert channel.
3. Inclusion of a new covert channel technique associated to the manipulation of the header fields into an other network protocol suite such as UDP, OSPF, RIP…etc.

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Study of Protein profile in patients with β-thalassemia

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Abstract:

Serum levels of total protein and protein profile were estimated in three groups of patients with β- thalassemia: 24 patients have splenectomy thalassemia major, 29 patients have non splenectomy thalassemia major and 19 patients have thalassemia intermedia. Data were compared to normal and pathological controls (anemia and minor). There was a significant decrease in serum total protein only in pateints with non splenectomy thalassemia as compared with controls, and there was a significant decrease in α_2 and β- fractions in all groups of patients studied as compared to normal and pathological controls.

In this paper the results indicate that a significant disturbance in the protein profile in sera of the patients with β- thalassemia in all groups studied.

الخلاصة:

تم تقدير مستوى البروتين الكلي وصورة البروتين في ثلاثة مجاميع من المرضى المصابين بالثلاسيميا نوع بيتا: 24 مريض بالثلاسيميا الحاد رافعي الطحال و29 مريض بالثلاسيميا الحاد غير رافعي الطحال و19 مريض بالثلاسيميا المعتدلة. تمت مقارنة النتائج بمجموعة من الاشخاص الأصحاء (الطبيعيين) ومجموعة سيطرة (مرضى فقر الدم والثلاسيميا الخفيفة) . بينت النتائج ان هناك انخفاضا معنويا في مستوى البروتين الكلي فقط في وصول مرضى المجموعة الثانية (غير رافعي الطحال) عند مقارنتها بمجموعة السيطرة وان هناك انخفاضا معنويا في الاجزاء α_2 (الفا 2)وبيتا في كل المجاميع المدروسة عند مقارنتها بمجموعة السيطرة والاشخاص الطبيعيين. تشير النتائج في هذا البحث الى وجود اضطراب معنوي في صورة البروتين في وصول مرضى الثلاسيميا نوع بيتا لكل المجاميع المدروسة.

Introduction:

Thalassemia is a heterogeneous group of genetic disorders in which the production of normal Hb is partly or completely suppressed because of defective synthesis of one or more globin chains[1,2]. β- thalassemia is the most familiar type[3], in which the β- globin chain synthesis is impaired[4]. The severity of the disease depends on the amount of HbA and HbF, which present[5].

It has been estimated that there are probably as many as (100,000) living patients with homozygous β- thalassemia in the world and over (150 million) peoples carry β- thalassemia gene, in Iraq, thalassemia carrier rate for the country was found to be (4-8%)[6].

The proteins are substances made up of 2 smaller building blocks called amino acids[7]. The major site of synthesis of the plasma proteins is the liver[8]. Total protein level depend on the balance between their synthesis and their catabolism or loss from body[9]. A total serum protein test measures total amount of protein in blood serum as well as the amounts of albumin and globulin[10].

Serum proteins has been studied as an indicator of liver disfunction parathyroid hormone disturbance in β- thalassemia[11], and hemoglobin SS disease[12].

The present study was carried out to study of protein profile and explore the fractions that is affected in sera of β- thalassemia major (splenectomy & non splenectomy) and β- thalassemia intermedia in comparison with normal subjects and pathological controls (anemia and minor).

Materials and Methods:

The study included 72 patients with β- thalassemia who admitted to thalassemia center in Ibn Al-Baladdy hospital to blood transfusion. Median age of patients ranged between (3 to 30) years. Patients were further classified according to their severity disease: thalassemia major (n=53) which is subdivided to splenectomy (n=24) and non- splenectomy (n=29) thalassemia major and thalassemia intermedia (n= 9).

The diagnosis of β- thalassemia was confirmed by measured Hbf, HbA, HbA_2 in Hb electrophoresis technique model: Bio Rad Variant. Patients suffering from any disease that may interfere with our study were excluded. Blood was also collected from (26) normal healthy control and (18) pathological control (anemia) and (15) of pathological control (miner) which is confirmed by Hb- electrophoresis technique too.

Total protein was performed using Randox kit (Biuret method[13], while the electrophoresis was by Sherwin and Kohn methods [14,15] and performed using cellulose stripe and barbital buffer with a pH of (8.6) and ionic strength (0.075). the buffer was prepared by dissolving (41.2) gm of sodium barbiton and (7.36) gm of diethyl barbituric acid in distilled water. Dye solution (Ponceau S) was made by dissolving (0.2) gm of (P) powder in (100ml) distilled water (solution A) and dissolving (3) gm of trichloro acetic acid in (100ml) distilled water (solution B), and solutions A and B were mixed together. The washing solution (5%) was done by diluting (5ml) of glacial acetic acid to (100ml) in distilled water.

The electrophoresis tank was filled with the buffer to the appropriate mark, the samples were placed in the cellulose acetate of the sheet. The sheet was placed in the tank, whereas the two end of the sheet floated in the buffer. Electrophoresis was done under constant current (150mA) for (30min). The sheet was removed and treated with a staining reagent for at least (15mins). Then the sheet was passed through a washing solution. And then allowed to dry. Finally the sheet was cut to five fractions and the dye was dissolved in appropriate solution (buffer). The colour was measured at (520)nm.

Statistical methods:

Results were analyzed statistically using student's "t" test [16] to determine the level of significance. The difference was considered to be significant only when "p" value was less than (0.05).

Result and discussion:

The total protein levels expressed as (mean ±SD) g% in sera of normal, pathological controls and patients with β- thalassemia major (splenectomy and non splenectomy) and intermedia, are shown in table (1).

Table (1): Total protein levels in sera of normal, pathological controls & patients with β- thalassemia

Groups	No.	TP g% mean ± SD
Normal control	26	7.28 ± 0.43
Pathological control:	33	
(1) anemia	18	7.26 ± 0.54
(2) minor	15	7.52 ± 0.58
Patients:	72	
(a) major	53	
(1) splenectomy	24	7.69 ± 0.60
(2) Non splenectomy	29	6.97 ± 0.46
(b) Intermedia	19	7.39 ± 0.45

Figures (1, 2) show the comparison and distribution of Tp levels in sera of all groups studied, respectively.

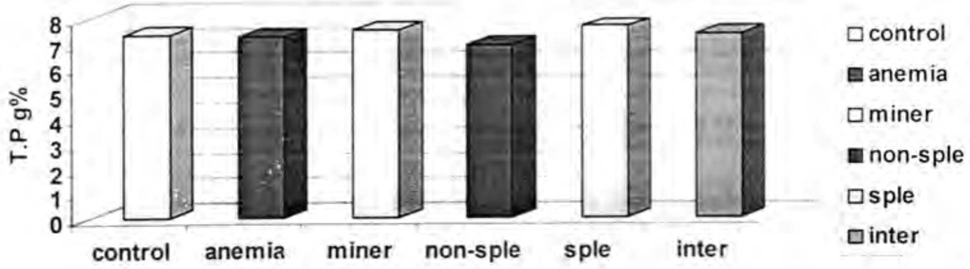

Fig. (1): The comparison of TP level in sera of normal, pathological controls, and thalassemia patients

Fig. (2): The distribution of TP level in sera of normal, pathological controls, and thalassemia patients

It is evident from the table and figures that there was a significant decrease ($P<0.002$) only in mean concentration of non splenectomy group in β- thalassemia as compared to normal and pathological controls.

The possible cause of decreased serum total protein secondarily decreased synthesis of protein by the liver[17].

The main aspect in management of sever β- thalassemia is to keep the patient on maintenance blood transfusion. The hyper transfusion regimen is the best, because the stimulus to unlimited bone marrow expansion, which causes much of the pathology, is reduced[18]. But the most serious side effect of lifelong transfusion therapy is iron overload[19]. The splenectomy β- thalassemia major and β- thalassemia intermedia in which the iron loading is less accelerated than that of non-splenectomy thalassemia major[1]. The same results were reported in sickle cell disease[20]. Similar reduction are shown in α- thalassemia hydrops fetalis but the cause is different. The resource is placental oedema[21].

Table 2 shows the result of serum protein electrephoresis & figure (3) shows the bands of serum protiens of normal and β- thalassemia (splenectomy , non splenectomy & intermedia patients) . In electrogram obtained from serum , usually only five bands (albumine , α_1 , α_2 , β & γ globulin) are seen. These results reveal that the mean values of albumin and α_1- globulin in sera of patients with β- thalassemia not significantly different as compared to normal and pathological controls. This agrees with Livrea et al.[22], who studied this parameter in patients with β-thalassemia major. Moreover it agrees with previous study in other hemoglobinopathies, VanDerdijs et al.[23], and Ojuawa et al.[24], studied this parameter in sickle cell disease, Dafallah et al.[25], studied it in glucose-6- phosphate dehydrogenase- deficient.

But the present study does not agree with Wanachiwanawin et al.[26], study in which the lower levels of serum albumin are seen, this is due to the presence of anti- HCV antibodies in sera of patients with β-thalassemia are studied.

Table (2): Results of serum protein electrophoresis

Groups	No.	Albumin g%	α_1 g%	α_2 g%	β g%	γ g%
Normal control	26	3.9 ± 0.49	0.360 ± 0.013	0.772 ± 0.186	0.82 ± 0.145	1.230± 0.315
Pathological anemia	18	3.78 ± 0.37	0.399 ± 0.014	0.847 ± 0.136	0.86± 0.165	1.340± 0.215
Pathological miner	15	4.13 ± 0.51	0.381 ± 0.020	0.727 ± 0.149	0.779± 0.186	1.470± 0.263
β- thalassemia splenectomy	24	4.07 ± 0.46	0.361 ± 0.015	0.670 ± 0.160	0.632± 0.165	1.855± 0.552
β- thalassemia non splenectomy	29	4.03 ± 0.48	0.360 ± 0.026	0.626 ± 0.182	0.526± 0.110	1.307± 0.354
β- thalassemia intermedia	19	4.37 ± 0.46	0.317 ± 0.019	0.551 ± 0.143	0.553± 0.176	1.567± 0.613

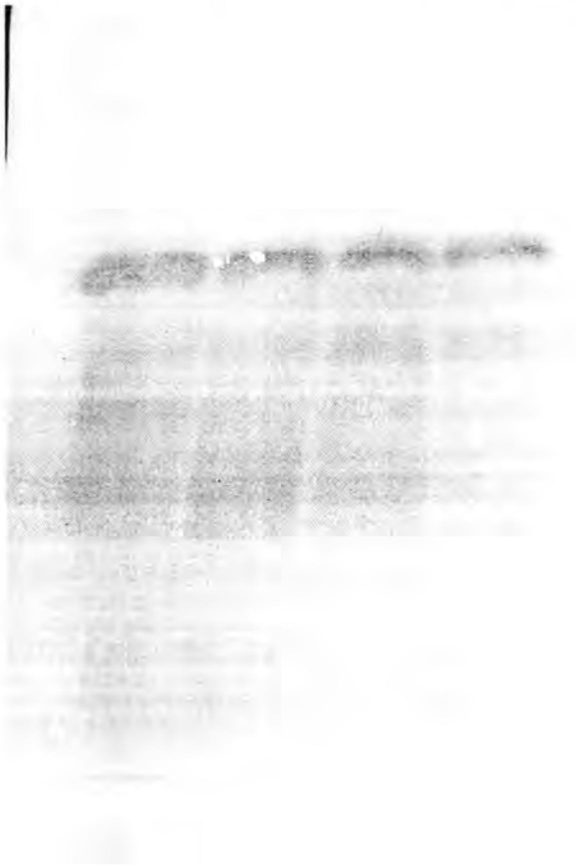

Fig. (3): sera electrophoresis pattern of normal, patients with β thalassemia (left):
1- normal. 2- splenectomy. 3- non splenectomy. 4- intermedia.

The comparison and distribution of albumin levels and α_1- globulin levels are shown in figures (4, 5, 6, 7) respectively.

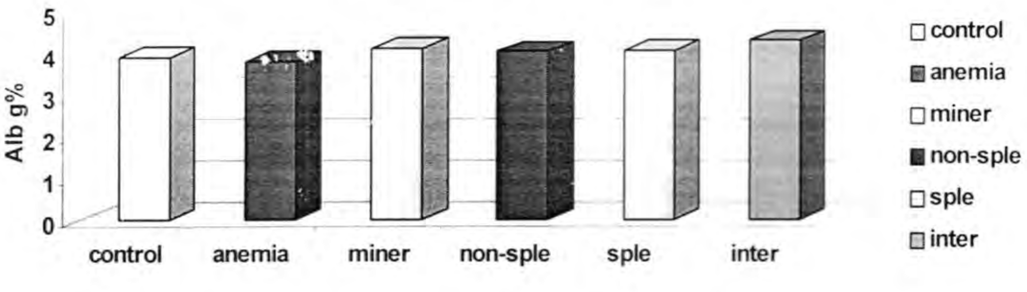

Fig. (4): comparison of albumin levels in sera of normal, pathological controls patients with β- thalassemia.

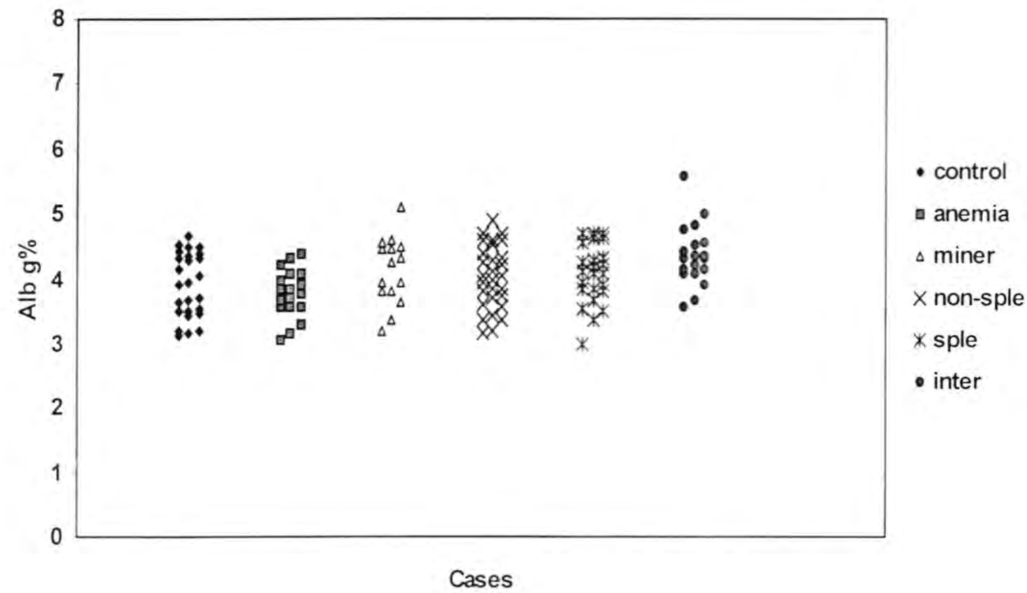

Fig. (5): Distribution of albumin levels in sera of normal, pathological controls patients with β- thalassemia.

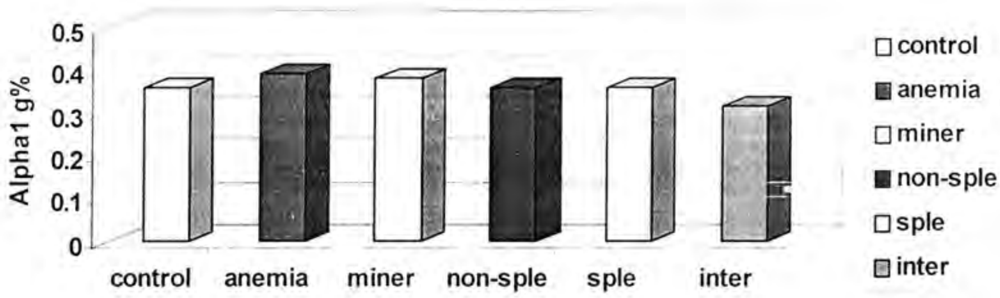

Fig. (6): comparison of Alpha α_1 levels in sera of normal, pathological controls patients with β- thalassemia.

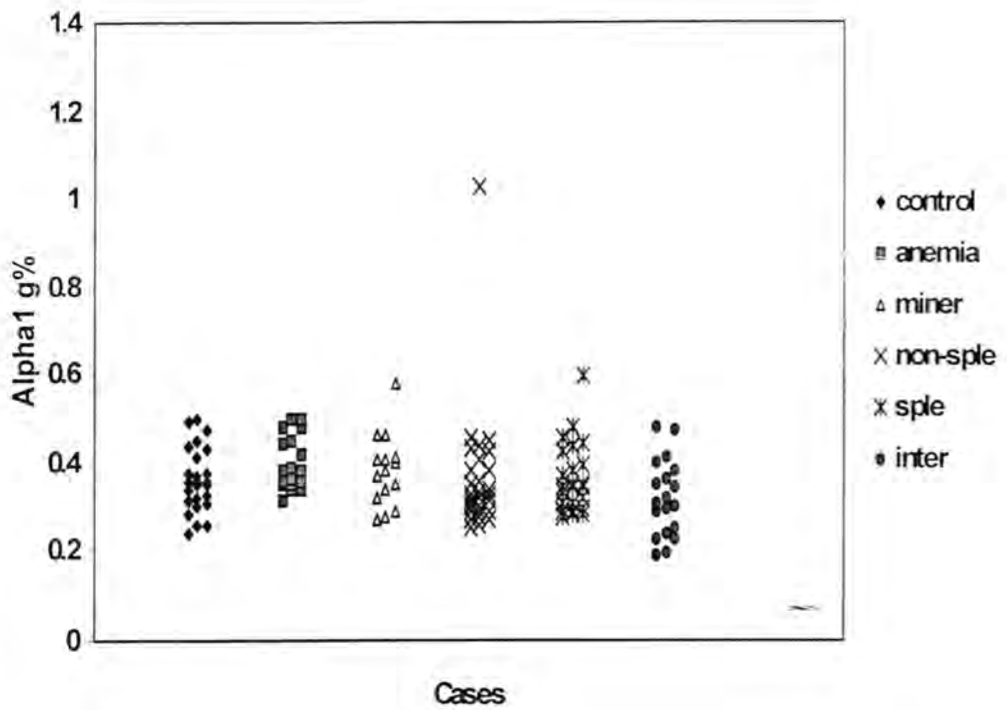

Fig. (7): Distribution of α1 levels in sera of normal, pathological controls, and patients with β- thalassemia

The reduction in α_2- globulin and β- globulin fractions are shown in table 2 and figures (8,9,10,11) respectively. The reasons of this reduction may be due to the presence of haptoglobin in α_2 band[27] and transferrin (the iron bound protein) in β- band[28], which is considered as to be the major component of the β- globin fraction and a pears as a distinct band on high – resolution serum protein electrophoresis[29].

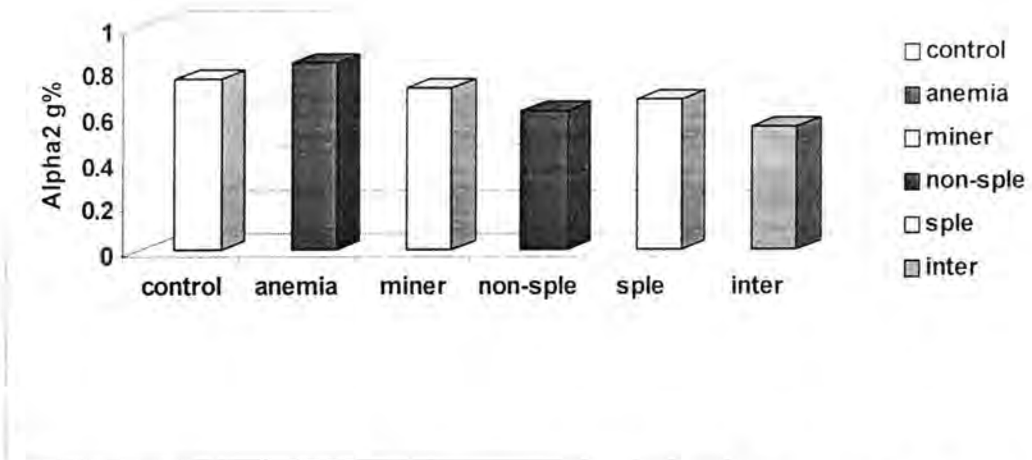

Fig. (8): Comparison of Alpha α_2 levels in sera of normal, pathological controls patients with β- thalassemia

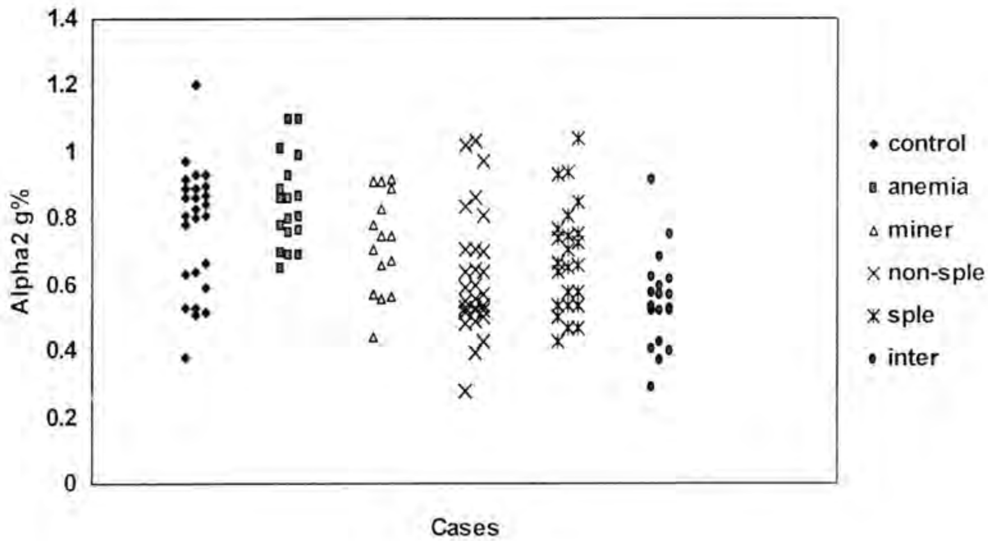

Fig. (9): Distribution of Alpha α_2 levels in sera of normal, pathological controls patients with β- thalassemia

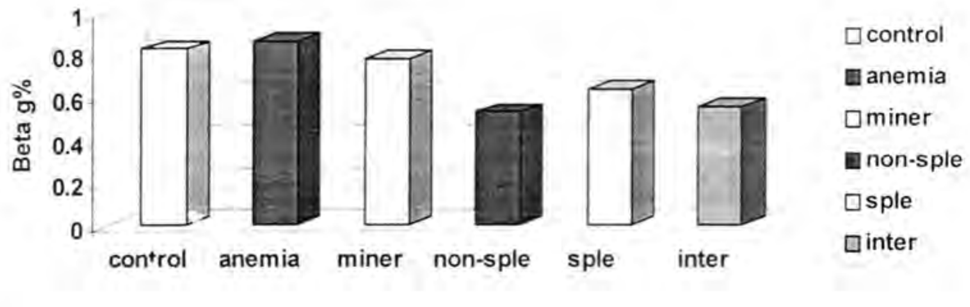

Fig. (10): Comparison of Beta globin levels in sera of normal, pathological controls patients with β- thalassemia.

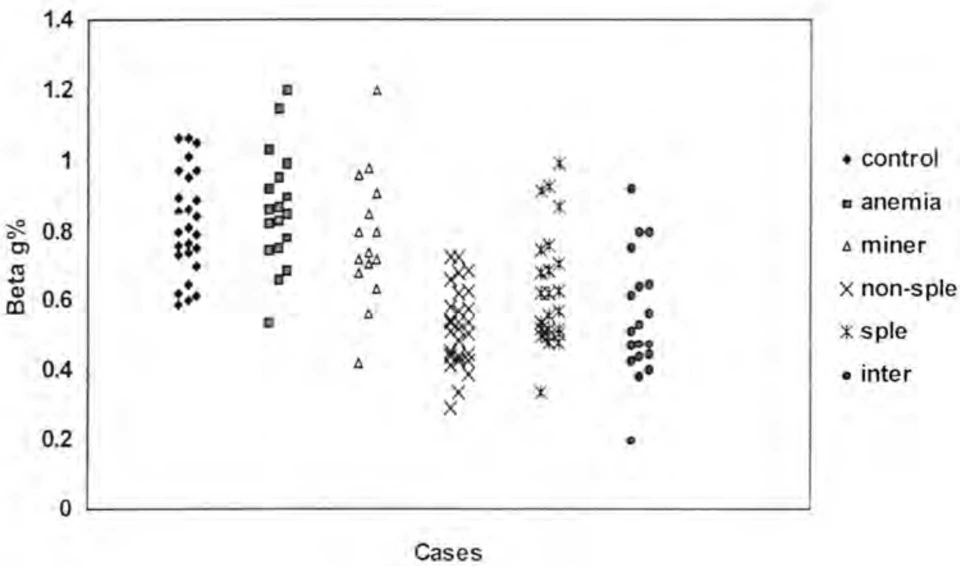

Fig. (11): Distribution of Beta globin levels in sera of normal, pathological controls patients with β- thalassemia

Highly significant elevation (P<0.001) was observed only in γ-globulin levels of patients with splenectomy β- thalassemia group as compared to normal and pathological controls. while other groups is significantly elevated , this shown in table 2 and figures (12,13) , the reasons may be due to that splenectomy thalassemia is classically associated with increased susceptibility to infection[30]. The major long-term risk after splenectomy is over welming sepsis. This a rises in γ-globulin levels were also demonstrated in patients with sickle cell anemia and genetic hemochromatosis by Rivero[31], Millard[32] and Fargion[33] respectively.

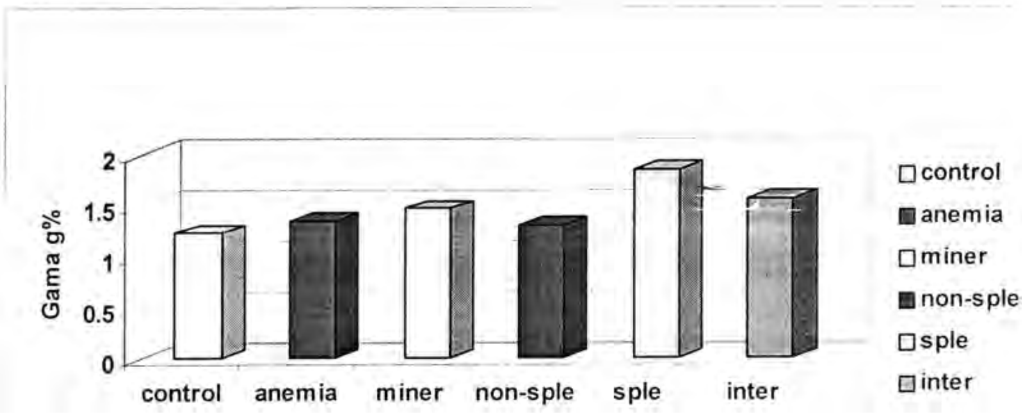

Fig. (3.12): Comparison of Gamma globulin levels in sera of normal, pathological controls patients with β- thalassemia.

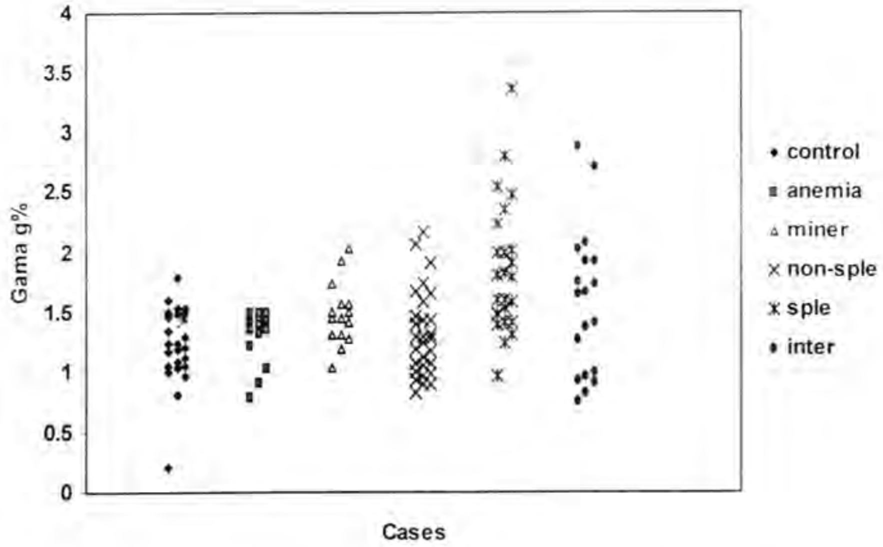

Fig. (3.13): Distribution of Gamma globulin levels in sera of normal, pathological controls patients with β- thalassemia.

In previous studies present that the risk of post splenectomy sepsis in thalassemia major is increased more than (30) fold in comparison with normal population[24]. On the other hand, the γ- globulin band consist of C- reactine protein[25], which is elevated as mush as (1000) fold in response to inflammation, also the acute nature of the inflammatory syndrome as assessed by CRP concentration was confirmed by high level of production of IgM and IgG antibodies[29, 30, 36].

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بسم الله الرحمن الرحيم

تعليمات النشر لمجلة علوم المستنصرية

٧٧٧٧٧٧٧٧٧٧٧

1. تقوم المجلة بنشر البحوث الرصينة التي لم يسبق نشرها في مكان آخر بعد إخضاعها للتقويم العلمي من قبل مختصين وبأي من اللغتين العربية او الانكليزية .
2. يقدم الباحث طلبا تحريريا لنشر البحث في المجلة على أن يكون مرفقا بأربع نسخ من البحث مطبوعة على الحاسوب ومسحوب بطابعة ليزرية وعلى ورق ابيض قياس (A4) مع قرص مرن (Disk) محمل بأصل البحث ويرفض البحث الذي يكون عدد صفحاته اكثر من **15** صفحة وبضمنها الاشكال والجداول على ان لايكون الحرف اصغر من قياس **12** .
3. يطبع عنوان البحث واسماء الباحثين (كاملة) وعناوينهم باللغتين العربية والانكليزية على ورقة منفصلة شرط ان لاتكتب اسماء الباحثين وعناوينهم في أي مكان اخر من البحث ، وتعاد كتابة عنوان البحث فقط على الصفحة الاولى من البحث .
4. تكتب اسماء الباحثين كاملة بحروف كبيرة وفي حالة استخدام اللغة الانكليزية وكذلك الحروف الاولى فقط من الكلمات (عدا حروف الجر والاضافة) المكونة لعنوان البحث ، وتكتب عناوين الباحثين بحروف اعتيادية صغيرة .
5. تقدم خلاصتان وافيتان لكل بحث ، احداهما بالعربية والاخرى بالانكليزية وتطبع على ورقتين منفصلتين بما لايزيد على **(250)** كلمة لكل خلاصة.
6. تقدم الرسوم التوضيحية منفصلة عن مسودة البحث ، وترسم على ورق شفاف (Tracing Paper) بالحبر الصيني الاسود ، وترفق ثلاث صور لكل رسم وتكتب المعلومات تحته على ورقة منفصلة.
7. يشار الى المصدر برقم يوضع بين قوسين بمستوى السطر نفسه بعد الجملة مباشرة وتطبع المصادر على ورقة منفصلة ، ويستخدم الاسلوب الدولي المتعارف عليه عند ذكر مختصرات اسماء المجلات.

8. يفضل قدر الامكان تسلسل البحث ليتضمن العناوين الرئيسة الاتية : المقدمة ، طرائق العمل ،النتائج والمناقشة ، الاستنتاجات ، المصادر، وتوضع هذه العناوين دون ترقيم في وسط الصفحة ولا يوضع تحتها خط وتكتب بحروف كبيرة عندما تكون بالانكليزية .

9. يتبع الاسلوب الاتي عند كتابة المصادر على الصفحة الخاصة بالمصادر: ترقيم المصادر حسب تسلسل ورودها في البحث ، يكتب الاسم الاخير (اللقب) للباحث او الباحثين ثم مختصر الاسمين الاولين فعنوان البحث ، مختصر اسم المجلة ، المجلد او الحجم ، العدد ، الصفحات ، (السنة) . وفي حالة كون المصدر كتابا يكتب بعد اسم المؤلف او المؤلفين عنوان الكتاب ، الطبعة ، الصفحات ، (السنة) الشركة الناشرة ، مكان الطبع .

10. بخصوص اجور النشر يتم دفع مبلغ (15000) خمسة عشر الف دينار عند تقديم البحث للنشر وهي غيرقابلة للرد ومن ثم يدفع الباحث (15000) خمسة عشر الف دينار اخرى عند قبول البحث للنشر وبهذا يصبح المبلغ الكلي للنشر ثلاثون الف دينار .

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الفهرسة

٥. تفوقت المعاملتان الملقحتان بالفطر *Gigaspora Spp.* والمسمدة بالمستوى ٢٠كغم فسفور /هـ وغير المسمدة(F_1P_0 ملقحة بفطريات المايكورايزا وغير مسمدة، F_1P_1 ملقحة بفطريات المايكورايزا ومسمدة بمستوى 20 كغم فسفور/هـ)في وزن الحاصل غم/نبات اذ كانت الزيادة معنوية في هذا القياس مقارنة ببقية المعاملات. مما يشير الى امكانية استخدام فطريات المايكورايزا لتقليل مستويات التسميد الفوسفاتي المضافة.

Abstract:

The research included a field experiment under different levels of phosphorus (0, 20, 40) Kgp/ha with and without presence of mycorrhizal fungi *Gigaspora Spp.* during the experimental period, five plants were taken randomly from each treatment at 50% flowering stage to measure shoots dry weights gm/plant. Concentration of N and p% percentage of root infection by mycorrhizal fungi, cotton productivity (hair) gm, number of balls/plant and weight of 100 seeds were determined. when the plants harvesting stage reached .

The results could be summarized as follows:

1. The highest shoot dry weight was recorded (54.30, 50.30)gm/plant in (F_0P_2 and F_1P_1) treatment respectively which indicated that mycorrhizal fungi might be used to reduce the application of phosphorous fertilizer.
2. Treatment that inoculated with *Gigaspora spp.* fungi and fertilized by 20kgp/ha gave the highest concentration of N and P as compared with other treatments.
3. The addition of 40Kg P/ha caused asignificant decrease in the percentage of root infection with mycorrhizal fungi as compared with treatment fertilized by 20KgP/ha and non-fertilized treatment.
4. The highest values for harvesting stage, cotton weight/plant, number of balls/plant and seed index (weight of 100 seeds) were (13.25, 8.56, 12.56) respectively for (F_1P_1) treatment with significant increase as compared with other treatments.
5. Inoculation with (*Gigaspora Spp*.) fungus with the addition of 20KgP/ha or without, showed of asignificant increase in cotton weight gm/plants as compared with other treatments. the significant difference between (F_1P_1 and F_1P_0) refers to the importance of fungi use .
6. The treatment that inoculated with (*Gigaspora Spp.* fungus and fertilized with 20KgP/ha or without fertilizer (F_1P_1 and F_1P_0) were significantly increased in weight of yield gm/plant compared with

المقدمة:

ان استخدام الاسمدة الحيوية (Biofertilizers) كفطريات المايكورايزا لزيادة نمو وانتاجية المحاصيل الحقلية كبديل عن بعض الأسمدة الكيمياوية أو لاختزال كميات الاسمدة المضافة لها اهمية اقتصادية وبيئية، وخاصة في ترب مثل ترب العراق التي تتصف بمحتواها العالي من كاربونات الكالسيوم وارتفاع الأس الهدروجيني الـ pH نسبياً مما يؤدي الى ارتفاع نسبة الفسفور الكلي وانخفاض الفسفور الجاهز في التربة حيث ان معظم الفسفور المضاف كاسمدة فوسفاتية يتحول الى فسفور غير جاهز في التربة.

وقد اشارت معظم الدراسات التي اجريت على فطريات المايكورايزا الحويصلة الشجيرية Vesicular-Arbuscular Mycorrhiza (VAM) إلى أن هذه الفطريات تزيد من نمو المحاصيل الحقلية وانتاجيتها نتيجة تشجيعها امتصاص العناصر الغذائية في الترب التي تعاني من نقص في هذه العناصر وخاصة الفسفور (١ و ٢) فضلاً عن ان هذه الفطريات لها تاثير ايجابي على تحسين العلاقات المائية في النبات العائل ومن ثم زيادة مقاومة النبات العائل للجفاف (٣ و ٤) ومالهذا من اهمية اخرى في قطرنا لكون العراق يقع ضمن المناطق الجافة وشبه الجافة. لذا يهدف البحث الحالي دراسة تأثير مستويات مختلفة من الفسفور(٠، ٢٠، ٤٠) كغم p/هـ بوجود وعدم وجود فطريات المايكورايزا المتمثل بالـ *Gigaspora Spp.* على نمو وإنتاج القطن أحد المحاصيل ذات الأهمية الاقتصادية في القطر.

المواد وطرائق العمل:

نفذت التجربة باستخدام القطاعات العشوائية الكاملة بعد حراثة التربة وتنعيمها، قسم الحقل الى ثلاث قطاعات (المكررات) كل منها احتوى على (٦) معاملات (الواح) ابعاد كل منها (١×٢)م^2 وكل لوح احتوى على مرزين للزراعة. اضيف السماد البوتاسي بمعدل (٨٠ كغم بوتاسيوم/هـ) والفسفوري بمعدل (٠، ٢٠، ٤٠) كغم p/هـ على هيئة سوبر فوسفات الكالسيوم الثلاثية عند موعد زراعة البذور بوضعها في اخاديد تحت مستوى الزراعة على عمق ١٠سم. اما النتروجين فقد تم اضافته بمعدل ٤٠ كغم N/هـ على دفعتين الاولى بعد عملية الخف مباشرة والثانية بعدها بشهر تقريباً. اما بالنسبة للقاح المايكورايزا المتمثل *Gigaspora Spp.* فقد تمت إضافته بشكل خليط من تربة مع جذور نباتات الذرة البيضاء المصابة بالفطر وحسب المعاملات بمعدل ٢٥٠غم/خط. اما المعاملات غير الملقحة بفطريات المايكورايزا فقد تم اضافة ٢٥٠غم/خط من خليط التربة مع جذور نباتات الذرة البيضاء غير

المصابة بالفطر. زرعت بذور القطن .*Gossypium hirscutum* L صنف لاشاتا في ٢٠٠٥/٣/٢٥ في جور بابعاد ٢٠سم على خط التعيير ووضع في كل جورة ٤-٥ بذرات. اجريت عملية الترقيع للجور الغائبة بعد ان ظهر فوق التربة ٧٠% من البادرات خففت النباتات إلى نباتين بالجورة بعد ان كان ارتفاع النبات ٢٥سم تقريباً وخلال فترة التجربة تم اخذ (٥) نباتات عشوائياً من كل لوح وذلك خلال مرحلة التزهير لحوالي ٥٠% من النباتات حيث اخذت النباتات وقطعت أجزائها الخضرية مع سطح التربة ثم ازيلت التربة من حول الجذور باستخدام تيار ماء ضعيف واجريت القياسات التالية:

١. الوزن الجاف للمجموع الخضري وفقاً لـ (٥).

٢. النسبة المئوية للجذور المصابة بفطريات المايكورايزا: تم فحص ٢٠ قطعة من الجذور الشعرية من كل نبات بطول (١سم) لكل قطعة تحت المجهر الضوئي بعد تصبيغها بصبغة (Acid Fuchsin) تبعاً لـ (٦) وتم حساب النسبة المئوية للاصابة حسب المعادلة الاتية:

$$\text{النسبة المئوية للإصابة} = \frac{\text{عدد القطع المصابة}}{\text{عدد القطع الكلية}} \times ١٠٠$$

٣. قياس تركيز النتروجين والفسفور في المجموع الخضري: تم طحن المجموع الخضري للعينات النباتية الجافة وهضمها حسب طريقة Stewart (٧) المبينة في (٨) بعدها تم تقدير كل من النتروجين باستخدام جهاز الماكروكلدال وفق طريقة (Brenner و Kenney)(٩) والفسفور باستخدام جهاز Spectrophotometer حسب طريقة (١٠).

كما اخذت بعض القياسات في مرحلة الحصاد وشملت:

١. حاصل القطن(الشعر): مجموع حاصل جنيتين من القطن (الشعر) محسوباً بالغرامات.

٢. عدد الجوزات للنبات الواحد: تم حسابها بجمع عدد الجوزات السليمة والمتفتحة لخمسة نباتات بصورة عشوائية وتقسيمها على عدد النباتات.

٣. معامل البذرة (غم): وزن مئة بذرة بالغرامات.

وقد اجري التحليل الاحصائي على نتائج التجربة واعتمد اختبار دنكن متعدد الحدود لاختبار المعنوية بين المعاملات المختلفة تبعاً للـ (١١).

النتائج والمناقشة:

تشير النتائج الموضحة في جدول (١) ان اعلى معدل للوزن الجاف للمجموع الخضري كان (٥٤.٣٠غم) والذي تفوق وبشكل معنوي على بقية المعاملات قد تم تسجيله في المعاملة (FoP_2) غير الملقحة والمسمدة بـ ٤٠كغمp /هـ والتي لم تختلف معنوياً عن

(50.30غم) لمعاملة (F_1P_1) الملقحة بفطر المايكورايزا والمسمدة بمستوى 20 كغم p/هـ في حين وصلت ادنى المعدلات وبشكل معنوي الى (20غم) لمعاملة (F_0P_0) غير الملقحة وغير المسمدة بالفسفور. وقد يعود السبب في زيادة معدل الوزن الجاف للنباتات غير الملقحة والمسمدة بالفسفور بمستوى 40كغم /هـ لكون الفسفور يعمل على زيادة انتاج المادة الجافة للجزء الخضري (12) كما انه من العناصر الغذائية المهمة (13) والذي يعزى له الزيادة في معدل الوزن الجاف لتلك النباتات، في حين قد تعود الزيادة في معدل الوزن الجاف للنباتات الملقحة والمعاملة بمستوى الفسفور الواطيء الى ان النباتات الملقحة بالفطر تستجيب بشكل افضل للمستويات الواطئة من الفسفور (14) في حين النباتات غير الملحقة فانها تستجيب للمستويات العالية للفسفور(14)، كما ان فطريات المايكورايزا تمتلك ميكانيكية خاصة لادراك مستويات الفوسفات التي ينظمها الموروثة الخاصة بذلك Phosphate Transporter gene (Gipt) الذي يتأثر بتركيز الفسفور المحيط بالهايفات الخارجية فضلاً عن حالة الفسفور في الجذور المصابة بالمايكورايزا(15). كما أن تعريض الجذور المصابة بالمايكورايزا إلى مستويات عالية من الفسفور فان جين (Gipt) استحث بضعف في الهايفات الخارجية كما ان الفوسفات لم تؤخذ من قبل الهايفات الخارجية (16) والماسيليوم (الهايفات الخارجية) مسؤولة عن اخذ الفسفور الذي ينتقل لاحقاً عن طريق الهايفات الداخلة إلى النبات(17). كما يلاحظ من نتائج جدول (1) ان اعلى نسبة مئوية لاصابة الجذور وصلت الى 90% في المعاملتين (F_1P_1) و (F_1P_0) وباختلاف معنوي عن بقية المعاملات، في حين كانت ادنى المعدلات (30%) للمعاملة (F_0P_0) غير الملقحة وغير المجهزة بالفسفور. وادت زيادة مستوى الفسفور الى 40كغم p/هـ الى نقص معنوي في النسبة المئوية لاصابة الجذور سواء للمعاملات الملقحة وغير الملقحة بالفطر. وربما يعود السبب في ذلك الى ان الية تثبيط الاصابة المايكورايزية تحت المستويات العالية من الفسفور ترتبط بمكونات الغشاء وبالتالي تسبب انخفاضاً في افرازات الجذور اذ ان مستوى الافرازات مرتبط مباشرة بالتغيرات في نفاذية غشاء الجذور التي تنظم بوساطة الفسفور(18) وعلى الرغم من تفوق المعاملات الملقحة بفطريات المايكورايزا معنويا في النسبة المئوية لاصابة الجذور مقارنة بمثيلاتها غير الملقحة (F_0) الا ان ذلك لا يمنع من وجود نسبة لا بأس فيها في المعاملات غير الملقحة ففطريات المايكورايزا توجد طبيعياً في معظم الترب ولها القدرة على اصابة معظم نباتات المحاصيل(19) كما ان 80% من النباتات الزهرية يمكن ان تقيم علاقات تعايشية مع هذه الفطريات(20).

كما يظهر الجدول (1) ان اعلى المعدلات لتركيز كل من N و P كان (3.68 و 0.62%) على التوالي لمعاملة (F_1P_1) والتي تفوقت معنويا على بقية المعاملات في حين

كانت ادنى المعدلات وبشكل معنوي (3.49 و 0.40%) لكل من تركيز N و P على التوالي لمعاملة (F_0P_0). كما يلاحظ من نتائج الجدول ذاته بان المعاملات الملقحة بفطريات المايكورايزا قد تفوقت على مثيلاتها غير الملقحة في تركيز كلا العنصرين مما يشير الى دور هذه الفطريات في زيادة امتصاص المغذيات اذ ان الهايفات الخارجية لفطريات المايكورايزا قادرة على اخذ الفسفور(21) والنتروجين(22) من التربة فضلاً عن ذلك فان فطريات المايكورايزا تزيد من سطح الامتصاص الفعال لجذور النبات المضيف بمقدار 10 مرات(23).

تشير نتائج الجدول (2) الى ان اعلى معدل لوزن الحاصل كان (11.93، 13.25) غم/نبات لمعاملة (F_1P_0 و F_1P_1) واللتان اختلفتا معنويا عن بقية المعاملات في حين كانت اقل المعدلات وبشكل معنوي (4.04 غم/نبات) لمعاملة F_0P_0. وربما يعود السبب في زيادة وزن الحاصل للمعاملات المصابة بفطر المايكورايزا الى ان هذه المعاملة كانت تملك اعلى نسبة مئوية لاصابة الجذور واعلى معدل لتركيز N و P كما ان النباتات المصابة بفطر المايكورايزا تمتص 15، 32 مرة كل من N و P على التوالي اكثر من غير المصابة (24) وبالتالي تعطي أعلى معدل للوزن كما ان النباتات المصابة بالمايكورايزا تكون افضل من ناحية الحجم من تلك عير المصابة وبالتالي تكون التغذية الفسفورية والعناصر الغذائية الاخرى افضل في تلك النباتات مقارنة بالنبات غير المصاب بالمايكورايزا وبالتالي تؤدي الى زيادة الحاصل(25). كذلك تفوقت معاملة (F_1P_1) وبشكل معنوي على بقية المعاملات في كل من معدل عدد الجوزات /نبات ومعامل البذرة (غم) حيث وصل معدلهما الى (8.65 و 12.56) على التوالي (جدول 2) وربما يعود السبب في هذه الزيادة الى ان النباتات الملقحة بفطر المايكورايزا عملت على اتاحة المغذيات النباتية لهذا المحصول خلال فصل النمو بشكل افضل مقارنة بالنباتات غير الملقحة بواسطة الاحياء المجهرية (فطريات المايكورايزا)(26) وما لهذا من اهمية كبرى وخاصة في ترب مثل ترب العراق.

كما يلاحظ من نتائج الجدول ذاته انخفاض في عدد الجوزات/نبات ومعامل البذرة(غم) عند زيادة مستوى الفسفور للنباتات الملقحة بالفطر وربما يعود ذلك ان للفسفور تاثير سلبي على فطريات المايحورايزا(27). ووصلت ادنى المعدلات (4.23 و 10.25) لمعاملة F_0P_0 لكل من عدد الجوزات/نبات ومعامل البذرة (غم) على التوالي.

لذا يمكننا القول من الضروري العمل على زيادة كثافة فطريات المايكورايزا في الترب العراقية حيث تساهم علاقة المايكورايزا في زيادة قابلية النبات في الحصول على فسفور اضافي عن طريق زيادة جاهزيته في التربة، اذ تملك فطريات المايكورايزا القدرة على تحليل صيغ الفسفور غير الذائبة في التربة فضلاً عن قابليتها في اعادة ذوبان الفوسفات

المثبت(28) وما لهذا من اهمية في زيادة انتاجية المحصول فضلاً عن الاهمية الاقتصادية والبيئية.

جدول (1)

المعاملة / القياس	الوزن الجاف للمجموع الخضري(غم/نبات)	تركيز N %	تركيز P %	النسبة المئوية للجذور المصابة %
F_0P_0	20.00e	3.49d	0.40d	30d
F_oP_1	35.00c	3.58bc	0.48c	50bc
F_0P_2	54.30a	3.55c	0.50bc	40cd
F_1P_0	27.80d	3.56c	0.51bc	90a
F_1P_1	50.30a	3.68a	0.62a	90a
F_1P_2	42.50b	3.62b	0.54b	60b

جدول (2)

المعاملة / القياس	وزن الحاصل (غم/نبات)	عدد الجوزات /نبات	معامل البذرة (غم)
F_0P_0	4.04d	4.23d	10.25d
F_oP_1	7.15c	4.98cd	11.00c
F_0P_2	8.89b	5.01cd	10.68cd
F_1P_0	11.93a	6.43bc	11.87b
F_1P_1	13.25a	8.56a	12.56a
F_1P_2	8.34bc	6.77b	12.03b

F_0: عدم التلقيح بفطريات المايكورايزا، F_1: التلقيح بالفطر *Gigaspora Spp.*

P_0: عدم اضافة السماد الفوسفاتي، P_1: اضافة 20كغم p/هـ

P_2 : اضافة 40 كغم p/هـ

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دراسة ميّزات التوزيعات الأسية المختلطة

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المستخلص

الهدف من البحث دراسة ميزات التوزيعات الأسية المختلطة من مجتمعين جزئيين للحالة المستمرة وكما تم استخدام طريقتي العزوم والامكان الأعظم في تقدير معلمات هذه التوزيعات (معلمة القياس ومعلمة النسبة) ، ومن ثم تم استخدام أسلوب المحاكاة لتوليد أرقام عشوائية ذات توزيع أسي بمعلمة مختلفة لكل مجتمع جزئي وتمت المقارنة بين المقدرات وقد تبين من نتائج أسلوب المحاكاة ان طريقة الامكان الأعظم كانت هي الأفضل لتقدير معالم التوزيع الأسي المختلط من مجتمعين جزئيين .

Abstract

The aim behind this research is to study of characteristics of mixture of exponential distributions and for two populations for the continuous case in addition to coming up to making formulas for the methods of estimating the parameters of these distributions (scale , proportion) by using is moment and maximum likelihood methods, then using simulation style to generate a random numbers with exponential distribution by a different parameter for each sub-population to compare between the simulations. In the exponential part, the researcher had came to the fact that the method of maximum likelihood was the best method to evaluate the significance of the mixture exponential distribution for two sub-populations.

المقدمة Introduction

تعدُّ التوزيعات المختلطة من التوزيعات الإحصائية المهمة في مجالات البايولوجية والفيزيائية وفي اختبارات الحياة، حيث تم افتراض ان المجتمع موضوع الدراسة متجانس يمتلك دالة توزيع $F(x,\theta)$ و ان الصيغة F معلومة لكن المعلمة θ غير معلومة.

في أكثر الأحيان يكون المجتمع تحت الدراسة غير متجانس. أي انه يحتوي على مجموعة من المجتمعات الجزئية وهي $s_{p_k},\ldots,s_{p_2},s_{p_1}$ مختلفة بنسب هي $p_k,\ldots,p_2,p_1$ على التوالي اذ ان دالة التوزيع في كل مجتمع جزئي هي $F_j(x,\theta_j)$ وان دالة الكثافة الاحتمالية p.d.f. هي $f_j(x,\theta_j)$ حيث $j=1,2,\ldots,k$.

فاذا سحبت عينة عشوائية $x_1, x_2, \ldots, x_n$ بحجم n من مجتمع مختلط موزعة على مجتمعات جزئية يمكن تحديد العينة بنوعين من الاختلاط [2]هما:

الحالة الأولى : في هذه الحالة من الممكن تحديد انتماء كل وحدة من وحدات العينة إلى أي مجتمع جزئي sp_j ، آي ان البيانات في هذه الحالة سوف تحتوي على n من المتغيرات العشوائية مجمعة كما يأتي :

$$\{(X_{11},X_{12},\ldots,X_{1n_1}),(X_{21},X_{22},\ldots,X_{2n_2}),\ldots,(X_{k1},X_{k2},\ldots,X_{kn_k})\}$$

اذ ان $(n_1, n_2, \ldots, n_k)$ حجوم العينات الجزئية للوحدات التي تنتمي إلى المجتمعات الجزئية $s_{p_1},s_{p_2},\ldots,s_{p_k}$ على التوالي. وان دالة الامكان الأعظم في هذه الحالة هي

$$L(x,n|\theta,p)=\frac{n!}{n_1!\ldots n_k!}p_1^{n1}\,p_2^{n2}\ldots p_k^{nk}\prod_{j=1}^{k}\left\{\prod_{i=1}^{n_j}f(x_{ji},\theta_j)\right\} \qquad (1)$$

حيث $n = n_1 + n_2 + \ldots + n_k$ و $p_1 + p_2 + \ldots + p_k = 1$

الحالة الثانية : في هذه الحالة لا نستطيع تحديد انتماء المفردة إلى أي مجتمع جزئي لذلك فان دالة التوزيع تعطى بالصيغة الاتية:

$$G(x)=p_r[X\le x]=\sum_{j=1}^{k}p_jF(x,\theta_j) \qquad (2)$$

هذا يؤدي الى ان تكون دالة كثافة الاحتمال p.d.f كالأتي:

$$g(x/\theta,p)=\sum_{j=1}^{k}p_jf(x,\theta_j) \qquad (3)$$

وان دالة الامكان للعينة هي :

$$L(x_1,x_2,\ldots,x_n/\theta,p)=\prod_{i=1}^{n}g(x\backslash\theta,p)=\prod_{i=1}^{n}\left\{\sum_{j=1}^{k}p_jf(x_i,\theta_j)\right\} \qquad (4)$$

التوزيعات الاسية المختلطة Mixture of exponential distributions

المتغير العشوائي X له توزيع اسي بمعلمة θ اذا كانت دالة كثافتـــه الاحتماليـــة هي[3]:

$$f(x|\theta)=\begin{cases}\frac{1}{\theta}\ e^{\frac{-x}{\theta}} & \text{if } x\geq 0\ ,\ \theta>0\\ 0 & \text{otherwise}\end{cases} \qquad (5)$$

حيث ان θ تمثل المعدل او معلمة القياس Scale parameter حيث يعد التوزيع الاسي من التوزيعات الاسية المختلطة عندما k=1 وكأن المجتمع مختلطاً مـــع نفســـه. امـــا اذا كـــان المجتمع المتجانس يحتوي على مجتمعين جزئيين (أي عندما k=2) وكانت دالة التوزيع لكل مجتمع جزئي هي:

$$F_j(x_i,\theta_j)=1-e^{\frac{-x_i}{\theta_j}} \quad ,\theta_j>0\ \ ,\ \ j=1,2$$

فان دالة الامكان للتوزيع الاسي المختلط في الحالة الاولى وباســـتخدام الصـــيغة (1) تكـــون بالشكل الاتي:

$$L(x_1,x_2,\ldots,x_n,n|\theta_j,p_j)=\frac{n!}{n_1!n_2!}\frac{p_1^{n_1}}{\theta_1^{n_1}}\frac{p_2^{n_2}}{\theta_2^{n_2}}\ \exp\left\{-\sum_{i=1}^{n_1}\frac{X_{1i}}{\theta_1}-\sum_{i=1}^{n_2}\frac{X_{2i}}{\theta_2}\right\} \qquad (6)$$

اما للحالة الثانية: فان دالة كثافة الاحتمال الحدية للمتغير العشوائي X معرفـــة كمـــا فـــي (3) فعندما k=2

$$f(x,\theta,p_1)=p_1f(x,\theta_1)+(1-p_1)f(x,\theta_2)$$

تكون دالة كثافة الاحتمال للتوزيع الاسي المختلط هي :

$$f(x,\theta,p_1)=p_1\frac{1}{\theta_1}e^{\frac{-x}{\theta_1}}+(1-p_1)\frac{1}{\theta_2}e^{\frac{-x}{\theta_2}} \qquad (7)$$

ميزات التوزيعات الأسية المختلطة[8]

Characterizations a Mixture exponential distributions

ميزات بوساطة العزوم للاحصاءيات المرتبة

Characterizations by moments of order statistics

لتكن $x_1, x_2, \ldots, x_n$ عينة عشوائية من الحجم $n\geq 2$ على فضاء احتمالي معطى لها دالة التوزيع و دالة كثافة احتمال ممثلة بوساطة F(x) و f(x) على التوالي.

سنمثل العزم k للاحصاءية المرتبة r بالشكل

$$M_{r,n}^{(k)}=E\{X_{r,n}^{(k)}\}$$

مبرهنة (1)

اذا كان X متغير عشوائي فان

$$M_{i+1,n}^{(2)} - M_{i,n}^{(2)} = (\frac{1}{\lambda_1} + \frac{1}{\lambda_2})(n-i)^{-1} M_{i+1,n} + (\frac{1}{\lambda_1} - \frac{1}{\lambda_2})(n-i)^{-1} (\lambda_1 \frac{\partial}{\partial \lambda_1} M_{i+1,n} - \lambda_2 \frac{\partial}{\partial \lambda_2} M_{i+1,n}) \quad (8)$$

لكل $i = 0,1,2,\dots,n = i,i+1,\dots$ اذا وفقط اذا كان F(x) خليط من توزيعين اسيين بمعدلين $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}$

البرهان : لبرهان الشرط الضروري

$$F(x) = 1 - p_1 e^{-\lambda_1 x} - (1-p_1) e^{-\lambda_2 x} \quad (9)$$

لايجاد العزم للاحصائية المرتبة i لعينة من الحجم n

$$M_{i,n}^{(1)} = nC_{i-1}^{n-1} \int_0^\infty x_i [F(x_i)]^{i-1} [1 - F(x_i)]^{n-i} f(x_i) dx_i$$

و بعد سلسلة من العمليات الحسابية نحصل بتعويض دالة كثافة الاحتمال المختلطة وبالتكامل بالتجزئة نحصل على

$$M_{i,n}^{(1)} = H \int_0^\infty \frac{(p_1 e^{-\lambda_1 x_i} + (1-p_1) e^{-\lambda_2 x_i})^{n-k}}{n-k} dx_i$$

حيث ان $H = nC_{i-1}^{n-1} \sum_{k=0}^{i-1} C_k^{i-1} (-1)^{i-1-k}$ وبإجراء عمليات حسابية مطولة وأجــراء التكامــل نحصل على

$$M_{i,n}^{(1)} = \frac{H}{(n-k)} R$$

حيث ان $R = \sum_{r=0}^{n-k} C_r^{n-k} \frac{p_1^{\,r} (1-p_1)^{n-k-r}}{\lambda_1 r + \lambda_2 (n-k-r)}$ و بالطريقة نفسها نجد أن

$$M_{i,n}^{(2)} = \frac{2H}{n-k} N$$

حيث ان $N = \sum_{r=0}^{n-k} C_r^{n-k} \frac{p_1^{\,r} (1-p_1)^{n-k-r}}{(\lambda_1 r + \lambda_2 (n-k-r))^2}$

ومن ثم وباجراء بعض العمليات الحسابية نحصل على

$$(n-i)(M_{i+1,n}^{(2)} - M_{i,n}^{(2)}) = 2GN$$

حيث ان $G = nC_i^{n-1} \sum_{k=0}^{i} C_k^i (-1)^{i-k}$

وبالتبسيط نحصل

$$(n-i)(M^{(2)}_{i+1,n}-M^{(2)}_{i,n})=\frac{1}{\lambda_1}\left[\frac{G}{(n-k)}R\right]+\frac{1}{\lambda_2}\left[\frac{G}{(n-k)}R\right]+\frac{(\lambda_1-\lambda_2)}{\lambda_1\lambda_2}\left[(n-k-r)\lambda_2-r\lambda_1\right]\left[\frac{G}{(n-k)}N\right]$$

وبمقارنة الناتج مع العزوم

$$(n-i)(M^{(2)}_{i+1,n}-M^{(2)}_{i,n})=(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})M_{i+1,n}+\frac{M^2_{i+1,n}}{2}\;\frac{(\lambda_1-\lambda_2)}{(\lambda_1\lambda_2)}\;[-\lambda_1 r+\lambda_2(n-k-r)]$$

وبتوزيع الحدود والقسمة على (n-i)

$$M^{(2)}_{i+1,n}-M^{(2)}_{i,n}=(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})(n-i)^{-1}M_{i+1,n}+(\frac{1}{\lambda_1}-\frac{1}{\lambda_2})(n-i)^{-1}\left[\lambda_1\frac{\partial}{\partial\lambda_1}M_{i+1,n}-\lambda_2\frac{\partial}{\partial\lambda_2}M_{i+1,n}\right]$$

لبرهان الشرط الكافي نكامل بالتجزئة المقدار .

$$M^{(2)}_{i+1,n}-M^{(2)}_{i,n}=nC^{n-1}_{i}\int_0^\infty x^2_{i+1}[F(x_{i+1})]^{i}[1-F(x_{i+1})]^{n-i-1}dF_{x_{i+1}}-nC^{n-1}_{i-1}\int_0^\infty x^2_i[F(x_i)]^{i-1}[1-F(x_i)]^{n-i}dF_{x_i}$$

$$M^{(2)}_{i+1,n}-M^{(2)}_{i,n}=2C^n_i\int_0^\infty x[F(x)]^{i}[1-F(x)]^{n-i}dx$$

كذلك

$$M_{i+1,n}=nC^n_i\int_0^\infty x_{i+1}[F(x_{i+1})]^{i}[1-F(x_{i+1})]^{n-i-1}dF(x_{i+1})$$

وباشتقاقها بالنسبة الى λ_1 ونضرب ونقسم على (n-i) وبما ان

$$d[F(x_{i+1})]^i=i[F(x_{i+1})]^{i-1}dF(x_{i+1})$$

وبحذف الحدود المتشابهة يكون :

$$\frac{\partial}{\partial\lambda_1}M_{i+1,n}=-(n-i)C^n_i\int_0^\infty[F(x_{i+1})]^{i}[1-F(x_{i+1})]^{n-i-1}\frac{\partial F(x_{i+1})}{\partial\lambda_1}d(x_{i+1})$$

و بالطريقة نفسها نجد .

$$\frac{\partial}{\partial\lambda_2}M_{i+1,n}=-(n-i)C^n_i\int_0^\infty[F(x_{i+1})]^{i}[1-F(x_{i+1})]^{n-i-1}\frac{\partial F(x_{i+1})}{\partial\lambda_2}d(x_{i+1})$$

وبالتعويض بالمعادلة (8) واستخراج $\int_0^\infty x_{i+1}[F(x_{i+1})]^i[1-F(x_{i+1})]^{n-i-1}dx_{i+1}$ كعامل مشترك يمكن كتابته بالشكل

$$2[1-F(x_{i+1})]=(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})\frac{\partial F(x_{i+1})}{\partial x_{i+1}}+\frac{\lambda_1}{x_{i+1}}(\frac{1}{\lambda_1}-\frac{1}{\lambda_2})\frac{\partial F(x_{i+1})}{\partial\lambda_1}+\frac{\lambda_2}{x_{i+1}}(\frac{1}{\lambda_2}-\frac{1}{\lambda_1})\frac{\partial F(x_{i+1})}{\partial\lambda_2}$$

لكن

$$\left[1-F(x_{i+1})\right]=p_1\left[1-F_1(x_{i+1})\right]+(1-p_1)\left[1-F_2(x_{i+1})\right]$$

وبمقارنة الحدود الخالية من p_1 في كلا الجانبين يكون:

$$2[\ 1-F_2(x_{i+1})\ ]=\frac{1}{\lambda_1}\left(\frac{\partial F_2(x_{i+1})}{\partial x_{i+1}}-\frac{\lambda_2}{x_{i+1}}\frac{\partial F_2(x_{i+1})}{\partial \lambda_2}\right)+\frac{1}{\lambda_2}\left(\frac{\partial F_2(x_{i+1})}{\partial x_{i+1}}+\frac{\lambda_2}{x_{i+1}}\frac{\partial F_2(x_{i+1})}{\partial \lambda_2}\right)$$

وبما ان $1-F_2(x_{i+1})$ لا تعتمد على λ_1 فان

$$\frac{\partial F_2(x_{i+1})}{\partial x_{i+1}}-\frac{\lambda_2}{x_{i+1}}\frac{\partial F_2(x_{i+1})}{\partial \lambda_2}=0 \qquad (10)$$

للسهولة نستبدل x_{i+1},λ_2,F بالمتغيرات X,Y,Z على التوالي حيث Z هي دالة لــ X,Y. لذلك المعادلة (18-2) تصبح بالشكل

$$x\frac{\partial z}{\partial x}-(y)\frac{\partial z}{\partial y}=0 \qquad (11)$$

العلاقة (11) يمكن كتابتها كما في [6]

$$Pp+Qq=R$$

حيــــث $P=x,Q=-y,R=0,\frac{\partial z}{\partial x}=p,\frac{\partial z}{\partial y}=q$ لـــذا باســـتخدام المعـــادلات المميـــزة (lagrange's characteristic equation)[5], [11] نحصل على

$$\frac{dx}{x}=\frac{dy}{-y}=\frac{dz}{0}$$

حيث f اختيارية وبما ان $x \geq 0\ ,\ y > 0$ فان

$$xy \geq 0 \Rightarrow 0 \leq xy < \infty$$
$$0 \leq 1-e^{-xy} < 1$$
$$0 < z \leq 1$$

اذن $z=1-e^{-xy}$ أي ان $x_{i+1} \geq 0$, $F_2(x_{i+1})=1-e^{-\lambda_2 x_{i+1}}$ و بالطريقة نفسها نجـد $x_{i+1} \geq 0$, $F_1(x_{i+1})=1-e^{-\lambda_1 x_{i+1}}$ وهذا يعني ان $F(x_{i+1})$ هي خليط من توزيعين اسيين بمعدلات $\frac{1}{\lambda_1}$ و $\frac{1}{\lambda_2}$.

مبرهنة (2)

لكل $i=1,2,\ldots;n=i,i+1,i+2,\ldots$ فانه .

$$n(M_{i,n}^{(2)}-M_{i-1,n-1}^{(2)})=\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)M_{i,n}+\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)\left(\lambda_1\frac{\partial}{\partial \lambda_1}M_{i,n}-\lambda_2\frac{\partial}{\partial \lambda_2}M_{i,n}\right)\ldots \quad (12)$$

إذا وفقط إذا كان F(x) هو خليط من توزيعين اسيين بمعدلات $\frac{1}{\lambda_1}$ و $\frac{1}{\lambda_2}$

البرهان :-باستخدام العلاقة التكرارية

$$(n-i)M^{k}_{i,n} + iM^{k}_{i+1,n} = nM^{k}_{i,n-1}$$
$$k = 1,2,\ldots; i = 1,2,\ldots; n = i+1, i+2,\ldots$$

حيث نختار k=2 ثم نستخدم (8) نحصل على(12)

ميزات باستخدام الفروق المطلقة لمتغيرين عشوائيين مستقلين ومتماثلي التوزيع [8]
Characterizations using the absolute difference of two (iid) random variables

مبرهنة(3)

ليكن X_1 و X_2 متغيرين عشوائيين مستقلين متماثلي التوزيع مع المتغير العشوائي X الذي له دالة التوزيع المختلطة فان الفرق $r=|X_1 - X_2|$ و X متماثلي التوزيع بـاختلاف النسب أي

$$F(r) = BF(r,\lambda_1) + (1-B)F(r,\lambda_2) \qquad ,0 < B < 1 \tag{13}$$

حيث $1-B \equiv (1-P_1)^2 + \frac{2P_1(1-P_1)\lambda_1}{\lambda_1+\lambda_2}$. $B \equiv P_1^2 + \frac{2P_1(1-P_1)\lambda_2}{\lambda_1+\lambda_2}$ اذا وفقط اذا

$$F(x,\lambda_i) = \exp f(\lambda_i, x) \qquad \forall i = 1,2,\ldots \tag{14}$$

البرهـــان : لـــنكن $Y_1 = \min(X_1, X_2)$ و $Y_2 = \max(X_1, X_2)$ و $u = Y_1$ و $r = Y_2 - Y_1 = |X_1 - X_2|$ لبرهان الشرط الضروري فان دالة كثافة الاحتمال المشتركة لـــ Y_1 و Y_2 هي

$$\Phi(y_1, y_2) = 2f(y_1)f(y_2)$$

لذلك نجد

$$g(r) = 2\int_0^{\infty} f(u)f(u+r)du \tag{15}$$

$$g(r) = 2\left[\frac{p_1^2\lambda_1 e^{-\lambda_1 r}}{2} + \frac{p_1(1-p_1)\lambda_1\lambda_2 e^{-\lambda_2 r}}{\lambda_1+\lambda_2} + \frac{p_1(1-p_1)\lambda_1\lambda_2 e^{-\lambda_1 r}}{\lambda_1+\lambda_2} + \frac{(1-p_1)^2\lambda_2 e^{-\lambda_2 r}}{2}\right]$$

وبما أن

$$f(r,\lambda_i) = \lambda_i e^{-\lambda_i r} \quad \forall i = 1,2$$

أذن

$$g(r) = (p_1^2 + \frac{2p_1(1-p_1)\lambda_2}{\lambda_1+\lambda_2})f(r,\lambda_1) + ((1-p_1^2) + \frac{2p_1(1-p_1)\lambda_1}{\lambda_1+\lambda_2})f(r,\lambda_2)$$

لبرهان الشرط الكافي نعوض (7) في (15) فنحصل على

$$(p_1^2 + \frac{2p_1(1-p_1)\lambda_2}{\lambda_1+\lambda_2}) f(r,\lambda_1) + ((1-p_1^2) + \frac{2p_1(1-p_1)\lambda_1}{\lambda_1+\lambda_2}) f(r,\lambda_2)$$
$$= 2\int_0^{\infty} [p_1 f(u,\lambda_1) + (1-p_1) f(u,\lambda_2)][p_1 f(u+r,\lambda_1) + (1-p_1) f(u+r,\lambda_2)] du$$

وبمقارنة الحدود الخالية من p_1 نحصل على

$$2\int_0^{\infty} [f(u,\lambda_2) f(u+r,\lambda_2)] du = f(r,\lambda_2) \qquad (16)$$

وبما انه $f(r,\lambda_2)$ يحقق (16) لكل $r \geq 0$ فان [9]

$$F(r,\lambda_2) = \exp\ f(\lambda_2, r)$$

و بالطريقة نفسها نجد

$$F(r,\lambda_1) = \exp\ f(\lambda_1, r)$$

اذن F(x) هو خليط من توزيعين اسيين

ميزات بوساطة التوقعات الشرطية [9]
Characterizations of Conditional Expectation

مبرهنة (4)

لكل متغير عشوائي غير سالب X بدالة توزيع F(x) وبدالة كثافة احتمال f(x) والتي تكون مشتقتها الثانية موجودة يكون توزيعها خليط من توزيعين اسيين بمعدلين $\frac{1}{\lambda_1}$ ، $\frac{1}{\lambda_2}$ اذا وفقط اذا كان

$$E\{X|X>y\} = y + (\frac{1}{\lambda_1} + \frac{1}{\lambda_2}) - \frac{1}{\lambda_1\lambda_2}\frac{f(y)}{1-F(y)}\ , y>0 \qquad (17)$$

البرهان : لبرهان الشرط الضروري ، بما ان دالة كثافة الاحتمال لـــ x معرفة كما فـــي (7) حيث

$$E\{X|X>y\} = \int_y^{\infty} x\frac{f(x)}{1-F(y)}\ dx$$

وبتعويض (7) نحصل على .

$$E\{X|X>y\} = \frac{1}{1-F(y)}\int_y^{\infty} x\left[p_1\lambda_1 e^{-\lambda_1 x} + (1-p_1)\lambda_2 e^{-\lambda_2 x}\right]\ dx$$

وبالتكامل بالتجزئة وباستخدام (9) نجد

$$E\{X|X>y\}=y+(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})-\frac{p_1\frac{1}{\lambda_2}e^{-\lambda_1 y}+(1-p_1)\frac{1}{\lambda_1}e^{-\lambda_2 y}}{1-F(y)}$$

أذن

$$E\{X|X>y\}=y+(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})-\frac{1}{\lambda_1\lambda_2}\frac{f(y)}{1-F(y)}$$

ولبرهان الشرط الكافي (17) نستطيع كتابتها بالشكل

$$\int_y^{\infty}xf(x)dx=\left[y+(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})\right][1-F(y)]-\frac{1}{\lambda_1\lambda_2}f(y)$$

نشتق بالنسبة الى Y

$$xf(x)\ ]_y^{\infty}=\left[y+(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})\right][-f(y)]+[1-F(y)]-\frac{1}{\lambda_1\lambda_2}f'(y)$$

وبما ان دالة كثافة الاحتمال معرفة كما في (7) فان

$$-yf(y)=-yf(y)-(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})f(y)+[1-F(y)]-\frac{1}{\lambda_1\lambda_2}f'(y)$$

$$(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})f(y)+\frac{1}{\lambda_1\lambda_2}f'(y)=[1-F(y)]$$

نشتق مرة اخرى بالنسبة الى Y

$$(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})f'(y)+\frac{1}{\lambda_1\lambda_2}f''(y)=-f(y)$$

$$\frac{1}{\lambda_1\lambda_2}f''(y)+(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})f'(y)+f(y)=0$$

وهي معادلة تفاضلية خطية من الرتبة الثانية وبحل هذه المعادلة نكتبها بدلالة المعادلة المميزة

$$\frac{1}{\lambda_1\lambda_2}m^2+(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})m+1=0$$

اذن

$f(y)=A\lambda_1e^{-\lambda_1 y}+B\lambda_2e^{-\lambda_2 y}$ حيث A ، B ثوابت موجبة وبما ان $\int_0^{\infty}f(y)dy=1$

اذن

$$1=\int_0^{\infty}(A\lambda_1e^{-\lambda_1 y}+B\lambda_2e^{-\lambda_2 y})dy$$

$1=A+B \quad \rightarrow \quad B=1-A$

$$f(y)=A\lambda_1 e^{-\lambda_1 y}+(1-A)\lambda_2 e^{-\lambda_2 y}$$

اذن $f(y)$ هي دالة كثافة الاحتمال للتوزيعات الاسية المختلطة بمعدلات $\frac{1}{\lambda_1}$ و $\frac{1}{\lambda_2}$ عندما k=2

بعض طرائق تقدير معلمات التوزيع الاسي المختلط

Some Methods of Estimating Parameters of Mixture of Exponential Distribution

طريقة الامكان الاعظم Maximum Likelihood Method

تعد هذه الطريقة من اهم طرائق التقدير النقطية وقد اقترحها العالم الاحصائي الشهير (Fisher) عام 1920، اذ تفترض ان المعلمة المراد تقديرها لمجتمع معين هي كمية ثابتة غير معروفة تقدر في ضوء معطيات العينة[7]. ان مقدر الامكان الاعظم (MLE) لمعالم لتوزيع الاسي المختلط وعندما k = 2 للمعادلة (7) يمكن الحصول عليه باشتقاق لوغاريتم دالة الامكان ومساواة الناتج بالصفر لذا

$$\log L=\log k+n_1\log p_1-n_1\log\theta_1+(n-n_1)\log(1-p_1)$$
$$-(n-n_1)\log\theta_2-\frac{\sum_{i=1}^{n_1}x_{1i}}{\theta_1}-\frac{\sum_{i=1}^{n_2}x_{2i}}{\theta_2}$$

حيث ان $k=\frac{n!}{n_1!n_2!}$

$$\frac{\partial}{\partial p_1}\log L(\underline{x},n|\theta_1,\theta_2,p_1,p_2)=\frac{n_1}{p_1}-\frac{(n-n_1)}{1-p_1}=0 \qquad (18)$$

$$\frac{\partial}{\partial \theta_1}\log L(\underline{x},n|\theta_1,\theta_2,p_1,p_2)=\frac{-n_1}{\theta_1}+\frac{\sum_{i=1}^{n_1}x_{1i}}{\theta^2_1}=0 \qquad (19)$$

$$\frac{\partial}{\partial \theta_2}\log L(\underline{x},n|\theta_1,\theta_2,p_1,p_2)=\frac{-(n-n_1)}{\theta_2}+\frac{\sum_{i=1}^{n_2}x_{2i}}{\theta^2_2}=0 \qquad (20)$$

وبحل المعادلات انفا نحصل على مقدر الامكان الاعظم للمعالم θ_1 و θ_2 و p_1 وكما في الصيغ الاتية

$$\hat{p}_1=\frac{n_1}{n}$$
$$\hat{\theta}_1=\bar{x}_1 \qquad (21)$$
$$\hat{\theta}_2=\bar{x}_2$$

حيث $\bar{x}_1=\frac{\sum_{i=1}^{n_1}x_{1i}}{n_1}$ و $\bar{x}_2=\frac{\sum_{i=1}^{n_2}x_{2i}}{n_2}$

$$\hat{p}_2 = 1 - \hat{p}_1 = \frac{n_2}{n}$$

ولايجاد تقديرات للمعلمات في الحالة الثانية وعندما k=2 للتوزيعات الاسية المختلطة باستخدام المعادلة (6)ومن ثم اخذ لوغاريتم لدالة الإمكان(6)

$$\log L(\underline{x}, \theta_1, \theta_2, p_1) = \sum_{i=1}^{n} \log\left(\frac{p_1}{\theta_1} e^{\frac{-x_i}{\theta_1}} + \frac{(1-p_1)}{\theta_2} e^{\frac{-x_i}{\theta_2}}\right)$$

$$\frac{\partial \log L(x, \theta_1, \theta_2, p_1)}{\partial p_1} = \sum_{i=1}^{n} \frac{\frac{1}{\theta_1} e^{\frac{-x_i}{\theta_1}} - \frac{1}{\theta_2} e^{\frac{-x_i}{\theta_2}}}{\frac{p_1}{\theta_1} e^{\frac{-x_i}{\theta_1}} + \frac{(1-p_1)}{\theta_2} e^{\frac{-x_i}{\theta_2}}} = 0 \qquad (22)$$

$$\frac{\partial \log L(\underline{x}, \theta_1, \theta_2, p_1)}{\partial \theta_1} = \sum_{i=1}^{n} \frac{\frac{p_1}{\theta^3_1} x_i e^{\frac{-x_i}{\theta_1}} - \frac{p_1}{\theta_1^2} e^{\frac{-x_i}{\theta_1}}}{\frac{p_1}{\theta_1} e^{\frac{-x_i}{\theta_1}} + \frac{(1-p_1)}{\theta_2} e^{\frac{-x_i}{\theta_2}}} = 0 \qquad (23)$$

$$\frac{\partial \log L(\underline{x}, \theta_1, \theta_2, p_1)}{\partial \theta_2} = \sum_{i=1}^{n} \frac{\frac{(1-p_1)}{\theta^3_2} x_i e^{\frac{-x_i}{\theta_2}} - \frac{(1-p_1)}{\theta_2^2} e^{\frac{-x_i}{\theta_2}}}{\frac{p_1}{\theta_1} e^{\frac{-x_i}{\theta_1}} + \frac{(1-p_1)}{\theta_2} e^{\frac{-x_i}{\theta_2}}} = 0 \qquad (24)$$

ان المعادلات انفا لا يمكن حلها بالطرائق الاعتيادية لذلك يمكن حلهــا باســتخدام احــدى الطرائق العددية لحل المعادلات غير الخطية مثل طريقة البحث المباشر(Direct search).

طريقة العزوم [4] Method of Moments

تعد طريقة العزوم من الطرائق الشائعة الاستخدام في حقل تقدير المعلمــات، اذ انهــا تتصف بسهولتها، وتعتمد على مساواة عزم المجتمع μ_k الذي سنقدره مع عــزم العينــة m_k وايجاد صيغة تقديرية للمعلمات. وقد اوجدها بيرسون (Person) في عام1894 [1].

للحالة الثانية وعندما تكون دالة كثافة الاحتمال للتوزيع الاسي المختلط لمجتمعــين معرفــة كما في (7) يمكن تقدير كل من θ_1 و θ_2 و P_1 باستخدام طريقة العزوم، وقد وجــد هــذه الطريقة (Rider) في عام (1961) للحصول على تقديرات للمعلمات بطريقة العــزوم لعينــة كاملة [12] . ان العزم k للتوزيع الاسي المختلط هو

$$M_k = k!(p_1\theta_1^k + (1-p_1)\theta_2^k) \qquad (25)$$

$$M_1 = p_1(\theta_1 - \theta_2) + \theta_2 \qquad (26)$$

$$\frac{M_2}{2} = p_1(\theta_1^2 - \theta_2^2) + \theta_2^2 \qquad (27)$$

$$\frac{M_3}{6} = p_1(\theta_1^3 - \theta_2^3) + \theta_2^3 \qquad (28)$$

بحل المعادلات (26) و (27) و (28) نجد

$$\begin{vmatrix} M_1 & \theta_1 - \theta_2 & \theta_2 \\ \frac{M_2}{2} & \theta_1^2 - \theta_2^2 & \theta_2^2 \\ \frac{M_3}{6} & \theta_1^3 - \theta_2^3 & \theta_2^3 \end{vmatrix} = 0$$

بعد إجراء العمليات الحسابية وبحساب المحدد

$$M_3 + 6\theta_1\theta_2 M_1 - 3M_2(\theta_1 + \theta_2) = 0$$

اذن

$$\theta_1 = \frac{M_3 - 3M_2\theta_2}{3M_2 - 6M_1\theta_2} \qquad (29)$$

وبحل المعادلات (26) و (27) انيا نحصل على

$$\theta_1 = \frac{M_2 - 2M_1\theta_2}{2(M_1 - \theta_2)} \qquad (30)$$

وبتساوي المعادلتين (29) و (30) نجد

$$6\theta_2^2(2M_1^2 - M_2) + 2\theta_2(M_3 - 3M_1M_2) - 2M_1M_3 + 3M_2^2 = 0 \qquad (31)$$

وبمساواة عزوم المجتمع M_1 و M_2 و M_3 المقدر مع عـزوم العينـة m_1 و m_2 و m_3 فـي المعادلة (31) نحصل على $\hat{\theta}_{21}$ و $\hat{\theta}_{21}^*$ كجذرين للمعادلة

$$6\theta_2^2(2m_1^2 - m_2) + 2\theta_2(m_3 - 3m_1m_2) - 2m_1m_3 + 3m_2^2 = 0$$

نعوض $\widetilde{\theta}_{21}$ في (29)

$$\widetilde{\theta}_{11} = \frac{m_3 - 3m_2\widetilde{\theta}_{21}}{3m_2 - 6m_1\widetilde{\theta}_{21}}$$

ومن (26) وبتعويض $\widetilde{\theta}_{11}$ و $\widetilde{\theta}_{21}$ نحصل على تقدير p_1

$$\widetilde{p}_1 = \frac{m_1 - \widetilde{\theta}_{21}}{\widetilde{\theta}_{11} - \widetilde{\theta}_{21}}$$

ان جذري المعادلة (31) قد لا يكونان كلاهما موجب او حقيقي وكلما ازداد حجـم العينـة تكون الجذور موجبة او حقيقية[10]

ويمكن ايجاد الحالة الثانية بطريقة (Person) للتقدير بطريقة العزوم [1]

$$p_1\theta_1 + (1 - p_1)\theta_2 = \overline{X} \qquad (32)$$

$$p_1\theta_1^2 + (1 - p_1)\theta_2^2 = \frac{\sum_{i=1}^{n} x_i^2}{2n} = b \qquad (33)$$

$$p_1\theta_1^3 + (1 - p_1)\theta_2^3 = \frac{\sum_{i=1}^{n} x_i^3}{6n} = c \qquad (34)$$

الجانب التجريبي

يعرض هذا البند من الدراسة مقارنة مقدرات المعالم التي توصلنا اليها باستخدام المحاكاة لكل من طريقة الامكان الاعظم وطريقة العزوم في تقدير معالم التوزيع الاسي المختلط ولدالة الامكان في الحالة الثانية باستخدام طريقة البحث المباشر (Direct search)

لقد تم اختيار حجوم عينة لكل مجتمع جزئي وهي (n_1=30,70,200) يمثل حجم العينة للمجتمع الجزئي الاول و(n_2=30,70,200) يمثل حجم العينة للمجتمع الجزئي الثاني اما($n=n_1+n_2$) يمثل حجم العينة اذ ان (n=60) يمثل اصغر الحجوم و (n=100,140,230,270) يمثل متوسط الحجوم و(n=400) يمثل اكبرالحجوم للعينة المستخدمة. لقد تم تحديد قيم افتراضية لمعالم القياس θ_1 و θ_2 حيث θ_1 تمثل معلمة القياس للمجتمع الجزئي الاول و θ_2 تمثل معلمة القياس للمجتمع الجزئي الثاني، وتم اجراء اربع تجارب، حيث كانت قيم المعالم مبينة ازاء كل تجربة وكما موضح في الجدول ادناه:

جدول (1)

يبين قيم المعالم الافتراضية

التجربة / المعلمة	1	2	3	4
θ_1	0.2	0.2	0.2	0.7
θ_2	0.2	0.7	1.5	1.5

القيمة الافتراضية للنسبة P يتم تحديدها من خلال العلاقة $\hat{P}=\frac{n_1}{n}$

ثم تم توليد أرقام عشوائية ذات توزيع اسي مختلط وذلك من خلال استخدام اسلوب التوزيع المعكوس (invers transformed method) **و** تم تقدير معالم القياس θ_1 و θ_2 والنسبة P لمجتمعين جزئيين في التوزيع الاسي المختلط في الصيغ السابقة (22) , (23) , (24) , (32) , (33),(34) ثم تمت المقارنة بين طرائق تقدير معالم القياس والنسبة لتوزيع الاسي المختلط ، وباستخدام المقياس الإحصائي (متوسط مربعات الخطا mean square error) لاجراء المقارنة بين طرائق تقدير معالم القياس والنسبة والذي يكون بالصيغة الاتية :

$$MSE(\hat{\theta})=\frac{1}{L}\sum_{i=1}^{L}(\hat{\theta}_i-\theta)^2$$

اذ ان

$\hat{\theta}_i$: تقدير معالم القياس او النسبة حسب الطرائق المستخدمة

L : عدد المكررات لكل تجربة حيث كانت $L=1000$

وتم اعداد برنامج بلغة (Visual Basic) للحصول على نتائج المحاكاة بعد تنفيذه على الحاسبة.

جدول (2)

يبين قيم متوسط مربعات الخطا لمختلف حجوم العينات الجزئية وقيم المعلمات الافتراضية بطريقة العزوم وطريقة الامكان الاعظم

الحجوم	قيم المعلمات الافتراضية			متوسط مربعات الخطا بطريقة العزوم			متوسط مربعات الخطا بطريقة الامكان الاعظم		
	P	θ_1	θ_2	p	Θ_1	θ_2	p	θ_1	θ_2
$n_1=30$ $n_2=30$	0.5000	0.2000	0.2000	0.2351	0.0400	0.0000	0.1297	0.0123	0.0003
	0.5000	0.2000	0.7000	0.0701	0.0400	0.0101	0.0172	0.0109	0.0000
	0.5000	0.2000	1.5000	0.0195	0.0400	0.0181	0.0013	0.0065	0.0022
	0.5000	0.7000	1.5000	0.1416	0.4900	0.0533	0.1136	0.2711	0.0382
$n_1=30$ $n_2=70$	0.3000	0.2000	0.2000	0.0250	0.0000	0.0000	0.0001	0.0000	0.0000
	0.3000	0.2000	0.7000	0.0480	0.0400	0.0104	0.1810	0.0123	0.0034
	0.3000	0.2000	1.5000	0.0205	0.0400	0.0325	0.0038	0.0097	0.0045
	0.3000	0.7000	1.5000	0.0818	0.4900	0.0513	0.0597	0.3583	0.0309
$n_1=30$ $n_2=200$	0.1304	0.2000	0.2000	0.0122	0.0000	0.0000	0.0005	0.0000	0.0000
	0.1304	0.2000	0.7000	0.7522	0.2004	5.6947	0.7361	0.1855	3.3874
	0.1304	0.2000	1.5000	0.7038	1.2298	3.3976	0.6420	1.6012	1.9909
	0.1304	0.7000	1.50000	0.7561	0.5378	5457.43	0.7409	0.4801	18.866
$n_1=70$ $n_2=70$	0.5000	0.2000	0.2000	0.2416	0.0001	0.2368	0.2284	0.0000	0.1953
	0.5000	0.2000	0.7000	0.1038	0.0187	0.2858	0.0761	0.0139	0.1639
	0.5000	0.2000	1.5000	0.0675	0.0770	0.8812	0.0074	0.0064	0.1181
	0.5000	0.7000	1.5000	0.1578	0.0851	2.3150	0.1863	0.1140	3.8884
$n_1=70$ $n_2=200$	0.2593	0.2000	0.2000	0.0254	0.0000	0.0000	0.1815	0.0000	0.0000
	0.2593	0.2000	0.7000	0.4373	0.1124	0.2948	0.4689	0.1208	0.4217
	0.2593	0.2000	1.5000	0.1600	0.3938	0.1966	0.0830	0.2214	0.1130
	0.2593	0.7000	1.5000	0.5452	0.3922	24.0271	0.5335	0.3535	15.9361
$n_1=200$ $n_2=200$	0.5000	0.2000	0.2000	0.1606	0.0000	0.0000	0.2473	0.2141	0.0000
	0.5000	0.2000	0.7000	0.0010	0.0044	0.0063	0.0001	0.0028	0.0070
	0.5000	0.2000	1.5000	0.0022	0.0009	0.0477	0.0029	0.0000	0.0432
	0.5000	0.7000	1.5000	0.2447	0.1284	3.1598	0.2353	0.1177	1.6884

نتائج المحاكاة

من ملاحظة نتائج تجارب المحاكاة في الجدول (2)نجد انه:

(ا)عند حجم العينة الصغيرة(n=60)

نلاحظ ان مقدرات الامكان الاعظم للمعالم وبالاعتماد علىمقياس متوسط مربعات الخطا كان افضل من مقدرات العزوم ولكن عندما تكون القيمة الافتراضيةلـ$\theta_1=0.2$, $\theta_2=0.2$, p=0.5

يكون مقدر العزوملـ θ_2 هو الافضل من مقدر الامكان الاعظم

(ب)عند حجوم العينة المتوسطة

1-عندما (n=100) نلاحظ ان مقدرات الامكان الاعظم للمعالم وبالاعتماد علىمقياس متوسط مربعات الخطا كان افضل من مقدرات العزوم ولكن عندما تكون القيمة الافتراضية لـ$\theta_2=0.7$, $\theta_1=0.2$,p=0.3يكون مقدرالعزوم لـp افضل من مقدر الامكان الاعظم

2-عندما(n=230) نلاحظ ان مقدرات الامكان الاعظم للمعالم وبالاعتماد علىمقياس متوسط مربعات الخطا كان افضل من مقدرات العزوم ولكن عندما تكون القيمة الافتراضية لـ$\theta_2=0.7$, $\theta_1=1.5$ ،p= 0.1304 يكون مقدر العزوم لـ θ_1 هو الافضل من مقدر الامكان الاعظم

3-عندما(n=140) نلاحظ ان مقدرات الامكان الاعظم للمعالم وبالاعتماد علىمقياس متوسط مربعات الخطا كان افضل من مقدرات العزوم ولكن عندما تكون القيمة الافتراضيةلـ $\theta_2=1.5$, $\theta_1=0.7$, p=0.5 يكون مقدر العزوم للمعالم هو الافضل من مقدر الامكان الاعظم

4-عندما (n=270)نلاحظ ان مقدر العزوم لمقياس متوسط مربعات الخطا وعندما تكون القيمة الافتراضيةلـ$\theta_2=0.2$, $\theta_1=0.2$, p=0.2593 و $\theta_2=0.7$, $\theta_1=0.2$, p=0.2593افضل من مقدر الامكان الاعظم

(ج)عند حجم العينة الكبيرة (n=400)

نلاحظ ان مقدر الامكان الاعظم لمتوسط مربعات الخطا هو الافضل من مقدرالعزوم

ومن خلال المقارنة بين مقدرات الامكان الاعظم لمتوسط مربعات الخطأ ومقدرات العزوم وبالاعتماد على مقياس متوسط مربعات الخطا. وجد ان مقدر الامكان الاعظم للجدول (3-2) ولجميع حجوم العينات الجزئية كان هو الافضل والسبب في ذلك يعود الى انه يمتلك اقل قيمة لمتوسط مربعات الخطا.

الاستنتاجات

1. يفضل اعتماد الحالة الثانية للتوزيع المختلط لانه لا يمكن دائما تحديد المفردة الى مجتمع جزئي معين.
2. يمكن تعميم بعض النظريات للتوزيع المختلط الذي يمتلك مجتمع جزئي واحد الى مجتمعين جزئيين او اكثر.
3. يمكن اعتبار أي توزيع (مستمر، متقطع) هو توزيع مختلط يمتلك مجتمع جزئي واحد.
4. نجد من خلال المقارنة بين طريقتي الامكان الاعظم والعزوم وبشكل عام بان طريقة الامكان الاعظم هي الافضل لتقدير المعالم. وذلك لاغلبية حجوم المجتمعات الجزئية.
5. عند حجوم العينات الصغيرة تظهر تقديرات للمعالم سالبة او غير حقيقية لذا يفضل استخدام حجوم عينة كبيرة في طريقة العزوم .
6. حجم العينه للمعلمه الاولى دائما ناخذه أصغر لانه سوف يعطي نتائج أدق وكلما كانت قيمه المعلمه صغيرة تكون أكثر دقه في أعطاء النتائج وعندما تكون كبيرة ستقترب من التوزيع المنتظم

التوصيات :-

1- اعتماد الحالة الثانية للتوزيعات المختلطة حيث لا يمكن تحديد المفردة للمجتمع الجزئي .
2- التوسع باستخدام طرائق اخرى في التقدير للمعلمات ومقارنتها مع الطرائق التي تم استخدامها في هذا البحث .
3- التوسع في جانب المحاكاة باستخدام ثلاثة مجتمعات جزئية للتوزيع الاسي المختلط او اكثر وبيان اثر ذلك في نتائج طرائق التقدير .

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