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CONTENT

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6. The same facts should not be given in tables and figures except when it is absoutely necessary to do so.
7. Reference numbers should be written between square brackets []. A list of referances should be given on a separate sheet of paper.
8. Where possible, Papers should follow the Pattern: Introduction, Experimental, Results and Discussion.

AL-MUSTANSIRIYAH JOURNAL OF SCIENCE

1980

C O N T E N T

A NEW TECHNETIUM SULPHIDE COLLOID WITH COPPER AS A CARRIER

Abdul Munim Al-Hilli, Adel F. Roomaya* And S. Al-Murab

A B S T R A C T

A new formulation for Technetium Sulphide Colloid, had been adopted using Cu II as a carrier.

The characteristics of ^{99m}Tc- eluates used in the preparation together with the method of preparing the colloid have been mentioned.

The choice of favourable conditions to secure the stability of Tc_2S_7 formed in the reaction mixture, and the radiochemical yield of the product had been investigated.

INTRODUCTION

The desirable physical characteristics of 99^{m}Tc, which are low gamma radiation energy (140 Kev), short half-life (6 hours) and the absence of beta-radiation [1], potentiate the use of this radionuclide as anexcellenttool for brain scanning. This however is not due to the selectivity of brain tissue to this nuclide, but to the capability of using a relatively larger amounts of activity, compared with other nuclides previously used. Its use will result in a decrease in scanning time and improved percision. Since the benefit of ^{99m}Tc in brain scanning depends on its exclusion from normal brain tissue [2].

Chemical review of ^{99m}Tc show that, this element has many oxidation states, but the most stable are the tetravalent and heptavalent states in aquoeus solution [3]. The heptavalent state is the most important for preparing the labelled compounds.

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The useful application of ^{99m}Tc- pertechnetrate in the eluates of the generator for preparation of the labelled compounds, is largely dependant on the quality of the eluates, which are rather variable from one generator to another, though these eluates should have a certain requirements to fulfil the purpose: a high radionuclidic purity, which puts the limit on the presence of long lived radionuclides, mainly ^{99}Mo from the possible breakthrough the generator column [4], in any case ^{99}Mo, should be less than 0.1 uci/1 mci ^{99m}Tc in the eluate [2,5], since its presence in larger amounts will add to the activity of the product and later to the dose received by the patient, besides that it will migrate to other sites in the body rather than the desirable organ. Morever ^{99m}Tc in the eluates should be at least 95% in the heptavalent state otherwise labelling would not be efficient and radiochemical impurities will appear in the product.

Different formulations had been adopted for the preparation of ^{99m}Tc-sulphide colloid, whereas in all of them, technetium heptasulphide and the stability of the colloid are the most important and could be summerized as follows:

1. The selection of a proper source of sulphide ions.
2. The incorporation of a suitable metal ion as a carrier for techmetium heptasulphide.
3. The use of a certain hydrophilic stabilizer for the colloid.
4. The adjustment of the final pH by a suitable buffering system to safeguard stability of the colloidal preparation.

MATERIALS AND METHODS

Reagents.

All the chemicals used in preparing the reagent solutions were of analytical grade.

Solution No. 1

$CuCl_2$	387	µg/ml
Gelatine	33.2	mg/ml

The gelatine soaked in cold distilled water, then heated on water bath to complete dissolution and passed through sintered glass (G.4) while hot. $CuCl_2$ dissolved to the final volume with distilled water. The two solutions were mixed together and completed.

Solution No. 2

$Na_2S_2O_3$	12.750 mg 1
Na_2CO_3	0.125 mg/ml

Solution No. 3

HCl	2 N.

Solution No. 4

Sodium citrate	143.8 gm/ml
Citric acid	29.3 mg/ml
NaOH	40.0 mg/ml

ASSAY

Assay of the previously prepared solutions were performed according to the conventional analytical methods, while the concentration of copper in solution No. 1 had been determined colorimetrically*, by forming a colored complex with sodium diethyldithiocarbamate in an ammoniacal solution.

* A modified method from procedures of laboratory application, pye unicam on the determination of copper in food stuffs.

The total citrate in the buffer solution No. 4 had been determined alkalimetrically after retaining the Na^+ present in the solution by Dowex-50 in a column.

Preparation of the Colloid:

5 ml of copper gelatine solution in 30 ml capacity vial, was placed in a lead container, then 5 ml ^{99m}Tc pertechnetate eluate had been delivered to the vial by means of a pipette, mixed well, followed by the addition of 2 Ml thiosulphate solution and 2ml HCL solution respectively, by means of two new pipettes, shaked well and left to stand for 5 minutes to initiate the reaction. The vial was taken from the container and placed in a hot water bath for 1.5 minute, then cooled in a stream of cold water. The vial was returned to the lead container, 5 ml of the citrate buffer was added at the end of the preparation and the colloid mixed thoroughly to insure homogenity.

Characteristics of the final product:

Volume	19 ml
Colour	brown
TcO_4^-	$< 5\%$
pH	5.2-5.7

Radiochemical purity investigation:

Six technetium sulphide colloidal solutions had been prepared from eluates of two different generators during two successive weeks (three preparations a week from one generator eluates).

An ascending paper chromatographic runs had been carried out of sample aliquots on three whatman No. 1 paper strips, for each preparation. 85% methanol in water was used as a solvent, the development time of the chromatograms was 3 hours. The separated radioactivity were counted after autoradiography, in a well-type scintillation detector connected to 512-channels pulse Height analyser

(Nuclear chicago), where the activity of the sulphide was expressed as percentage of the total activity of the chromatogram. The Rf. of the separated activity on the strips were: zero for Tc_2S_7 colloid and 0.5 - 0.6 for TcO_4^- (radiochemical impurities).

Another eight heptasulphide colloids were prepared by the previously mentioned method from eluates of one generator, but the buffer added to each preparation in the final step contains variable concentrations of NaOH, which gave a different pH values to the final colloids ranging from 4.4 - 6.2. Radiochemical purity run for each colloid was determined by ascending paper chromatography and the pH of each solution was measured by a calibrated pH-meter (Radiometer-Copenhagen, type 26) with a combined glass-calomel electrode assembly. A graph was obtained showing the effect of pH variation on the radiochemical purity (Fig. 1).

RESULTS AND DISCUSSION

The usefulness of Technetium sulphide preparation for liver scanning purposes is greatly dependant on the colloid characters of the product, since the presence of millicurie quantities of pertechnetate in the generator eluates, involve such a minute amounts of ^{99m}Tc in the order of 1.153×10^3 atoms/10 millicuries, that chemically Tc_2S_7 formed in the reaction mixture does not form a precipitate or even an opalescence in the usual sense. Therefore the use of a carrier becomes an important factor to secure the colloidal properties of the preparation. Moreover Tc_2S_7 alone without a carrier hydrolyzes easily to soluble pertechnetate in aqueous solutions [6].

Patton et al [7], have introduced Technetium sulphide colloid for clinical use by acid reduction of thiosulphate in presence of rhenium in form of perrhenate as a carrier, since it has resemblance

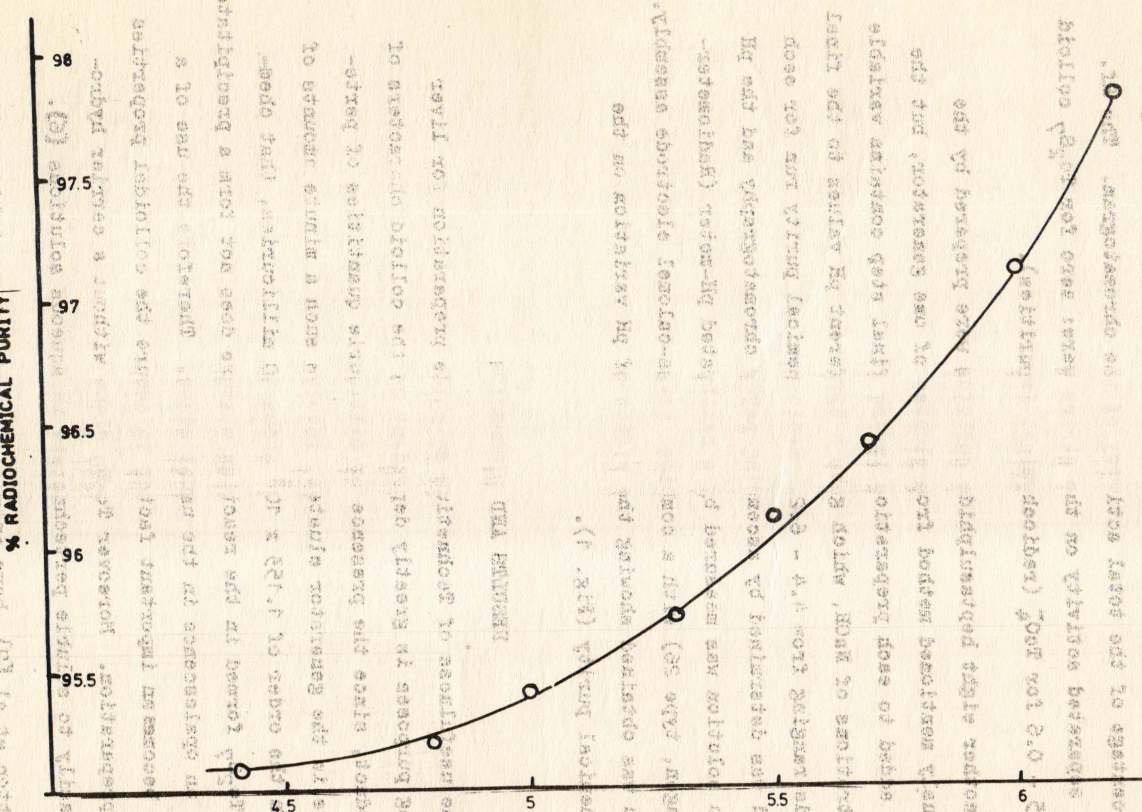

FIG. 1. SHOWING THE RELATION BETWEEN PH & RADIOCHEMICAL PURITY OF THE PREPARED ^{99m}TC_SULPHIDE COLLOID AT PH HIGHER THAN 5.7, DISSOLUTION OF THE COLLOIDAL PARTICLES HAVE BEEN STARTED

in chemical properties to technetium. This Technique was devised for the coprecipitation of Tc_2S_7 with Re_2S_7, where the latter present work we have tried CuII in form of $CuCl_2$ as an alternative to Re carrier. The Tc_2S_7 formed had been coprecipitated with CuS where te the latter acts as a collector [8] for the heptasulphide of technetium.

Furthermore we would like to point out that traces of copper are not harmful to the living body and even essential for normal growth [9] while rhenium are not.

The results of radiochemical purity investigations have shown that higher than 95% of the redioactivity was in the sulphide form, which reveals a good indication for the stability of Tc_2S_7 in the preparation and a high radiochemical yield been obtained in the product. Although the presence of gelatine in the mixture can serve as a hydrophilic stabilizer for the colloid and prevents coalescence.

Trials had been made with polyviyhlpyrrolidone (PVP) as a substitute of gelatine, since PVP is a safe and compatable chemical to body fluids, with a powerful hydrophilic properties. The biological characters of PVP stabilized colloids seem to be equal or better suited for clinical application [10]. The results were unsuccessful, since the yield of radioactivity in the sulphide form was low, where this stabilizer interfere with the formation of Tc_2S_7. It has been found that the more powerful the stabilizer the more distinct is the effect [6].

The results obtained in Fig. 1 have shown that at pH lower than 5, part of Tc_2S_7 formed in the mixture had been easily hydrolyzed to soluble pertechnetate giving a relatively low radiochemical yield. While at higher pH approching 6, dissolution of the colloidal particles had been noticed and a clear solution resulting in a short period of time.

Experimental runs have indicated that best results were obtained at pH 5.2 - 5.7 which give a favourable stability of the colloid and a high radiochemical yield, without appreciable hydrolysis.

CONCLUSION

Technetium sulphide colloid prepared with a copper carrier giving a promising results, mainly the Tc_2S_7 formed in the product have shown a good stability for a reasonable length of time. The concentration of solutions used in the reaction mixture had been chosen from the study of kinetics of the reaction between the principal reactants without leaving an excess in the product. The order of mixing the reagent solutions in a certain sequence (as mentioned under preparation of the colloid) is an important factor to safeguard the radiochemical yield, since the addition of the reagent solutions in any other sequance give a poor yield.

The reagent solutions used in the preparation should not be kept more than one month otherwise deterioration of some of their constituents will takes place and the product not be satisfactory, especially the thiosulphate solution which impart a bluish coloration to the product when stored for a long time, Overheating the reaction mixture gives the same result and should be avoided.

From the work mentioned in this Paper, it has been shown that, the prepared technetium sulphide colloid, according to this formulation, has passed the chemical control satisfactorily but still not advised for routine application until it would satisfy the biological control, in particular the distribution and localization of radioactive material in the living oxganism, which will be considered in the near future.

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RECENT FORMINIFERIDA FROM THE SEA SHORES OF YEMEN ARAB REPUBLIC PART 2 THE GENUS TRILOCULINA

H. A. EL-NAKHAL*

ABSTRACT

More than 100 foraminiferidal species have been recorded from four sandy samples collected from different localities along the nearshore some of the southeastern Red Sea, Yemen Arab Republic. Twenty six species of these belong to the genus Quincueloculine which were discussed in part 1 (El-Nakhal, in press), and twenty one species belong to the genus Triloculina. The remaining forms which belong to the other foraminiferidal genera, will be treated in part 3.

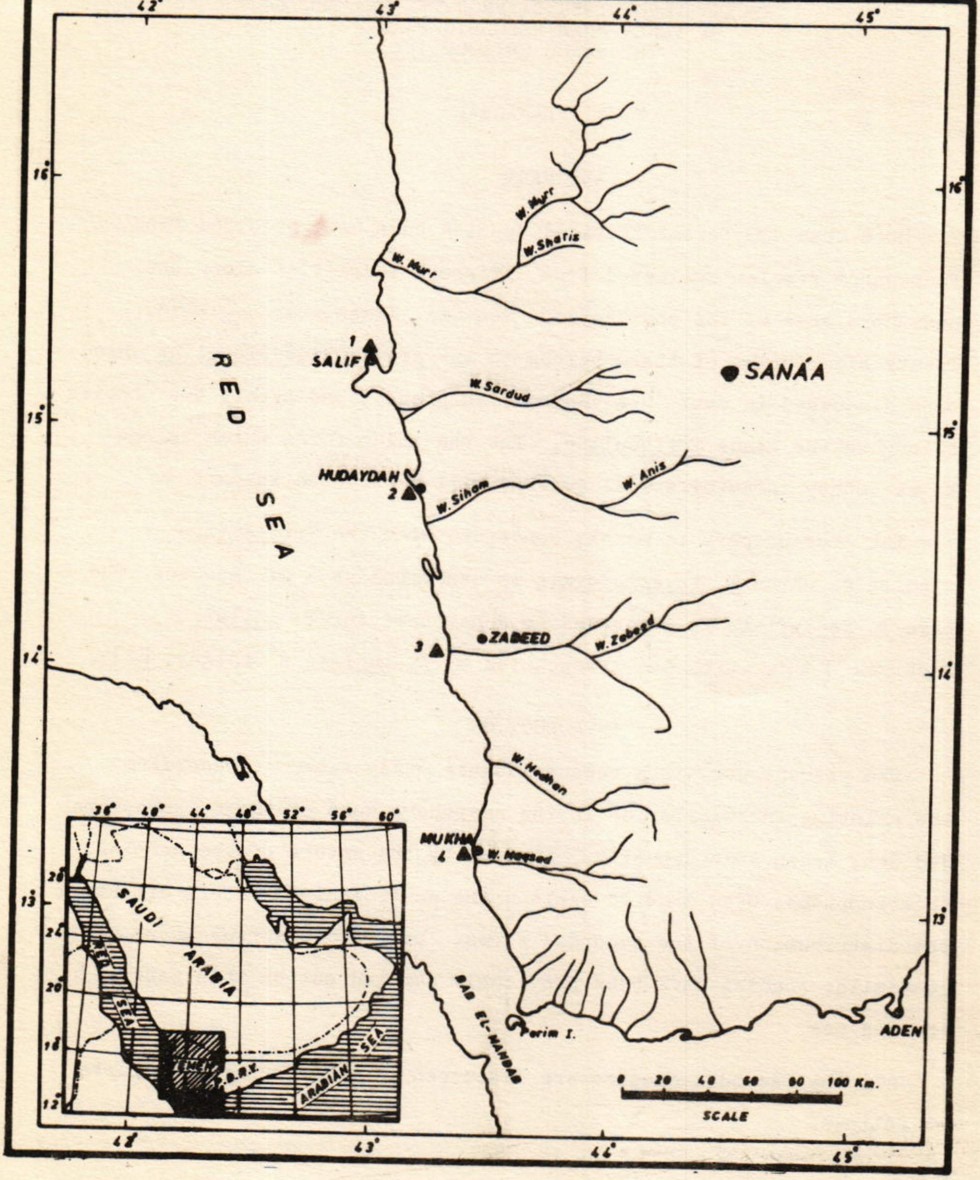
The present part is mainly concerned with the Triloculine species of which T. reversaformis is described as a new species. The name T. neoinflate is suggested as a new name for T. inflata Deshayes [1] , which was preoccupied by T. inflata d'Orbigny, [2] .

INTRODUCTION

The present work is a reconnaissance study aimed at recording the existing foraminiferida in the nearshore zone of the southeastern Red Sea, Yemen Arab Republic. Limited by the nature of the samples, no attempt has been made to explain the enviromental factors affecting the distribution of the recorded forms. Literature survey shows that no similar studies have been previously carried out on this part of the Red Sea.

All the recorded species are discussed in three successive parts as follows:

* Department of Geology, University of Sana's, Yemen Arab Republic.



Text-fig. 1: Sample collecting localities, Yemen Arab Republic.

Species \ Sample no. / Locality	1 Salif	2 Hudaydah	3 Zabeed	4 Mukha
T. affinis	F	F	A	F
brongniartiana	-	R	R	-
howchini	-	R	R	-
inflata	F	F	A	A
linnaiana	R	R	F	A
litteralis	F	A	F	F
longidentata	F	A	A	F
mindenensis	-	R	R	R
neoinflata	R	R	R	R
oblonga	R	F	R	R
peroblonga	-	-	R	-
quadrata	R	R	F	R
reversaformis	A	A	A	A
retunda	R	F	A	R
subgranulata	R	F	A	R
suttuensis	R	R	F	R
terquemiana	F	A	A	A
tricarinata	F	A	A	A
trigonula	A	A	A	A
trihedra	-	F	F	R
tubiformis	R	-	R	-

Table 1: The frequency of the recorded species in the studied localities.

R = Rare: 1-4 specimens.

F = Frequent: 5-15 specimens.

A = Abundant: over 15 specimen.

Part 1: concerned with the genus Quincueloculina d'Orbigny (El-Nakhal, in press).

Part 2: (The present part): is devoted to the genus Triloculina d'Orbigny.

Part 3: Will treat the remaining genera.

METHODS OF STUDY

Four sandy samples were collected during the period February-April 1978, from the sea shore at Salif, Hudaydah, Zabeed and Mukha (Fig. 1). These samples were collected from nearshore areas at about 1.5 m depth. One hundred grams of each of the original sandy samples were treated for studying their foraminiferidal content and they were hand picked under the binocular microscope. Several species were identified by using the catalogue of foraminifera [3]. All the recorded species were drawn by the author from camera lucida. The frequency of these forms in the different localities, is shown in (Fig. 1). The illustrated specimens are deposited in the Department of Geology, University of Sana'a.

SYSTEMATIC DESCRIPTIONS

The systematic position of the genus Triloculina d'Orbigny is according to the classification of Loeblich and Tappan [4]

Order Foraminiferida Eichwald, 1830

Suborder Miliolina Delage and Herouard, 1896

Superfamily Miliolacea Ehrenberg, 1839

Family Miliolidae Ehrenberg, 1839

Subfamily Quinqueloculininae d'Orbigny 1826

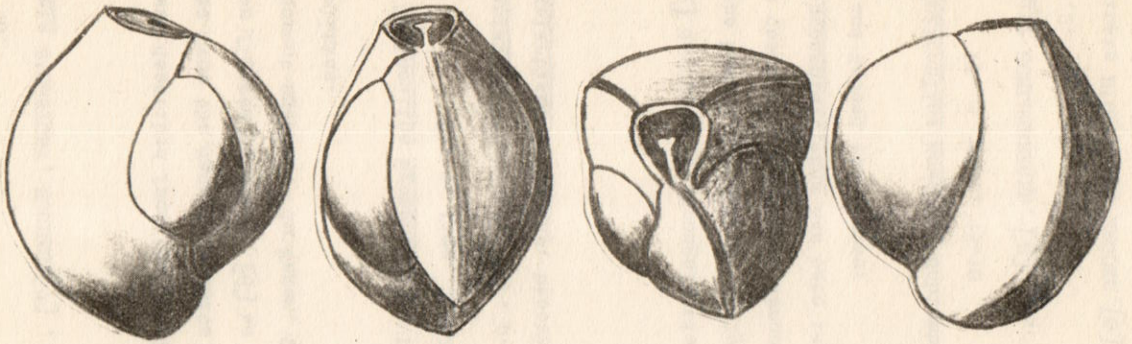
Genus Triloculina d'Orbigny, 1826

Triloculina affinis d'Orbigny

Pl. 2, figs. 5-8

Triloculina affinis D'ORBIGNY, [2], P. 133 (299) nom.mud.

Triloculina affinis D'ORBIGNY, [5], P. 161, Pl. 1, fig. 1.



5 6 7 8

Figs. 5-8 T. affinis d'Orbigny, from Zabeed sea shore, X 105.

PLATE 2

Triloculina Baldai Bermudez and Seiglie, BROOKS, [6] , P. 406, Pl. 6, figs. 5, 10.

Triloculina affinis D'ORBIGNY, HUGHES,[7] , P. 48 (no figs.).

Remarks:

T. affinis was described from the Tertiary of France [5] . It was later recorded from the Recent deposits of Solomon Islands [7] . The form which was figured by Brooks [6] as T. Baldai, most probably belongs to the present species. In Yemen, T. affinis occurs in the four studied localities.

Triloculina Brongniatiana D'Orbigny

Pl. 4, figs. 10-12

Triloculina brongniartii D'Orbigny, [2] , p. 300

Triloculina brongniartiana D'Orbigny, Brooks, [6], p. 406, pl. 6, figs. 3-4.

Remarks:

D(Orbigny [2] described T. brongniartiana as a fossil form as well as from the Recent deposits of Italy. It was later recorded from the southern coast of Puerto Rico (Brooks, [6]). In the present study, T.brongníartianna has been recorded as a rare form in both Rudaydah and Zabeed sea shores.

Triloculina howchini Schlumberger

Pl. 3, figs. 13-16

Triloculina howchini SCHLUMBERGER, [8] , p. 119, pl. 3, fig 6; text-figs. 1,2.

Triloculina bicarinata D'Orbigny, BROOKS, [6] , p. 406, pl. 6, figs. 11, 12.

Remarks:

T.howchini was described from the Late Eocene of Australia and Phiĺippine Islands, [8] . It was later recorded from the Recent

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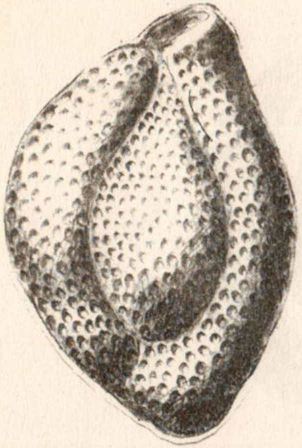
Figs. 10-12 T. brongniartiana d'Orbigny, from Hudaydah sea shore X 55.

PLATE 4



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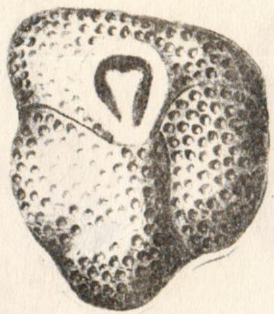

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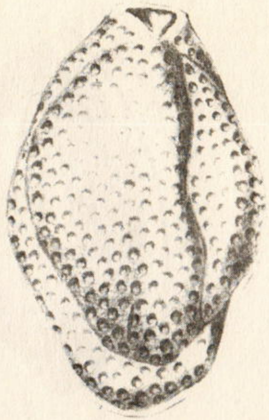

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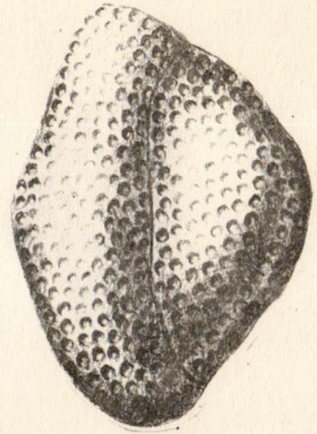

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Figs. 13-16 T. howchini Schlumberger, from Hudaydah seashore, X 90.

PLATE 3

deposits of Puerto Rico, as T.bicarinata d'Orbigny [6] . In the studied localities, T.howchini occurs as a rare form in Hudaydah and Zabeed sea shores.

Triloculina inflata d'Orbigny

Pl. 1, figs. 7-9

Triloculina inflata d'Orbigny, [2] , p. 300, pl. 8, fig. 16; pl. 17. figs. 13-15

Non Triloculina inflata Deshayes [1] , p. 251, pl. 4, figs. 1-3.

Triloculina inflata d'Orbigny, [9] , p. 130, pl. 2, fig. 18

Triloculina sp. [6] , p. 402, pl. 3, figs. 4-6.

Remarks:

The present species was recorded from Italy and France as a fossil and from the Recent sediments of the Mediterranean Sea. It was later recorded from the coasts of western North America [9] , and Puerto Rico [6] . In the present study. T. inflata occurs in the four localities

Triloculina linneiana d'Orbigny

Pl. 4, figs. 7-9

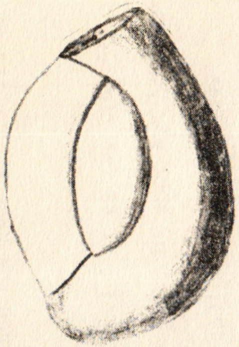
Triloculina linneiana [10] , p. 173, pl. 9 figs 11-13.

Triloculina linneiana d'Orbigny, [6] , p. 408, pl. 7, figs. 3,4.

Triloculina linneiana d'Orbigny, [7] , p. 48 (no figs.).

Remarks:

T.linneiana was described from the Recent deposits of Cuba and Jamaica [10] . It was later recorded from the southern coast coast of Puerto Rico [6] and from Solomon Islands [7] . The present species occurs in the four studies samples.



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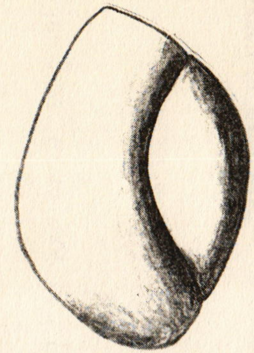

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Figs. · 7-9 T. inflata d'Orbigny, from Zabeed sea shore, X 85.

PLATE I

7 8 9

Figs. 7-9 T. linneiana d'Orbigny, from Mukha Sea shore, X 90.

PLATE 4

Triloculina litteralis Collins

Pl. 4, figs. 4-6

Triloculina littoralis [11] , p. 369, pl. 3, fig. 12.

Remarks:

The present form was originally described from the Recent deposits of the Great Barrier Reef of Australia [11] . T.littoralis is abundant in Hudaydah sea shore and frequent in the remaining three localities.

Triloculina longidentata Bandy

Pl. 3, Figs. 1-3

Triloculina inornata d'Orbigny var. longidentata [12] , p. 178, pl. 21, fig. 2.

Remarks:

T.longidentata was recorded from the shallow water off the mouth of the Tijuana River, San Diego Country, California [12] . In Yemen, T.longidentata occurs as an abundant form in Hudaydah and Zabeed, Whereas it is frequent in Salif and Mukha sea shores.

Triloculina mindemensis Howe

pl. 1, figs. 13-15

Triloculina mindenensis [13] , p. 37, pl. 3, figs. 11-13.

Remarks:

Howe [13] described T.mindenensis from the Eocene of Louisiana, U.S.A. In Yemen, T.mindenensis occurs as a rare form in Hudaydah, Zabeed and Mukha sea shores.

Triloculina meoinflata new name for

Triloculina inflata Deshayes, 1833

pl. 1, figs. 10-12

non Triloculina inflata [2] , p. 300, pl. 8, fig. 16; pl. 17, figs. 13-15.

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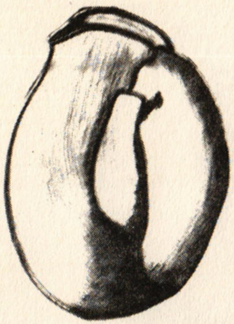
Figs. 4-6 T. littoralis Collins, from Hudaydah seashore, X 60.

PLATE 4



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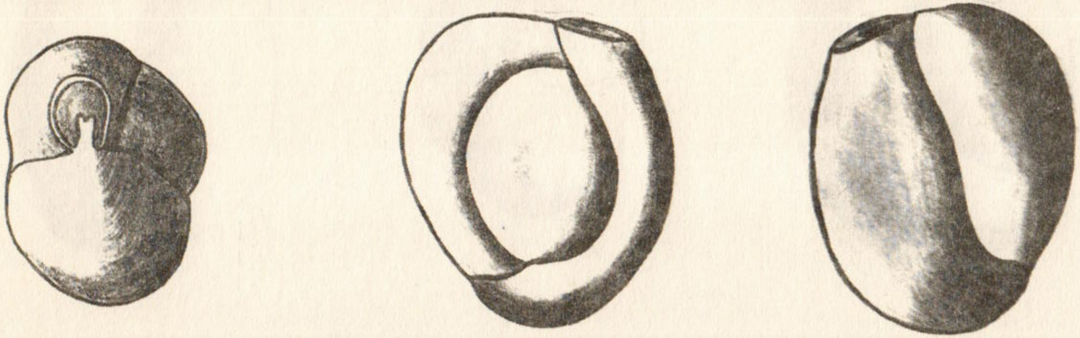
Figs. 1-3 T. <u>longidentata</u> Bandy, from Hudaydah sea shore, X 65.

PLATE 3

13 14 15

Figs. 13-15 T. mindenensis Howe, from Hudaydah sea shore, X 55.

PLATE I



10 11 12

Figs. 10-12 T. neoinflata El-Makhal, from Hudaydah sea shore, .
X 90.

PLATE I

Triloculina inflata DESHAYES [1] , p. 251, pl. 4, figs. 1-3.

Remarks:

Deshayes [1] described T.inflata from the Eocene of France. Due to the fact that the name T.inflata Deshayes [1] was preoccupied by T.inflata[2] , the new name T.meoinflata is suggested in the present study for Deshayes's form. In Yemen, T.meoinflata occurs as a rare form in the four stduied localities.

Triloculina oblonga (Montagu)

pl. 1, figs. 1-3

Vermiculum oblongum MONTAGU, 1803, p.522, pl. 14, fig. 9.
Triloculina sp. of. T.oblonga (Montagu), [14] , p. 35 (no figs.).
Triloculina oblonga (Montagu), [15] , p. 485 (no figs.).
Triloculina oblonga (Montagu), [16] , p. 414, pl. 1, fig 13.
Triloculina oblonga (Montagu), [7], p. 48, (no figs.)
Triloculina oblonga (Montagu), [17] , p. 120, pl. 2, figs 17, 18.

Remarks:

T. Oblonga was described from the Recent deposits of Devonshire, England (Montagu, 1803). It was later recorded from southeastern Louisiana, U.S.A. [14] , the English Channel [15] , eastern coast of India [16] , Solomon Islands [7] and the Central Arctic Ocean [17] . The present species occurs as a frequent form in Hudaydah sea shore and rene in Salif, Zabeed and Mukha.

Triloculina peroblonga Cushman

pl. 1, figs. 16-18

Triloculina peroblonga [18] , p. 143, pl. 34, figs. 4, 5.

Remarks:

T. peroblonga was originally described from the Early Oligocene of Marianna Limestone [18] . In Yemen, T. peroblonga has been recorded as a rare form in Hudaydah sea shore.

1 2 3

Figs. 1-3 T. oblonga (Montagu), from Zabeed sea shore, X 65.

PLATE I

16

17

18

Figs. 16-18 T. *peroblonga* Cushman, from Zabeed sea shore, X 65.

PLATE I

Triloculina quadrate Colline

pl. 3, figs. 7-9

Triloculina quadrata [11] , p. 369, pl. 3, figs. 13.

Remarks:

Collins [11] described the present species from the Recent deposits of the Great Barrier Reef of Australia. In the studied localities, T. quadrata occurs as a rare form in Salif and Hudaydah and Mukha whereas, it is frequent in Zabeed sea shore.

Triloculina reversaformis sp.nov.

pl. 4, figs. 1-3

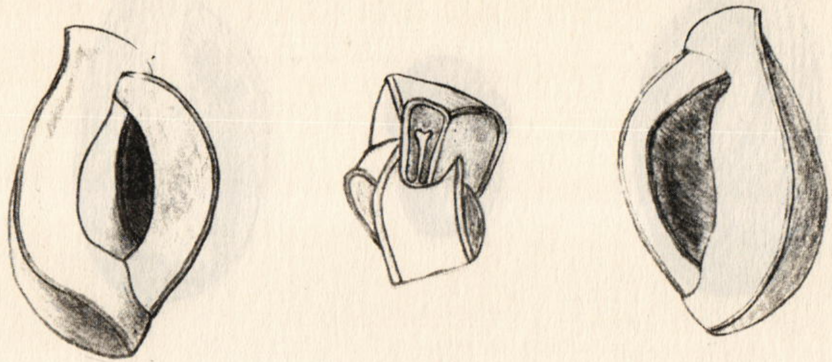
Description:

Test free, large, elongate, ovate in outline; equatorial periphery triangular, axial periphery rounded; chambers three at the last whorl, arranged in a triloculine manner, subcrescentic, the last chamber projects into a neck at the pertural end; sutures curved, slightly depressed; aperture subcircular, terminal at the end of a short prominent apertural neck, with bifid tooth; wall calcareous, prorcelaneous; surface ornamented by longitudinal costae which are slightly oblique.

Remarks:

T. reverseaformis sp. nov. most closely resemles both T. reversa d'Orbigny and T. littoralis Collins. It differs from T. reversa in having shorter and less oblique apertural neck. T. littoralis has compressed chambers and acute axial periphery, whereas the present species has inflated chambers and rounded periphery.

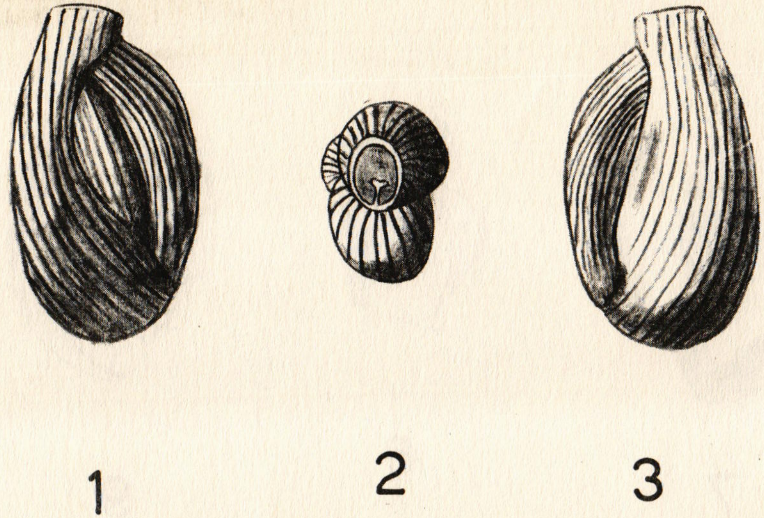
T. reversaformis occurs as an abundant form in all of the studies localities in Yemen. It has been previously recorded as T. sp. of. T T. reversa, in the Recent deposits of northern Arabian Guls (Shoblaq, unpublished study, Kuwait University, personal communications).



7 8 9

Figs. 7-9 T. quadrata Collins, from Zabeed sea shore, X 60.

PLATE 3



Figs. 1-3 T. reversaformia sp. nov., holotype, from Hudayda sea shore, X 65.

PLATE 4

Type locality: Nearshore zone, about four kms south Hudaydah port.

Type specimens: Holotype and paratypes are deposited in the Department of Geology, University of Sana'a.

Distribution: Occurs as an abundant form in the nearshore zone of Salif, Hudaydah, Zabeed and Mukha.

Age: Recent.

Dimentions of the holotype: Length 0.57 mm.
Width 0.31 mm.
Thickness 0.18 mm.

The species name refers to the resemblance to T. reversa.

Triloculina rotunda d'Orbigny

pl. 4, figs. 13-15

Triloculina rotunda [2] , p. 299 (nom.mud.).

Triloculina rotunda [10] , p. 64. [19]

Triloculina of. rotunda d'Orbigny, [19] , p. 18, pl. 2, fig. 1.

Triloculina rotuda d'Orbigny, [6] , p. 402, pl. 3, figs. 1-3.

Triloculina rotunda d'Orbigny, [17] , p. 120, pl. 1, fig. 21.

Remarks:

T. rotunda was originally described from the Recent deposits the Adriatic Sea, Italy [2] ,[10] . It was later recorded from the shore sands of western India [19] , the coast of Puerto Rico [6] , and the Central Arctic Ocean [17] . In Yemen, T. rotunda occurs in all of the studied samples.

Triloculina subgranulata Cushman

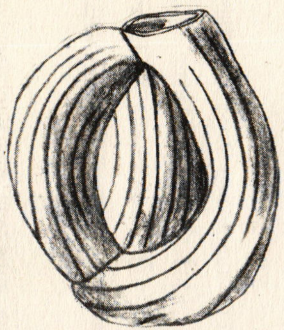
Pl. 3, figs. 10-12

Triloculina subgranulata [20] , p. 290, pl. 96, fig. 4.

Triloculina subgranulata Cushman, [7] , p. 48 (no figs.).

Remarks:

Cushman [20] described the present species from the Recent deposits of the Great Barrier Reef, Australia. It was later recorded



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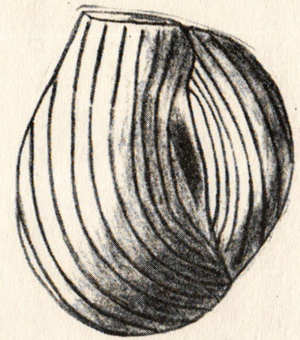

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Figs. 13-15 T. rotunda d'Orbigny, from Zabeed sea shore, X 95.

PLATE 4

10 11 12

Figs. 10-12 T. subramulata Cushman, from Zabeed sea shore, X 55.

PLATE 3

from Honiara Bay, Solomon Islands, southwest Pacific Ocean [7] . In the present study T. subgranulata occurs in the four localities.

Triloculina suttuensis Asano
pl. 2, figs. 1-4

Triloculina suttuensis [21] , p. 621, pl. 33, fig. 2.

Remarks:

Originally this species was recorded from the Neogene of Japan [21] . In Yemen, T. suttuensis is frequent in Zabeed sea shore where as it is rare in the other three localities.

Triloculina terquemiana (Brady)
pl. 4, figs. 16-19

Miliolina terquemiana [22] , p. 166, pl. 14, fig. 1.
Triloculina terquemiana (Brady), [19] , p. 19, pl. 2, fig.3.

Remarks:

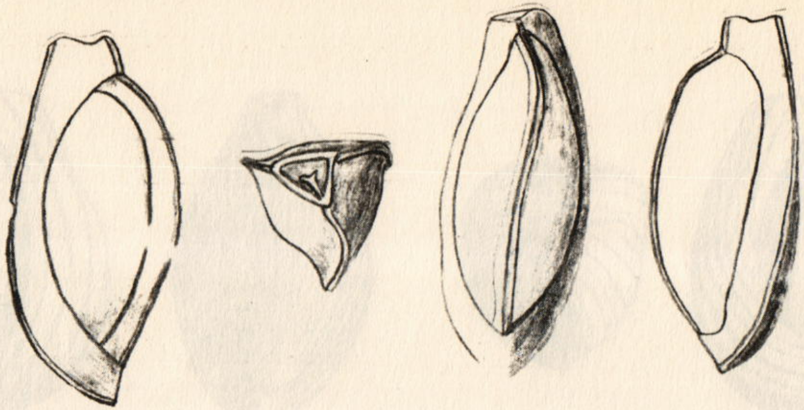
Brady [22] described T. terquemiana from the shallow water of Ceylon and Madagascar. It was later recorded from the shore sands of western India [19] . The present species occurs in all of the studied samples.

Triloculina tricarinata d'Orbigny
pl. 2, figs. 9-12.

Triloculina tricarinata [2] , p. 299, No. 94.
Triloculina tricarinata d'Orbigny, [19] , p. 19, pl. 1, fig. 16.
Triloculina tricarinata d'Orbigny, [23] , p. 86, (no figs.).
Triloculina tricarinata d'Orbigny, [7] , p. 48, pl. 3, figs. 59, 60.

Remarks:

T. tricarinata was described from the Red Sea [2] . It was later recorded from western coast of India [19] , the deposits of the Grand Banks, Newfoundland [23] , and Solomon Islands, southwest

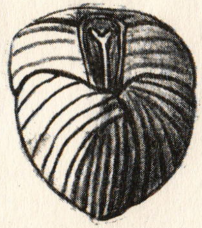


1 2 3 4

Figs. 1-4 T. suttensis Asano, from Zabeed sea shore, X 60.

PLATE 2

16



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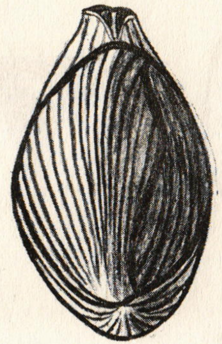

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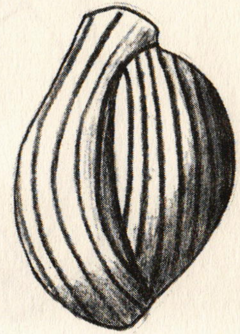

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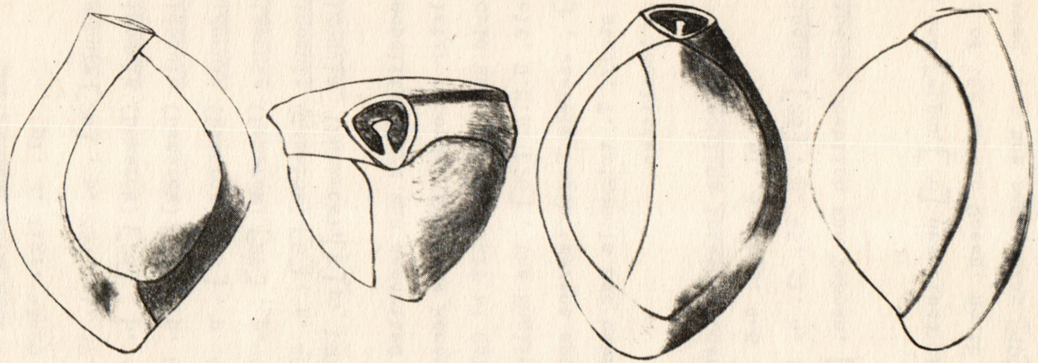
Figs. 16-19 T. terquemiana (Brady), from Hudaydah sea shore, 80.

PLATE 4



9 10 11 12

Figs. 9-12 T. tricarinata d'Orbigny, from Hudaydah sea shore, X 55.

PLATE 2

Pacific Ocean [7] . T. tricarinata is frequent in Salif seashore and abundant in the remaining localities.

Triloculina trigonula (Lamarck)

pl. 2, figs. 13-20

Miliolites trigonula [24] , p. 351, pl. 17, fig. 4.

Triloculina trigonula (Lamarck) [25] , p. 17 (no figs.).

Triloculina trigonula (Lamarck) [26] , p. 416 (no figs.).

Triloculina trigonula (Lamarck) [15] , p. 485 (no figs.).

Triloculina trigonula (Lamarck) [27] , p. 157, pl. 20, fig. 4.

Triloculina trigonula (Lamarck) [6], p. 400, pl. 2, figs. 13-16.

Triloculina trigonula (Lamarck), [9], p. 130, pl. 2, fig. 17.

Remarks:

This cosmopolitan form was described from the Eocene of France [24] . It was later recorded from the Recent deposits of several parts of the world such as: the Gulf of California [25] , the Atlantic Continental Shelf, U.S.A. [26] , the English Channel (Murray, 15), east India [27] , western North America and south Puerto Rico [6]. In the present study, T. trigonula has been recorded as an abundant form in the four localities.

Triloculina trihedra Loeblich and Tappan

pl. 3, figs. 4-6

Triloculina trihedra [28] , p. 45, pl. 4. fig. 10.

Triloculina trihedra Loeblich and Tappan, [17] , p. 120, pl. 1, fig. 10.

Remarks:

Loeblich and Tappan [28] described the present species from the Recent deposits of the Arctic Ocean, north Alaska and Greenland. It was later recorded from the same ocean [17] . In Yemen, T. trihedra is frequent in Hudaydah and Zabeed, rare in Mukha, and absent in Salif sea shore.

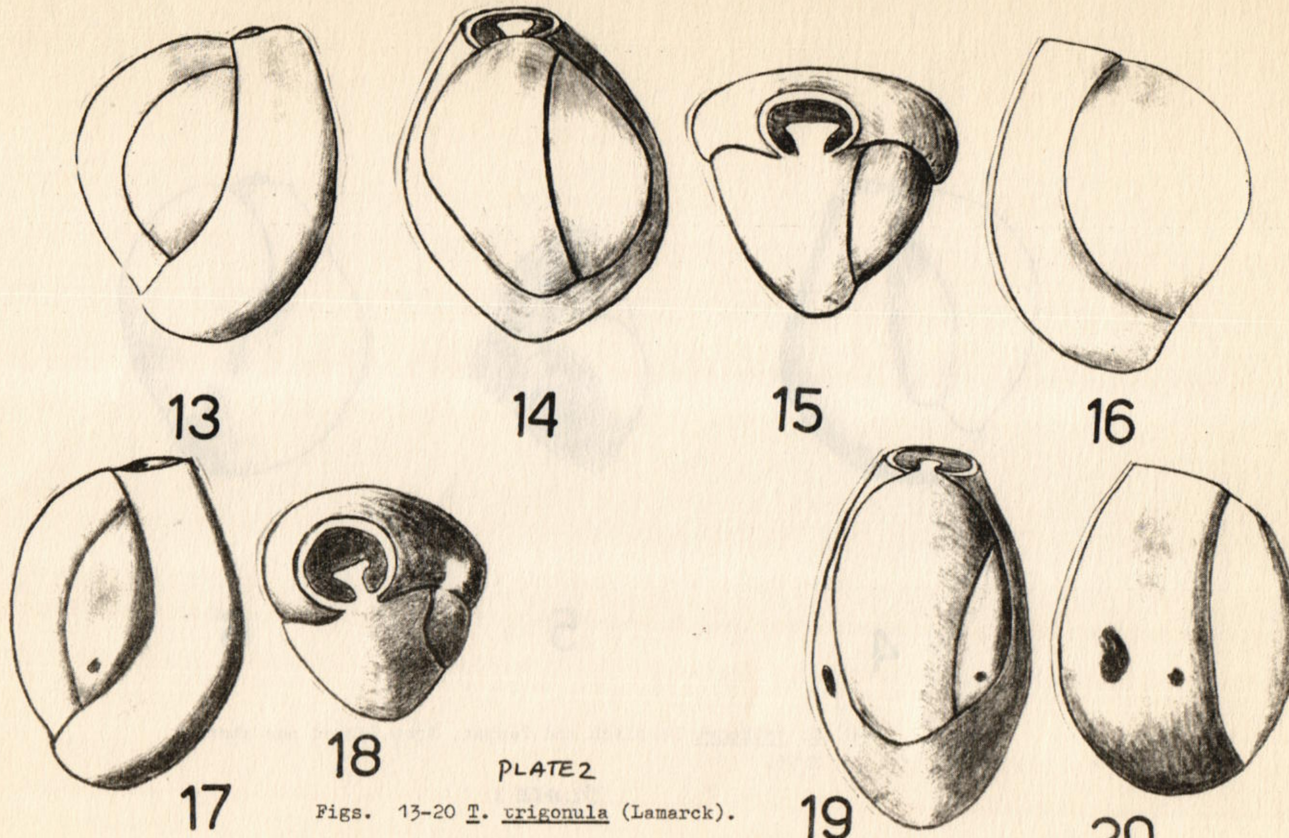
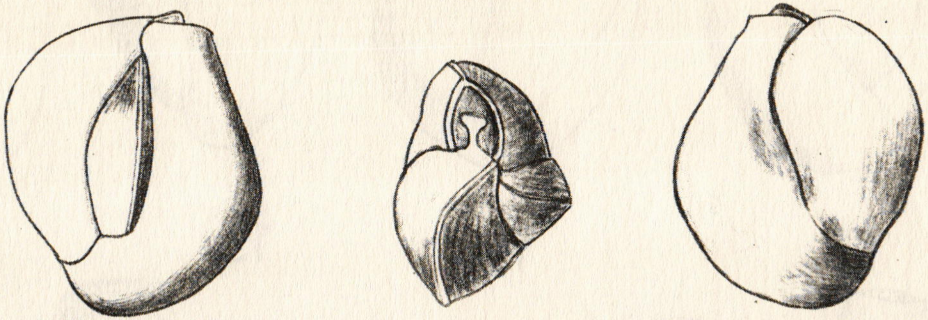


PLATE 2

Figs. 13-20 T. trigonula (Lamarck).

13-16 From Zabeed sea shore, X 75.

17-20 From Mukha sea shore, X 60.



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Figs. 4-6 T. trihedra Loeblich and Tappan, from Zabeed sea shore, X 75.

PLATE 3

<u>Triloculina tubiformis</u> Yabe and Asano

pl. 1, figs. 4-6

Triloculina tubiformis [29] , p. 116, pl. 17, fig. 9.

Remark:

T. tubiformis was described from the Pliocene of west Java, Netherland Indies [29] . The present species occurs as a rare form in both Salif and Zabeed seashore.

ACKNOWLEDGEMENTS

I am grateful to Dr. Z. El-Naggar of Qatar University, for introducing me to the present study. Appreciation is also due Dr. S. Abdel Razaq for allowing the use of the catalogue of foraminifera at Kuwait University. The author gratefully acknowledges Dr. H. El-Shatoury of Sana'a University, for his help in collecting sample No. 3.

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ON DOUBLE STAGE ESTIMATION OF THE MEAN VECTOR USING PRIOR INFORMATION*

ZUHAIR A. HAMID** AND H.A. AL-BAYYATI***

A B S T R A C T

A double-stage procedure using shrinkage technique is used for the estimation of the mean vector $\underline{U}$ of the normal population when the covariance matrix Σ is known or unknown using total or partial prior information.

INTRODUCTION

Katti [3] considered the problem of estimating the mean θ of the normal population by double sampling plan when there is a prior information θ_o about θ in the form of initial value. Arnold, and Al-Bayyati, [2] considered a double stage shrinkage estimator (DSSE) of the mean of the normal distribution in the univariate case. Their proposed estimation technique is to obtain a sample of size n_1 (the minimum value of n_1 is K^2N) to compute $\bar{X}_1$ and then construct a region R based on the prior information U_o. If $\bar{X} \in R$ we stop the sampling and shrink $\bar{X}_1$ toward U_o, if $\bar{X}_1 \notin R_1$; $n_2 = N-n_1$ additional, observation are taken and use the mean of all $N = n_1 + n_2$ observations is used as an estimate of U. Hence, the DSSE of the mean is given by

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* The article is based on the M.Sc. thesis of Zuhair A. Hamid.

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$$\widetilde{U} = \begin{cases} K \ (\bar{X}_1 - U_o) + U_o & \text{if} \ \ \bar{X}_1 \in R \\ \bar{X} = \dfrac{n_1\bar{X}_1 + n_2\bar{X}_2}{N} & \text{if} \ \ \bar{X}_1 \notin R \end{cases}$$

K : The shrinkage factor; $0 < K \leqslant 1$.

The region R which minimized the mean squared error (MSE) of $\widetilde{U}$ is given by :

$$R = \left(U_o \pm \sqrt{\frac{(1 + K)\,\sigma^2}{(1 - K)\,N}} \right) \qquad (1)$$

Shrinkage techniques have been investigated also by Waikar and Katti [3]; using double samples their estimator of the mean vector of the normal populations is:

$$\widetilde{\underline{U}}_{PT} = \begin{cases} \bar{\underline{X}}^{(1)} & \text{if } \bar{\underline{X}}^{(1)} \in R \\ \bar{\underline{X}} = \dfrac{n_1 \bar{\underline{X}}^{(1)} + n_2 \bar{\underline{X}}^{(2)}}{N} & \text{if } \bar{\underline{X}}^{(1)} \notin R \end{cases}$$

The region R which minimized the MSE ($\widetilde{\underline{U}}_{PT/} \underline{U}_o$) is the sphere in the space of $\bar{\underline{X}}^{(1)}$ which is given by

$$R = \left[\bar{\underline{X}}^{(1)} \ \ \bar{\underline{X}}^{(1)} \leqslant P/ \ (2n_1 + n_2) \right] \qquad (2)$$

The objective of the above estimators is to decrease the sample size by utilizing prior information in the form of initial estimate and yet to have high efficiency. The purpose of this paper is to estimate the mean vector of the normal population using total and partial prior information in case the co-variance matrix is known and also in the case if it is unknown.

ESTIMATION OF THE MEAN VECTOR WHEN THE CO-VARIANCE MATRIX IS KNOWN

Let $\underline{X} = (X_1, X_2, \ldots, X_p)'$ be a random vector where the sample of size n_1 for each variable is $\underline{X}_i = (X_{i1}, X_{i2}, \ldots, X_{in_1})'$ $i = 1, 2, \ldots, P$ having a p - normal distribution with unknown mean vector $\underline{U} = (U_1, U_2, \ldots, U_p)'$ and a known covariance matrix $\sum = (\sigma_{ij})$ $i,j = 1, 2, \ldots , p$. Suppose that our prior information is $\underline{U}_o = (U_{10}, U_{20}, \ldots , U_{po})'$ of $\underline{U}$. We wish to estimate $\underline{U}$ not only by using the observations $(X_{i1}, X_{i2}, \ldots, X_{in_1})$ but also using the prior information $\underline{U}_o$. The proposed estimation technique is to obtain a sample of size n_1, compute $\bar{\underline{X}}^{(1)}$ and then construct a region R_1. If $\bar{\underline{X}}^{(1)} \in R_1$, our estimate $\tilde{\underline{U}}$ is $K (\bar{\underline{X}}^{(1)} - \underline{U}_o + \underline{U}_o$. If $\bar{\underline{X}}^{(1)} \notin R_1$, we obtain n_2 additional observation, then calculate $\bar{\underline{X}}$ based on $N = n_1 + n_2$ observation. Therefore ;

$$\tilde{\underline{U}} = \begin{cases} K \ (\bar{\underline{X}}^{(1)} - \underline{U}_o) + \underline{U}_o & \text{if } \bar{\underline{X}}^{(1)} \in R_1 \\ \bar{\underline{X}} = \dfrac{n_1 \bar{\underline{X}}^{(1)} + n_2 \bar{\underline{X}}^{(2)}}{N} & \text{if } \bar{\underline{X}}^{(1)} \notin R_1 \end{cases}$$

K : is the shrinkage factor; $0 < K \leq 1$

The region R_1 which minimized the MSE ($\tilde{\underline{U}} / \underline{U}_o$) is given below:

$$R_1 = \left[(\bar{\underline{X}}^{(1)} - \underline{U}_o)' \ (\bar{\underline{X}}^{(1)} - \underline{U}_o) \leq \frac{n_2 \, tr. \sum}{N^2 K^2 - n_1^2} \right] \quad (3)$$

For a procedure to obtain such a region R_1, see [5].

The mean squared error expression of $\tilde{\underline{U}}$ is given by:

$$\text{MSE}\ (\tilde{\underline{U}}/\ \underline{U},\ R_1) = \int_{R_1} \Big[(K^2 - \frac{n_1}{N^2})(\bar{\underline{X}}^{(1)} - \underline{U})^1(\bar{\underline{X}}^{(1)} - \underline{U}) - \frac{n_1^2\ \text{tr.}\Sigma}{N^2} \Big] dF(\bar{\underline{X}}^{(1)})$$

$$+ \int_{R_1} \Big[(\underline{U}_o - \underline{U})(1-K)^2\ (\underline{U}_o - \underline{U})' + K(1-K)(\bar{\underline{X}}^{(1)} - \underline{U})'\ (\bar{\underline{X}}^{(1)} - \underline{U}_o)$$

$$+ (1-K)K(\underline{U}_o - \underline{U})'\ \ (\bar{\underline{X}}^{(1)} - \underline{U}) \Big]\ dF(\bar{\underline{X}}^{(1)}) + \frac{\text{tr.}\Sigma}{N} \qquad (4)$$

and the expected sample size is:

$$E(n\,/\,\underline{U};\ R_1) = N - (N - n_1).\ \Pr(\bar{\underline{X}}^{(1)} \in R_1) \leqslant N \qquad (5)$$

The bias expression of $\tilde{\underline{U}}$ is

$$B(\tilde{\underline{U}}/\underline{U};\ R_1) = \int_{R_1} \Big[(K - \frac{n_1}{N})(\bar{\underline{X}}^{(1)} - \underline{U}) + (1-K)(\underline{U}_o - \underline{U}) \Big] dF(\bar{\underline{X}}^{(1)}) \qquad (6)$$

Then the expression of efficiency of $\tilde{\underline{U}}$ relative to $\bar{\underline{X}}$ is given by:

$$E^{ff}(\tilde{\underline{U}}/\underline{U};\ R_1) = \frac{\text{tr.}\Sigma}{\text{MSE}(\tilde{\underline{U}}/\underline{U};R_1)\ E(n/\underline{U};R_1)} \qquad (7)$$

The region R_1 of the shrinkage estimator which is given in (3) is more general than the region of $\tilde{\underline{U}}_{PT}$ mean is given in (2). If $\Sigma = I$, $\underline{U} = \underline{0}$ and $K = 1$, the mean squared error expression and the region of the proposed estimator reduces to the mean squared error and the region of Waikar and Katti's estimator.

THE CHOICE OF THE SHRINKAGE FACTOR K

In many problems the choice of the shrinkage factor $K (0 < K \leqslant 1)$ is left to the experimenter. In this section we consider the estimation value of K. Thompson [4] estimate of θ the value of K which minimize the MSE of $K(\hat{\theta} - \theta_o) + \theta_o$ is:

$$K = (\hat{\theta} - \theta_o)^2 \Big/ (\hat{\theta} - \theta_o)^2 + Var(\hat{\theta}) \qquad (8)$$

Al-Bayyati and Arnold [1] indicated that K = 0.6 yield high efficiency of the shrinkage estimator in the univariate case. The value of the shrinkage factor K which minimize MSE of $K(\underline{\bar{X}}^{(1)} - \underline{U}_o) + \underline{U}_o$ is given by ;

$$K = (\underline{U}_o - \underline{U})' (\underline{U}_o - \underline{U}) \Big/ (\underline{U}_o - \underline{U})^1 (\underline{U}_o - \underline{U}) + E(\underline{\bar{X}}^{(1)} - \underline{U})^1 (\underline{\bar{X}}^{(1)} - \underline{U}) \qquad (9)$$

Note that K is a function of the parameters $\underline{U}$. Hence, the estimate of the value of K is

$$K = (\underline{U}_o - \underline{U})^1 (\underline{U}_o - \underline{U}) \quad (\underline{U}_o - \underline{U})^1 \ (\underline{U}_o - \underline{U}) + E(\underline{\bar{X}}^{(1)} - \underline{U})^1 (\underline{\bar{X}}^{(1)} - \underline{U}) \qquad (10)$$

ESTIMATION OF THE MEAN VECTOR WHEN THE COVARIANCE MATRIX IS UNKNOWN

In section (2), we aasumed that $\sum$ is known. When if it is not assumed to be known the following estimation procedure for the mean vector $\underline{U}$ is proposed. We start with the sample size $n_1 < N$ and compute $\underline{\bar{X}}^{(1)}$ and $\hat{\sum}_1$, where

$\hat{\sum}_1 = E(\underline{X}_i^{(1)} - \underline{\bar{X}}^{(1)}) (\underline{\bar{X}}_i^{(1)} - \underline{\bar{X}}^{(1)})'$ and construct a region S_1 around $\underline{U}_o$. If $\underline{\bar{X}}^{(1)} \in S_1$ we use the shrinkage estimator $K(\underline{\bar{X}}^{(1)} - \underline{U}_o) + \underline{U}_o$ as an estimate of $\underline{U}$. If $\underline{\bar{X}}^{(1)} \notin S_1$ we take additional sample of size $n_2 = N - n_1$ and estimate $\underline{U}$ by pool $\underline{\bar{X}}^{(1)}$ and $\underline{\bar{X}}^{(2)}$. Therefore, the DSSE of $\underline{U}$ is given by

$$\tilde{\underline{U}} = \begin{cases} K(\underline{\bar{X}}^{(1)} - \underline{U}_o) + \underline{U}_o & \text{if } \underline{\bar{X}}^{(1)} \in S_1 \\ \underline{\bar{X}} = \dfrac{n_1\underline{\bar{X}}^{(1)} + n_2\underline{\bar{X}}^{(2)}}{N} & \text{if } \underline{\bar{X}}^{(1)} \in S_1 \end{cases}$$

The region S_1 which minimized the $MSE(\underline{\tilde{U}} / \underline{U}_o)$ is given below:

$$S_1 = \left[(\underline{\bar{X}}^{(1)} - \underline{U}_o)' (\underline{\bar{X}}^{(1)} - \underline{U}_o) \leqslant \frac{n_2 \, tr. \hat{\Sigma}_1}{N^2 K^2 - n_1^2} \right] \qquad (11)$$

The mean squared error expression of $\underline{\tilde{U}}$ in this case is given by:

$$MSE(\underline{\tilde{U}} / \underline{U}; S_1) = \int_0^{\infty}\int_{S_1} \Big[(K^2 - \frac{n_1^2}{N^2})(\underline{\bar{X}}^{(1)} - \underline{U})^1 (\underline{\bar{X}}^{(1)} - \underline{U})$$

$$- \frac{n_2 \, tr. \hat{\Sigma}_1}{N^2} \Big] dF(\underline{\bar{X}}^{(1)} / \hat{\Sigma}_1) dF(\hat{\Sigma}_1 / \Sigma_1)$$

$$+ \int_0^{\infty}\int_{S_1} \Big[(1-K)^2 (\underline{U}_o - \underline{U})' (\underline{U}_o - \underline{U}) + K(1-K)(\underline{\bar{X}}^{(1)} - \underline{U})' (\underline{U}_o - \underline{U})$$

$$+ K(1-K)(\underline{U}_o - \underline{U})' (\underline{\bar{X}}^{(1)} - \underline{U}) \Big] - dF(\underline{\bar{X}}^{(1)} / \hat{\Sigma}_1) dF(\hat{\Sigma}_1 / \Sigma_1)$$

$$+ \frac{tr. \hat{\Sigma}_1}{N} \qquad (12)$$

Where

$$dF(\hat{\Sigma}_1 / \Sigma_1) = \frac{\left| n_1 \hat{\Sigma}_1 \right|^{\frac{1}{2}(n_1 - P - 1)} \exp. - \frac{1}{2} tr. \, n_1 \hat{\Sigma}_1 \Sigma_1^{-1}}{2^{\frac{1}{2} n_1 P} \; \pi^{P(P-1)/4} \; |\Sigma|^{\frac{1}{2} n_1} \; \prod_{i=1}^{P} \Gamma\left[\frac{1}{2}(n_1 + 1 - i)\right]}$$

The expected sample size is

$$E(n / \underline{U}; S_1) = N - (N - n_1) \, P_1(\underline{\bar{X}}^{(1)} \in S_1) \leqslant N \qquad (14)$$

and the bias expression of $\underline{\tilde{U}}$ is

$$B(\underline{\tilde{U}} / \underline{U}; S_1) = \int_0^{\infty}\int_{S_1} \Big[(K - \frac{n_1}{N})(\underline{\bar{X}}^{(1)} - \underline{U})$$

$$+ (1-K)(\underline{U}_o - \underline{U}) \Big] \, dF(\underline{\bar{X}}^{(1)} / \hat{\Sigma}_1) dF(\hat{\Sigma}_1 / \Sigma) \qquad (15)$$

Finally the expression of efficiency is given by

$$B^{ff}(\underline{\tilde{U}} / \underline{U}) = \frac{tr. \hat{\Sigma}_1}{MSE(\underline{\tilde{U}} / \underline{U}; S_1) E(n / \underline{U}; S_1)} \qquad (16)$$

ESTIMATION OF THE MEAN VECTOR WITH PARTIAL PRIOR INFORMATION WHEN THE COVARIANCE MATRIX IS KNOWN

Let $\underline{X} = (X_1, X_2 \ldots X_p)^1$ be a random P = variables where $\underline{X}_i = X_{i1}, X_{i2} \ldots X_{in_1})'$, i = 1,2, ... P having a P-normal populations with $\underline{U} = (U_1, U_2, \ldots U_p)'$ and a known covariance matrix $\Sigma = \{\sigma_{ii}\}I$. Let us assume that the prior information is about a part of the mean vector. We want to estimate the mean vector using partial prior information. Devide the input X matrix and the mean vector on a basis of a given prior information. Rearrange $\underline{U}$ in two portions such that $\underline{U} = [(1)\,\underline{U}\,,\,(2)\,\underline{U})]'$ where $(1)\,\underline{U}$ is the portion on which we have the prior information $(1)\underline{U}_o$. Also the corresponding division in $X = [(1)X,\,(2)X]$ and $\Sigma = \begin{pmatrix} \Sigma_{ii} & 0 \\ 0 & \Sigma_{22} \end{pmatrix}$ where:

$$\Sigma_{ii} = E\left((i)\underline{X} - (i)\underline{U}\right)\left((i)\underline{X} - (i)\underline{U}\right)' \quad ; \quad i = 12 \qquad (17)$$

Let $(1)\overline{\underline{X}}$ and $(2)\overline{\underline{X}}$ be the L.S.E. of $(1)\underline{U}$ and $(2)\underline{U}$; and $(i)\,\overline{\underline{X}}^{(j)}$ i,j = 1,2 be the L.S.E. of the ith portion based on j th sample, where:

$$E\left((i)\ \overline{\underline{X}}^{(j)}\right) = (i)\ \underline{U}, i = 1,2 \qquad (18)$$

The proposed estimation technique is to obtain a sample of size n_1, compute $(1)\overline{\underline{X}}^{(1)}$ and $(2)\underline{X}^{(1)}$, and then construct a region R_2 based on $(1)\underline{U}$ only. If $(1)\underline{X}^{(1)} \in R_2$, our estimate is $K(\ (1)\overline{\underline{X}}^{(1)} - \underline{U}_o) + (1)\underline{U}_o$ of $(1)\underline{U}$ and $(2)\overline{\underline{X}}^{(1)}$ of $(2)\underline{U}$. If $(1)\overline{\underline{X}}^{(1)} \notin R_2$, $n_2 = N - n_1$ additional observations are taken and we calculate $(1)\overline{\underline{X}}$ and $(2)\overline{\underline{X}}$ based on N observation as estimates of $(1)\underline{U}$ and $(2)\underline{U}$ respectively. Therefore, the DSSE of $\underline{U} = (\ (1)\underline{U}, (2)\underline{U})'$ is given by

$$\tilde{\underline{U}} = \begin{cases} \begin{pmatrix} K((1)\bar{\underline{X}}^{(1)} - (1)\underline{U}_o) + (1)\underline{U}_o \\ (2)\bar{\underline{X}}^{(1)} \end{pmatrix} & \text{if } (1)\bar{\underline{X}}^{(1)} \in R_2 \\ \bar{\underline{X}} = \begin{pmatrix} (1)\bar{\underline{X}} = \dfrac{n_1 (1)\bar{\underline{X}}^{(1)} + n_2 (1)\bar{\underline{X}}^{(2)}}{N} \\ (2)\bar{\underline{X}} = \dfrac{n_1 (2)\bar{\underline{X}}^{(1)} + n_2 (2)\bar{\underline{X}}^{(2)}}{N} \end{pmatrix} & \text{if } (1)\bar{\underline{X}}^{(1)} \notin R_2 \end{cases}$$

The region R_2 which minimize MSE $(2)\tilde{\underline{U}}/(2)\underline{U}_o)$ is given by:

$$R_2 = \left(((1)\bar{\underline{X}}^{(1)} - (1)\underline{U}_o)'((1)\bar{\underline{X}}^{(1)} - (1)\underline{U}_o) \leq \frac{n_2 tr. \Sigma_{11}}{N^2K^2 - n_1^2} \right) \tag{19}$$

The mean squared error of $(1)\tilde{\underline{U}}$ is

$$\text{MSE}((1)\tilde{\underline{U}}/(1)\bar{U}; R_2) = \int_{R_2} \left[(K^2 - \frac{n_1^2}{N^2})((1)\bar{\underline{X}}^{(1)} - (1)\underline{U})'((1)\bar{\underline{X}}^{(1)} - (1)\underline{U}) - \frac{n_2 tr. \Sigma_{11}}{N^2} \right] \cdot$$

$$dF((1)\bar{\underline{X}}^{(1)}) + \int_{R_2} \left[(1-K)^2 ((1)\underline{U}_o - (1)\underline{U})'((1)\underline{U}_o - (1)\underline{U}) + K(1-K)((1)\bar{\underline{X}}^{(1)} - (1)\underline{U})' \right.$$

$$\left. ((1)\underline{U}_o - (1)\underline{U}) + K(1-k)((1)\underline{U}_o - (1)\underline{U})'((1)\bar{\underline{X}}^{(1)} - (1)\underline{U}) \right] dF((1)\bar{\underline{X}}^{(1)}) + \frac{tr. \Sigma_{11}}{N} \tag{20}$$

and the mean squared error expression of $(2)\tilde{\underline{U}}$ is

$$\text{MSE}((2)\tilde{\underline{U}}/(2)\underline{U}; R_2) = \int_{R_2} \left[(1 - \frac{n_1^2}{N^2})((2)\bar{\underline{X}}^{(1)} - (2)\underline{U})'((2)\bar{\underline{X}}^{(1)} - (2)\underline{U}) - \frac{n_2 tr. \Sigma_{22}}{N^2} \right] \cdot$$

$$dF((2)\bar{\underline{X}}^{(1)}) + \frac{tr. \Sigma_{22}}{N} \tag{21}$$

The mean squared error of $\tilde{\underline{U}} = ((1)\tilde{\underline{U}}, (2)\tilde{\underline{U}})'$ can be **expressed as:**

$$\text{MSE}(\tilde{\underline{U}}/\underline{U}, R_2) = \text{MSE}((1)\tilde{\underline{U}}/(1)\underline{U}; R_2) + \text{MSE}((2)\tilde{\underline{U}}/(2)\underline{U}; R_2) \tag{22}$$

and the bias expression of $(1)\tilde{\underline{U}}$ and $(2)\tilde{\underline{U}}$ are given respectively by:

$$B({}_{(1)}\tilde{\underline{U}}/{}_{(1)}\underline{U}\ ; R_2) = \int_{R_2} \left[(K - \frac{n_1}{N})({}_{(1)}\overline{\underline{X}}^{(1)} - {}_{(1)}\underline{U}) + (1-K)({}_{(1)}\underline{U}_0 - {}_{(1)}\underline{U}) \right] dF({}_{(1)}\overline{\underline{X}}^{(1)}) \quad (23)$$

$$B({}_{(2)}\tilde{\underline{U}}/{}_{(2)}\underline{U}\ ; R_2) = \int_{R_2} (1 - \frac{n_1}{N})({}_{(2)}\overline{\underline{X}}^{(1)} - {}_{(2)}\underline{U})\, dF({}_{(1)}\overline{\underline{X}}^{(1)}) \quad (24)$$

and the expected sample size in this case is given by:

$$E(n/{}_{(1)}\underline{U}\ ; R_2) = N - (N - n_1) P_r({}_{(1)}\overline{\underline{X}}^{(1)} \in R_2) \leq N \quad (25)$$

The efficiency expression of $\tilde{\underline{U}} = ((1)\tilde{\underline{U}}\ , (2)\underline{U})'$ can be written as:

$$\mathrm{Eff}(\tilde{\underline{U}}/\underline{U}) = \frac{tr.\Sigma}{MSE(\tilde{\underline{U}}/\underline{U}; R_2)\, E(n/{}_{(1)}\underline{U}; R_2)} \quad (26)$$

ESTIMATION OF THE MEAN VECTOR WITH PARTIAL PRIOR INFORMATION WHEN THE COVARIANCE MATRIX IS KNOWN

In Section (5) we assumed that Σ is known. When Σ is not known the following estimate for the mean vector $\underline{U} = ((1)\underline{U}\ , (2)\underline{U})'$ is proposed. We make same arrangement on $\underline{U}$ and input X matrix and Σ which were given in the above section. We start with the sample of size n_1 compute $(1)\overline{\underline{X}}^{(1)}$ and $(2)\overline{\underline{X}}^{(1)}$ and $\hat{\Sigma} = \begin{pmatrix} \hat{\Sigma}_{11} & 0 \\ 0 & \Sigma_{22} \end{pmatrix}$ construct a region S_2 around $(1)\underline{U}_0$. If $(1)\overline{\underline{X}}^{(1)} \in S_2$ we use the shrinkage estimator $K((1)\overline{\underline{X}}^{(1)} - (1)\underline{U}_0) + (1)\underline{U}_0$ as an estimate of $(1)\underline{U}$ and $(2)\overline{\underline{X}}^{(1)}$ of $(2)\underline{U}$. If $(1)\overline{\underline{X}}^{(1)} \notin S_2$ we take n_2 additional observation and calculate $(1)\overline{\underline{X}}$ and $(2)\overline{\underline{X}}$ based on N observation. Hence, the DSSE of $\underline{U} = ((1)\underline{U}\ , (2)\underline{U})'$ is

$$\tilde{\underline{U}} = \begin{cases} \begin{pmatrix} K({}_{(1)}\overline{\underline{X}}^{(1)} - {}_{(1)}\underline{U}_0) + {}_{(1)}\underline{U}_0 \\ {}_{2}\overline{\underline{X}}^{(1)} \end{pmatrix} & \text{if } {}_{(1)}\overline{\underline{X}}^{(1)} \in S_2 \\ \overline{\underline{X}} = \begin{pmatrix} {}_{(1)}\overline{\underline{X}} = \dfrac{n_1 {}_{(1)}\overline{\underline{X}}^{(1)} + n_2 {}_{(2)}\overline{\underline{X}}^{(2)}}{N} \\ {}_{(2)}\overline{\underline{X}} = \dfrac{n_1 {}_{(2)}\overline{\underline{X}}^{(1)} + n_2 {}_{(2)}\overline{\underline{X}}^{(2)}}{N} \end{pmatrix} & \text{if } {}_{(1)}\overline{\underline{X}}^{(1)} \notin S_2 \end{cases}$$

The region S_2 which minimized MSE ($(1)\tilde{\underline{U}}/(1)\underline{U}_0$) is given by:

$$S_2 = \left(((1)\overline{\underline{X}}^{(1)} - (1)\underline{U}_0)'((1)\overline{\underline{X}}^{(1)} - (1)\underline{U}_0) \leq \frac{n_2 tr. \hat{\Sigma}_{11}}{N^2 K^2 - n_1^2} \right) \qquad (27)$$

The mean squared error of $(1)\underline{U}$ is given by:

$$MSE((1)\tilde{\underline{U}}/(1)\underline{U}; S_2) = \int_0^{\infty}\int_{S_2} \left[(K^2 - \frac{n_1^2}{N^2})((1)\overline{\underline{X}}^{(1)} - (1)\underline{U})'((1)\overline{\underline{X}}^{(1)} - (1)\underline{U}) - \frac{n_2 tr \hat{\Sigma}_{11}}{N^2} \right]$$

$$dF((1)\overline{\underline{X}}^{(1)}/\hat{\Sigma}_{11})\, dF(\hat{\Sigma}_{11}/\Sigma_{11}) + \int_0^{\infty}\int_{S_2} \left[(1-K)^2((1)\underline{U}_0 - (1)\underline{U})'((1)\underline{U}_0 - (1)\underline{U}) \right.$$

$$\left. + K(1-K)((1)\overline{\underline{X}}^{(1)} - (1)\underline{U})'((1)\underline{U}_0 - (1)\underline{U}) + (1-K)K((1)\underline{U}_0 - (1)\underline{U})'((1)\overline{\underline{X}}^{(1)} - (1)\underline{U}) \right]$$

$$dF((1)\overline{\underline{X}}^{(1)}/\hat{\Sigma}_{11})\, dF(\hat{\Sigma}_{11}/\Sigma_{11}) + \frac{tr. \hat{\Sigma}_{11}}{N} \qquad (28)$$

Where

$$dF(\hat{\Sigma}_{11}/\Sigma_{11}) = \frac{|n_1 \hat{\Sigma}_{11}|^{\frac{1}{2}(n_1-p-1)} \exp. -\frac{1}{2} tr. n_1 \hat{\Sigma}_{11} \Sigma_{11}^{-1}}{2^{\frac{1}{2}np}\, \pi^{p(p-1)/4}\, |\Sigma_{11}|^{\frac{1}{2}n_1 p} \prod_{i=1}^{p} \Gamma\left[\frac{1}{2}(n_1 - 1 - i)\right]} \qquad (29)$$

The mean squared error of $(2)\tilde{\underline{U}}$ is:

$$MSE((2)\tilde{\underline{U}}/(2)\underline{U}; S_2) = \int_0^{\infty}\int_{S_2} \left[(1 - \frac{n_1^2}{N^2})((2)\overline{\underline{X}}^{(1)} - (2)\underline{U})'((2)\overline{\underline{X}}^{(1)} - (2)\underline{U}) - \right.$$

$$\left. \frac{n_2 tr. \hat{\Sigma}_{22}}{N^2} \right] dF((2)\overline{\underline{X}}^{(1)}/\hat{\Sigma}_{22}) \cdot dF(\hat{\Sigma}_{22}/\Sigma_{22}) + \frac{tr \Sigma_{22}}{N} \qquad (30)$$

and the mean squared error expression of $\tilde{\underline{U}} = ((1)\tilde{\underline{U}}, (2)\tilde{\underline{U}})'$ can be expressed as:

$$MSE(\tilde{\underline{U}}/\underline{U}; S_2) = MSE((1)\tilde{\underline{U}}/(1)\underline{U}; S_2) + MSE((2)\tilde{\underline{U}}/(2)\underline{U}; S_2) \qquad (31)$$

The bias of $(1)\tilde{\underline{U}}$ and $(2)\tilde{\underline{U}}$ are given respectively by:

$$B((1)\tilde{\underline{U}}/(1)\underline{U}; S_2) = \int_0^{\infty}\int_{S_2} \left[(K - \frac{n_1}{N})((1)\overline{\underline{X}}^{(1)} - (1)\underline{U}) + (1-K) \right.$$

$$\left. ((1)\underline{U}_0 - (1)\underline{U}) \right] \cdot dF((1)\overline{\underline{X}}/\hat{\Sigma}_{11})\, dF(\hat{\Sigma}_{11}/\Sigma_{11}) \qquad (32)$$

$$B((2)\underline{\tilde{U}}/(2)\underline{U}\,;S_2) = \int_0^{\infty}\int_{S_2} (1-\frac{n_1}{N})((2)\underline{\bar{X}}^{(1)} - (2)\underline{U})\,dF((2)\underline{\bar{X}}^{(1)}/\hat{\Sigma}_{22})\,dF(\frac{\hat{\Sigma}_{22}}{\Sigma_{22}}) \quad (33)$$

and the expected sample size is:

$$E(n/(1)\underline{U}\,;S_2) = N-(N-n_1)P_r((1)\underline{\bar{X}}^{(1)} \in S_2) \leq N \quad (34)$$

The efficiency expression of $\underline{\tilde{U}} = (\ (1)\underline{\tilde{U}}\ ,\ (2)\underline{\tilde{U}}\ )'$ is given by:

$$\text{Eff}\,(\underline{\tilde{U}}/\underline{U}) = \frac{tr.\,\hat{\Sigma}}{MSE\,(\underline{\tilde{U}}/\underline{U}\,;S_2)\,E(n/(1)\underline{U}\,;S_2)} \quad (35)$$

If the prior information $\underline{U}_0$ is equal to the true value of $\underline{U}$ ($\underline{U}_0 = \underline{U}$), then we get different expressions for the mean squared error, bias, expected sample size and efficiency of any shrinkage estimator by substituting $\underline{U}_0$ in place of $\underline{U}$. These expressions are different from the corresponding expressions in case of $\underline{U} \neq \underline{U}_0$ which were given in the above sections.

E X A M P L E S

In this section we will obtain the DSSE of the means of the random variables X_1 and X_2 which are independent and normally distributed with $\underline{U} = (U_1\ ,\ U_2)'$ and unknown co-variance matrix $\Sigma = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix}$ where

X_1 denote the Length of the Crab-fish, and

X_2 denote the diameter of the Crab-fish.

The prior information about the parameters $\underline{U} = (U_1\ ,\ U_2)'$ is $\underline{U}_0 = (21.065,\ 33.313)'$. We draw a sample of size $n_1 = 90$. The DSSE of $\underline{U}$ with total and partial prior information when Σ is unknown is considered below:

CASE (A):

Consider the prior information $\underline{U}_o = (21.065, 33.313)'$. From the first sample we estimate the value of the mean vector and the co-variance matrix. To test whether we use the first sample or not, we use the formula for the region S_1 which is given in (11). For specified values of K we shall get different regions which are given in Table (1).

Table: 1 - Regions for Different Choice of K

K	S_1
0.509	$1.2964 < 313.59$
0.600	$1.2964 < 28.508$
0.700	$1.2964 < 13.066$
0.800	$1.2964 < 8.640$
0.900	$1.2964 < 7.87$

From the Table (1), we see that the first sample is acceptable, so we obtain the shrinkage estimator which are given in Table (2).

Table : 2 - **Shrinkage Estimation for Different Value of K**

K	$\tilde{\underline{U}} = K\,\bar{\underline{X}}^{(1)} + (1-K)\,\underline{U}_0$
0.509	$\begin{pmatrix} 20.8023 \\ 33.8304 \end{pmatrix}$
0.600	$\begin{pmatrix} 20.7554 \\ 33.9228 \end{pmatrix}$
0.700	$\begin{pmatrix} 20.6522 \\ 33.0243 \end{pmatrix}$
0.800	$\begin{pmatrix} 20.6522 \\ 34.1258 \end{pmatrix}$
0.900	$\begin{pmatrix} 20.6006 \\ 34.2273 \end{pmatrix}$

CASE (B) :

Let us assume that there is a prior information about U_1 only. For a specified values of K_1 we shall get different regions which are given in Table (3).

Table : 3 - **Regions for Different Choice of K_1**

K_1	S_2
0.506	20.544 ; 21.586
0.600	20.511 ; 21.619
0.700	20.4056 ; 21.7242
0.800	20.23393; 21.89607
0.900	19.8575 ; 22.2725

Using the formula (27); the above table shows that we use the shrinkage estimator based on the first sample. The shrinkage estimator of the means thus calculated as given in Table (4).

Table : 4 - Shrinkage Estimator with Partial Prior Information

K_1	$\tilde{\underline{U}} = \begin{pmatrix} K_1 \bar{X}^{(1)} + (1-K_1)U_{10} \\ \bar{X}_2^{(1)} \end{pmatrix}$
0.509	$\begin{pmatrix} 20.8023 \\ 34.3288 \end{pmatrix}$
0.6	$\begin{pmatrix} 20.7554 \\ 34.3288 \end{pmatrix}$
0.6	$\begin{pmatrix} 20.7038 \\ 34.3288 \end{pmatrix}$
0.8	$\begin{pmatrix} 20.6522 \\ 34.3288 \end{pmatrix}$
0.9	$\begin{pmatrix} 20.6006 \\ 34.3288 \end{pmatrix}$

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SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS AND ITS ERRORS

Haytham Z. Muhialdin*

A B S T R A C T

The Haytham elimination method has been successfully applied to the numerical solution of simultaneous linear equations in n unknowns using matrix method. This method is presented, implemented and discussed for selection problem.

The remainder of the paper presents in a conditioning and source of error.

INTRODUCTION

Many of the problems of numerical analysis can be reduced to the problem of solving linear systems of equations. Among the problems which can be so treated are the solution of system of equations. We will concern with the solution of n simultaneous linear equations in n unknowns.

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \quad i=1,2\ldots,n \qquad (1.1)$$

The use of matrix notation is not only convenient, but extremely powerful, in bringing out fundamental relationship.

Methods of solution of simultaneous linear equations belong essentially to either the class of direct methods or the class of iterative method. The best example for the direct method are Gaussian

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elimination, Gauss-Jordan elimination LU decomposition and for the iterative method are Jacobi and Gauss-Seidel iterations.

There is no problem in finding an analytic solution of (1.1). Cramers rule gives us such a solution.

As mentioned before there are many methods to find the solution of simultaneous linear equations but we cannot compute the sum of even or odd position elements directly by any method.

SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS

One of the most basic and important problems in science and engineering is the accurate and efficient simultaneous solution of a system of n linear equations in n unknowns. This problem is written in the form.

$$
\begin{array}{c}
a_{11}x_1 + a_{12}x_2 + \cdots\cdots\cdots + a_{1n}x_n = b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots\cdots\cdots + a_{2n}x_n = b_2 \\
\vdots \qquad\qquad\qquad\qquad \vdots \qquad \vdots \\
a_{n1}x_1 + a_{n2}x_2 + \cdots\cdots\cdots + a_{nn}x_n = b_n
\end{array} \qquad (2.1)
$$

Where each $a_{ij}, 1 \leqslant i \leqslant n,\ 1 \leqslant j \leqslant n$, and each b_i are known values and $x_j,\ 1 \leqslant j \leqslant n$ are the unknowns. We can find these unknowns easily by elimination of variables.

In matrix notation this set of equation can also be written in the form

$$Ax = b , \qquad (2.2)$$

Where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1}. & & \cdots\cdots & a_{nn} \end{bmatrix}, \; X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \; b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

and we shall use this alternative notation wherever convenient.

Unlike most problems in numerical analysis there is no difficulty in finding an analytical solution of the system (2.2). Assuming that det $(A) \neq 0$ the formula $x = A^{-1}b$ gives us an analytical solution.

Numerical methods for solving linear systems are divided into direct and iterative methods, direct methods are those which, in the absence of round-off or other error will yield the exact solution in a finite number of elementary arithemtic operations. In practice, because a computer works with a finite word length, direct methods do not lead to exact solution. Indeed, errors from roundoff, instability and loss of significance may lead to extremely poor or even useless results.

Iterative methods are those which start with an initial approximation and which by applying a suitably chosen algorithm, lead to successively better approximation.

Now there is new direct method, i.e. Haytham elimination method, this method has been very successfully applied to the numerical solutions of simultaneous linear equation inmunknowns using matrix method.

HAYTHAM ELIMINATION

The aim of this new method is to find the solution of n simultaneous linear equations in n unknowns by a matrix method numerically.

The idea of this method is to reduce the system of equations to upper triangular by systematic elimination of the elements below the diagonal, after that at the column which is in the upper traingular matrix starting from diagonal elements to make zero elements between each two elements above the diagonal which is in the same column, then finally introduce zero element between each two elements at the rows matrix. The system (1.1) is convently written in the matrix form (2.2) where $A = a_{ij}$ is the $n_x n$ nonsingular matrix of coefficients, $x^T = (x_1, --\ldots, x_n)$ and $b^T = (b_1, b_2, ---, b_n)$ with T denoting the transpose. We shall use matrix algebra & matrix notation extensively but not exclusively in this method.

We write out the system (1.1) in the form

$$a_{11}x_1 + a_{12}x_2 + \cdots\cdots + a1_n x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots\cdots + a_{2n}x_n = b_2$$

$$\cdots\cdots\cdots\cdots\cdots\cdots, \quad (2.3)$$

$$a_{n1}x_1 + a_{n2}x_2 \cdots\cdots\cdots + a_{nn}x_n = b_n$$

We assume, of course, that the matrix of coefficients is nonsingular. Suppose $a_{11} \neq 0$. Substract the multiple $\frac{a_{i1}}{a_{11}}$ of the first equation from the ith equation, i=2...,n, to get the first derived system

$$a_{11}x_1 + a_{12}x_2 + \cdots\cdots + a_{1n}x_n = b_1^{(1)}$$

$$a_{n2}^{(1)}x_2 + \cdots\cdots + a_{2n}^{(1)} x_n = b_2^{(1)} \quad . (2.4)$$

....

$$a_{n2}^{(1)}x_2 + \cdots\cdots\cdots + a_{nn}^{(1)}x_n = b_n^{(1)}$$

The new coefficients $a_{ij}^{(1)}$ are given by

$$a_{ij}^{(1)} = a_{ij} - m_{i1}\, a_{ij} \qquad i = 2, \ldots, n, \qquad (2.5)$$

$$\text{note}; \text{i}b_2^{(1)} = b_2^{(2)} \qquad j = 2, \ldots, n+1$$

Where $m_{i1} = \frac{a_{i1}}{a_{11}}$, $i = 2, \ldots, n$. If $a_{11} = 0$, then, because A is nonsingular, we can get a nonzero, element in the upper left-hand by interchanging two rows of (2.3). We can also interchange two columns of A to achieve the same effect.

Now, if $a_{22}^{(1)}$ in (2.4) is nonzero, we subtract $m_{i2} = \frac{a_{i2}^{(1)}}{a_{22}^{(1)}}$ times the second equation from the ith equation in (2.4), $i = 1,3,4, \ldots, n$, and get the second derived system

$$\begin{aligned}
a_{11}x_1 + \qquad & a_{13}^{(2)}x_1 + \cdots\cdots + a_{1n}^{(2)}x_n = b_1^{(2)} \\
a_{22}^{(1)}x_2 + {} & a_{23}^{(1)}x_3 + \cdots\cdots + a_{2n}^{(1)}x_n = b_2^{(2)} \\
& a_{33}^{(2)}x_3 + \cdots\cdots + a_{3n}^{(2)}x_n = b_3^{(2)}, \quad (2.6) \\
& \cdots\cdots\cdots\cdots\cdots\cdots \\
& a_{n3}^{(2)}x_3 + \cdots\cdots + a_{nn}^{(2)}x_n = b_n^{(2)}
\end{aligned}$$

$$\text{where } a_{ij}^{(2)} = a_{ij}^{(1)} - m_{i2}\, a_{2j}^{(1)} \quad i = 1,3,4,\ldots,n \; . \qquad (2.7)$$

$$j = 3,\ldots,n+1$$

Again, if $a_{22}^{(1)} = 0$, we can interchange two rows or columns to get a nonzero element in the (2.2) position.

Let us subtract the multiple $m_{i3} = \frac{a_{i3}^{(2)}}{a_{33}^{(2)}}$ times the third equation from the ith equation in (2.6), $i = 2,4,5 \ldots\ldots n$, we get the third derived system

$$
\begin{array}{llllll}
a_{11}x_1 + & & + a_{13}^{(2)}x_3 + & a_{14}^{(2)}x_4 + & \cdots\cdots + & a_{1n}^{(2)}x_n = b_1^{(3)} \\
 & a_{22}^{(1)}x_2 + & & a_{24}^{(3)}x_4 + & \cdots\cdots + & a_{2n}^{(3)}x_n = b_2^{(3)} \\
 & & a_{33}^{(2)}x_3 + & a_{34}^{(2)}x_4 + & \cdots\cdots + & a_{3n}^{(2)}x_n = b_3^{(3)} \\
 & & & a_{44}^{(3)}x_4 + & \cdots\cdots + & a_{4n}^{(3)}x_n = b_4^{(3)} \\
 & & & \vdots & & \vdots \qquad \vdots \\
 & & & a_{n4}^{(3)}x_n + & \cdots\cdots + & a_{nn}^{(3)}x_n = b_n^{(3)}
\end{array}
\quad ,(2.8)
$$

$$
\text{Where } a_{ij}^{(3)} = a_{ij}^{(2)} - m_{i4}a_{4j}^{(2)} \qquad i = 2,4,5,6, \ldots ,n \qquad (2.9)
$$
$$
j = 4,5, \ldots\ldots , n+1
$$

Now subtracting the multiple $m_{i4} = \dfrac{a_{i4}^{(3)}}{a_{44}^{(3)}}$ times the fourth equation from the ith equation in (2.8).

$i=1,3,5,6, \ldots, n$. we get the fourth derived system.

$$
\begin{array}{llllll}
a_{11}x_1 + & & a_{13}^{(2)}x_3 + & & + a_{15}^{(4)}x_5 + \cdots\cdots + & a_{1n}^{(4)}x_n = b_1^{(4)} \\
 & a_{22}^{(1)}x_2 + & & a_{24}^{(3)}x_4 & + a_{25}^{(3)}x_5 + \cdots\cdots + & a_{2n}^{(3)}x_n = b_2^{(4)} \\
 & & a_{33}^{(2)}x_3 + & & a_{35}^{(4)}x_5 + \cdots\cdots + & a_{3n}^{(4)}x_n = b_3^{(4)} \\
 & & & a_{44}^{(3)}x_4 + & a_{45}^{(3)}x_5 + \cdots\cdots + & a_{4n}^{(3)}x_n = b_4^{(4)} \\
 & & & & a_{55}^{(4)}x_5 + \cdots\cdots + & a_{5n}^{(4)}x_n = b_5^{(4)} \\
 & & & & \vdots & \vdots \qquad \vdots \\
 & & & & a_{n5}^{(4)}x_5 + \cdots\cdots + & a_{nn}^{(4)}x_n = b_n^{(4)}
\end{array}
\quad ,(2.10)
$$

Where $a_{ij}^{(4)} = a_{ij}^{(3)} - m_{i4}\, a_{4j}^{(3)}$ $i= 1,3,5,6,\ldots,n$
$j= 5,6,\ldots\ldots,n+1$

Continuing with this process through n steps, we arrive at the final system

$$
\begin{array}{llllllll}
a_{11}x_1+ & & a_{13}^{(2)}x_3+ & & a_{15}^{(4)}x_5+ & & a_{1n-1}^{(n-2)}x_{n-1} & =b_1^{(n-1)} \\
& a_{22}^{(1)}x_2+ & & a_{24}^{(3)}x_4+ & & a_{26}^{(5)}x_6+ & a_{2n}^{(n-1)}x_n & =b_2^{(n-1)} \\
& & a_{33}^{(2)}x_3+ & & a_{35}^{(4)}x_5+ & & a_{3n-1}^{(n-2)}x_{n-1} & =b_3^{(n-1)} \\
& & & a_{44}^{(3)}x_4 & & a_{46}^{(5)}x_6+ & a_{4n}^{(n-1)}x_n & =b_4^{(n-1)} \\
& & & & a_{55}^{(4)}x_5 & & a_{5n-1}^{(n-2)}x_{n-1} & =b_5^{(n-1)} \\
& & & & & a_{66}^{(5)}x_6+ & a_{6n}^{(n-1)}x_n & =b_6^{(n-1)} \\
& & & & & \ddots & \vdots & \vdots \\
& & & & & & a_{nn}^{(n-1)}x_n & =b_n^{(n-1)}
\end{array}
\quad ,(2.11)
$$

with the diagonal elements all nonzero and where

$$a_{ij}^{(k)} = a_{ij}^{(k-1)} - m_{ik}\, a_{kj}^{(k-1)} . \qquad \begin{array}{l} k = 0(2)\ n-2 \\ k = 1(2)\ n-1 \quad , (2.12) \\ j = k+1,\ldots,n+1 \\ i = k+1,\ldots,n \end{array}$$

with $m_{ik} = \dfrac{a_{ik}^{(k-1)}}{a_{kk}^{(k-1)}}$, given (2.11), the solution is easily calculated as

$$x_i = \frac{1}{a_{ii}^{(i-1)}} \left[b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)} x_j \right] \quad i=n,\ldots,1. \qquad (2.13)$$

Notes:

1 - $a_{ij}^{(o)} = a_{ij}$

HERE WE USE THE CONVENTION $\sum_{j=n+1}^{n} = 0$

In this method if the diagonal elements in the odd row. We elimenate the elements in the even rows which is in the same column and simillar for the diagonal element in the even row we elemenate the elements in the odd rows which is in the same column.

The Haytham elimination Algorithm is

```
┌──── for K    = 1(2) n-1     (step count)
│┌─── for K    = 0(2) n-2     (step count)
││┌── for i    = K + 1 (1)n   (row count)
│││
│││   m_ik     = a_ik^(k) / a_kk^(k)
│││
│││┌─ for j    = K + 1 (1)n   (column count)
││││
││││  a_ij^(K) = a_ij^(K-1) - m_ik a_kj^(k-1)
││││
│││└─ repeat
│││   b_i^(K)  = b_i^(K-1) - m_ik b_k^(K-1)
││└── repeat
│└─── repeat a step step
└──── repeat a step step
```

Notes:

(i) In computer program superfix notation not required.

(ii) In computer program the m_{ik} may be stored as a_{ik} .

(iii) In computer program the b_i may be stored as a_i.

(iv) Leave the first process K = 0, start from the second step K = 1. and then the third step K = 2.

Now in the Haytham Back substitution Algorithm the solution is given by following Algorithm

```
x_n = b_n/a_nn

┌─ for i = n-1 (1)
│     sum = 0
│  ┌─for j = i + 1 (1)n
│  │    sum = sum + a_ij x_j
│  └─repeat  j
│     x_i = (b_i - sum)/a_ii
└─ repeat  i
```

How to calculate the total number of multiplications and divisions in solving an nxn system of equations Ax = b by Haytham elimination. To eliminate x_k, from one equation we require one division, to form the multiplier, and $(n-K'+1)$ multiplications, to form the new cofficients $x'_{k+1}, x'_{k+2}, \ldots, x_n$ and the new value on the right-hand side. Since we need to eliminate x'_k, not from one equation but from $(n-K')$ equation, we therefore require $(n-K)$ division and $(n-K)(n-K'+1)$ multiplications. Hence total number of multiplications and divisions required for eliminations, process is

$$\sum_{k=1}^{k=\frac{n}{2}} \left[(n-k) + (n-k)(n-k'+1)\right] \qquad k' = 1,2, \ldots, n \, , \qquad (2.14)$$

We may illustrate Haytham elimination by solve the following system:

$$2x_1 - x_2 + 3x_3 - x_4 = 7 \quad , \qquad (2.15)$$

$$-\frac{1}{2}x_1 - x_2 + 4x_3 - 2x_4 = 5 \quad , \qquad (2.16)$$

$$-\frac{3}{2}3x_1 + 2x_2 + x_3 + 4x_4 = 31 \quad , \qquad (2.17)$$

$$-\frac{4}{2} \quad 4X_1 - 3x_2 + 3x_3 - 3x_4 = -5 \quad , \qquad (2.18)$$

Firstly add multiples of equations (2.16), (2.17), (2.18) to eliminate x_1. The appropriate multiples in this case are $\frac{1}{2}$, $-\frac{3}{2}$, $-\frac{4}{2}$ which are conveniently noted in the left hand column thus the number of operations to derive this system are:

Number of divisions	Number of multiplications
$(n - K)$	$(n-K)(n-K'+1)$ $K=1,\ldots,\frac{1}{2}n$
$(4 - 1) = 3$	$(4-1)(4-1+1)$ $K'=1,\ldots,n$
	$3 \times 4 = 12$

The first derived system is

$$\begin{aligned}
-2 \quad & 2x_1 - x_2 + 3x_3 - x_4 = 7 \\
& -\frac{1}{2}x_2 + \frac{5}{2}x_3 - \frac{3}{2}x_4 = \frac{3}{2} \\
7 \quad & \frac{7}{2}x_2 - \frac{7}{2}x_3 + \frac{11}{2}x_4 = \frac{41}{2} \\
-2 \quad & -x_2 - 3x_3 - x_4 = -19
\end{aligned} \quad , \qquad (2.19)$$

Number of divisions	Number of multiplications
$(n-K)$	$(n-K)(n-K'+1)$
$(4-1) = 3$	$(4-1)(4-2+1)$
	$3 \times 3 = 9$

The second derived system is

$$\begin{aligned}
\frac{2}{14} \quad & 2x_1 \qquad -2x_3 + 2x_4 = 4 \\
-\frac{5}{28} \quad & -\frac{1}{2}x_2 + \frac{5}{2}x_3 - \frac{3}{2}x_4 = \frac{3}{2} \\
& 14x_3 - 5x_4 = 31 \\
\frac{8}{14} \quad & -8x_3 + 2x_4 = -22
\end{aligned} \quad , \qquad (2.20)$$

Number of divisions	Number of multiplications
$(n-K)$	$(n-K)(n-K'+1)$
$(4-2) = 2$	$(4-2)(4-3+1)$
	$2 \times 3 = 6$

The third derived system is

$$\begin{array}{llllll} \frac{14}{6} & 2x_1 & & -2x_3 & +\ 2x_4 & = 6 \\ & & -\frac{1}{2}x_2 & & \frac{17}{28}x_4 & = \frac{113}{28} \\ \frac{-35}{6} & & & 14x_3 & -\ 5x_4 & = 31 \\ & & & & -\frac{6}{7}x_4 & = \frac{30}{7} \end{array} \quad , \qquad (2.21)$$

Number of divisions	Number of multiplications
$(n-K)$	$(n-K)(n-K'+1)$
$(4-2) = 2$	$(4-2)(4-4+1)$
	$2 \times 1 = 2$

The final derived system is

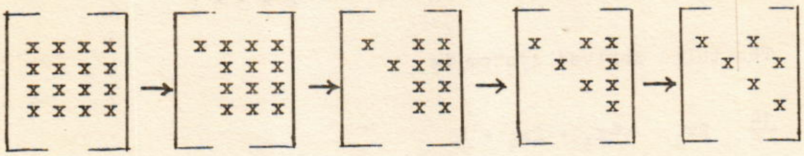
$$\begin{array}{lllll} 2x_1 & & 2x_3 & & = 6 \\ & -\frac{1}{2}x_2 & & \frac{17}{28}x_4 & = 1\ \frac{113}{28} \\ & 4x_2 & & & = 56 \\ & & & -\frac{6}{7}x_4 & = -\frac{30}{7} \end{array}$$

which by back substitutions gives $x_4 = 5$, $x_3 = 4$, $x_2 = 2$, $x_1 = 1$.

<u>Note</u>:

The number of division are equals at the first and second step of elimination and also at the third and fourth. Step of elimination, which leads to the conclusion that at each twice step of elimination we repeat the same number of K twice.

In Haytham elimination the matrix of coefficients on the left hand side is converted into upper triangular form with some zero elements as shown below.



CONDITIONING AND SOURCE OF ERROR

We introduce the idea of ill conditioning by considering the the problem of solving by some method the set of equations

$$Ax = b \quad , \qquad (3.1)$$

our aim, of course, is to obtain the exact or true solution x_t which satisfies

$$Ax_t = b \quad , \qquad (3.2)$$

but in practice, we obtain, instead, a computed solution x_c which is such that

$$b - Ax_c = r \quad , \qquad (3.3)$$

Where r is called residual vector. From (3.2) and (3.3)

$$Ax_t - Ax_c = r \quad , \qquad (3.4)$$

Hence $$x_t - x_c = A^{-1}r \quad , \qquad (3.5)$$

Therefore, if some elements of A^{-1} are large, a small component of r can still mean a large difference between x_t and x_c, or conversely, x_c may be far from x_t but r can nevertheless still be small. This implies that we cannot test the correctness of a computed solution of (3.1) merely by substituting the result into the equations and calculating the residuals. Or to put it another way, an accurate solution, i.e. a small difference between x_c and x_t will always produce small residuals if the matrix A is normalized, but small residuals do not guarantee an accurate solution.

There are three sources of error in the solution of systems of linear equations. The first caused by errors in the coefficients and the elements of b. When such errors occur, we must live with them because these quantities are empirical. If abound on the empirical errors is known we can do no more than use this to get bounds on the errors in the solution. We can control this source of error by using double precision arithmetic if necessary. The coefficient and vector b must be rounded when are inserted into the computer.

The second source of error is the roundoff error introduced in calculating the solution. The third source is truncation error.

CONCLUSIONS

Haytham elimination is one in the class of direct methods for solving linear system. Direct methods are those which, in the absence of round-off or other errors, will yield the exact solution in a finite number of elementary arithmetic operations. In practice, because a computer works with a finite word length, the direct methods do not lead to exact solutions. Indeed, errors arising from round-off, instability and loss of significance may lead to extremely poor or even useless results.

The solution of Haytham elimination method by back substition is more faster than Gaussian elimination and involves a little computation than Gauss-Jorden elimination. Further the solution of linear system by Haytham elimination is accurate.

If the matrix of coefficients can be entered into the fast store of computer then Haytham elimination is quick and more accurate. If also the matrix of coefficients has some special property or structure it is usually possible to increase the number of equations that can be solved by very efficient programming. Haytham elimination method are preferable when

(i) Several sets of equations with the same coefficient matrix but different right hand sides have to be solved.

(ii) The matrix is nearly singular. In this case small residuals do not imply small errors in the solution. This can be easily seen since.

Therefore, $(x_t - x_c)$ will have large components when the components of the residual vector are small because some of the elements of A^{-1} will be large, if A is nearly singular.

Finally in the Haytham elimination we can find that the odd rows contains just the odd position elements and in the even rows contains just the even position elements. Therefore we can compute the sum of odd position elements and the sum of even position elements directly.

APPENDICES

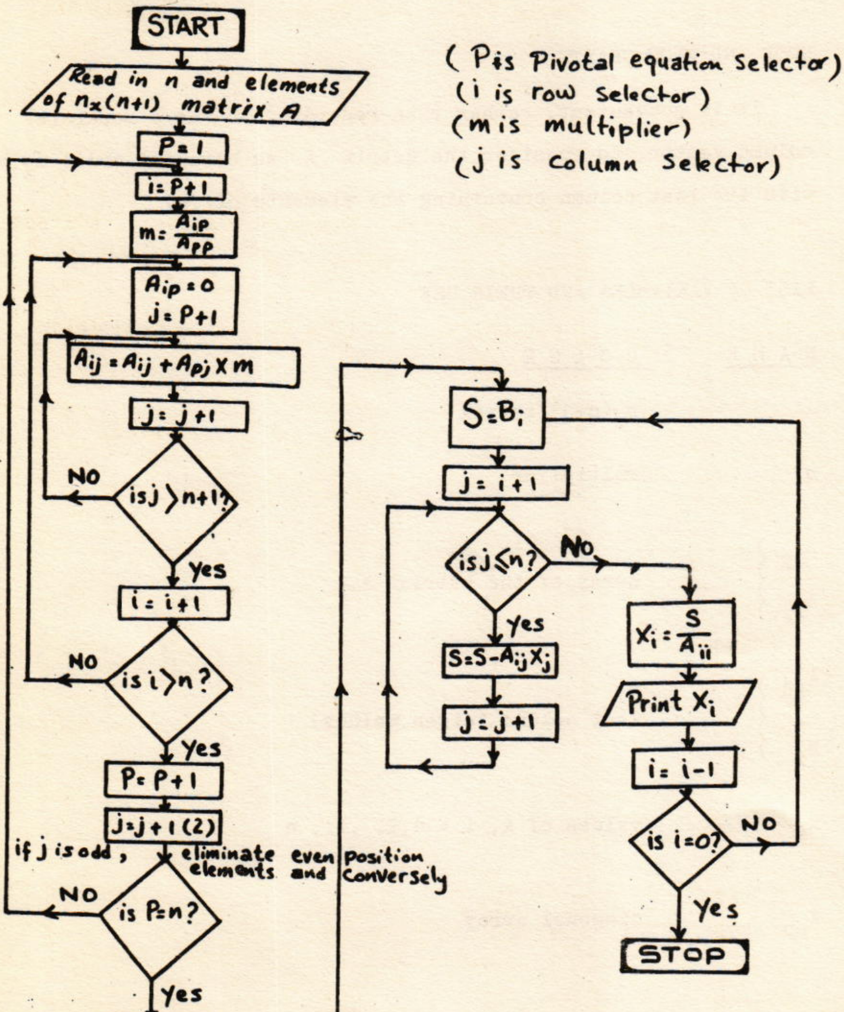
NOTE AND FLOW CHARTS

It is convenient, rather than regarding b as a separate column vector, to consider the matrix A as being of order $n_x(n+1)$ with its last column containing the elements of b.

LIST OF VARIABLES AND THEIR USE

NAME	USAGE
A	$n_x(n+1)$ matrix
m	multiplier
A_{ip}, A_{ij}	array of the matrix A.
A_{pj}, B_i	last column (eigen values)
x_i	values of x, i = 1,2, ..., n
A_{pp}	diagonal array

FLOW CHART FOR SOLVING, by HAYTHAM ELIMINATION, THE $n \times n$ SYSTEM OF EQUATIONS $Ax = b$.



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A MATHEMATICAL MODEL
OF A ONE-SEX POPULATION GROWTH

A.R. AL-RAWI*

A B S T R A C T

A study of the age composition of population with changing age specific birth and death rates are considered. It is assumed that birth and death rates are functions of age and time. With these assumptions, we derive our mathematical model which is a Voltera integral equation of the second kind. Two methods of solution are given for this model.

INTRODUCTION

In this paper we shall study one-sex (female) population growth which is common to most of the demographic work. The present problem considers the age structure of the population and the number of individuals of different ages at any time. This problem has been tackled by Coale [1] , Cooke and York [5] , Keyfitz [2] , and Pollard 4 with different assumption. Their main assumptions were that birth and death are functions of the size of population or age, and in particular, are not dependent upon time.

We assume that we have a closed population which means that there is no imigration in to or out of the population. All other phenomenas which could effect the population are allowed to take place. Finally the birth rate, death rate, and age-density function are considered to be functions of age and time.

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It is common knowledge that the probability of brith of a female daughter depends upon the age of the prospective mother and the probability of death of a female depends on her age. The available data for birth rate is different at different times for the same age group. Thus we define the age specific birth rate m(x,t) by:

$$m(x,t) = \lim h^{-1} \text{ pr}\Big\{\text{a female of age } x \text{ at time } t \text{ gives birth to a female daughter in the time interval } (t,t+h)\Big\},$$

where pr $\{.\}$ stands for the probability of the event described in the brackets. By this definition of m(x,t) is a continuous function with respect to age and time, because age propagates with velocity 1 with respect to time. Ofcourse this function is not easy to be represented analytically by elementary functions which might be easy to handle. However, it might be possible to find functions which would represent it approximately.

In the next section we will derive our mathematical model which happen to be a Voltera integral equation of the second kind. A method of solution of this integral equation is considered in Section 3.

POPULATION GROWTH MODEL

Let a and b($0 < a < b$) be the minimum and maximum age of child bearing and let

k(x,t)dx = number of female population between age x and x + dx at time t,

B(t)dt = total number of females born during (t,t+dt).

Clearly k(0,t) = B(t). Also let the probability of survival p(x,t) be defined by ,

$$p(x,t) = \text{pr}\Big\{\text{a female born at time to survives to age } x \text{ or to time } t + x\Big\} .$$

From this definition it follows that

$$k(x,t) = B(t-x)\,p(x,t-x), \quad \ldots\ldots\ldots\ldots \quad (1)$$

$p(o,t) = 1$ and $p(L,t) = o$ where L is the maximum life span of individuals in the population. The function $p(x,t)$ is a monotonically decreasing when we follow the same age group fram age o to L. With all these assumptions and using Figure 1, we develop our mathematical model.

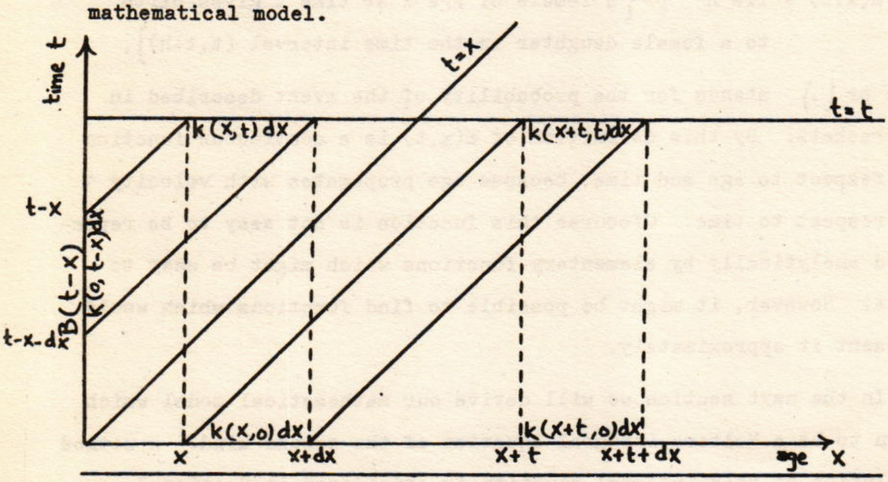

Figure 1.

From equation (1) we have

$$k(x,o) = B(-x)p(x,-x) \text{ , and}$$

$$k(x+t,t) = B(-x)p(x+t,-x)$$

which follows that

$$k(x+t,t) = \frac{k(x,o)\,p(x+t,-x)}{p(x,-x)} \quad \ldots\ldots\ldots\ldots \quad (2)$$

Thus, the number of daughters born to those women between age x+t and x+t+dx at time t will be

$$k(x+t,t)\,m(x+t,t)\,dx = \frac{k(x,o)\,p(x+t,-x)}{p(x,-x)}\,m(x+t,t)dx.$$

Hence, the total number of daughters born between time t and t+dt to those women who were born before time t=o will be denoted by G(t) and it is given by

$$G(t) = \int_{a-t}^{b-t} \frac{k(x,o)\ p(x+t,-x)}{p(x,-x)}\ m(x+t,t)dx \ .... (3)$$

Where a and b are as given above. We should note that $G(t)=o$ for $t \geqslant b$, because if $t \geqslant b$ then $x+t \geqslant b$ and hence $m(x+t,t) = o$.

However, females born after time t = o will also start to give birth at $t > a$ and they will dominate the region $t > x$ in Figure 1. At time t ($t > o$), the women whose ages between a and b will also give birth. From equation (1) and Figure 1 we have, the total number of daughters born to those women who were born after time t = o will be:

$$k(x,t)\ m(x,t)dx = \int_a^t B(t-x)\ P(x,t-x)\ m(x,t)dx$$

Therefore, B(t) is given by

$$B(t) = G(t) + \int_a^t B(t-x)\ p(x,t-x)\ m(x,t)dx \ .... (4)$$

which is a linear Voltera integral equation of the second kind. Note that if B(t) is known them from (1) we can determine the population density for any age x at any time $t > o$. Thus knowing B(t) we can know the age structure of the population.
Equation (4) will represent our mathematical model.

METHODS OF SOLUTIONS

Given a population, we can determine a,b, k(x,o), p(x,t) and m(x,t). Ofcourse a and b are different in different populations. Also from the birth and death date of the given population we can constract an explicit representation of m(x,t) and p(x,t). Finally the dat of the age structure of the population at time t=o is required to determine k(x,o). Note that any time t could be considered to be the starting point, that means time t=o. For example, 1900 or any other calander year could be considered to be t=0. The best choice of time t=0 is the smallest calander year of which birth and death data are available.

If a,b, k(x,o), p(x,t) and m(x,t) are given, then one can find G(t) from (3) and then equation (4) can be solved by the well known method of picard successive approximations which is described very well in Miller [3]. The second method of solution is given as follows:

Since $b > a$, there exist a positive integer $N > 1$ such that

$$Na \leqslant b < (N+1)a$$

Now, for k = 1,2,3, ..., we define $B_k(t)$ by

$$B_k(t) = B(t) \quad \text{for} \quad 0 \leqslant t \leqslant ka.$$

Then we have

$$B_1(t) = G(t) \quad \text{for} \quad 0 \leqslant t \leqslant a$$

and

$$B_2(t) = \begin{cases} B_1(t) & \text{for } 0 \leqslant t \leqslant a \\ G(t) + \int_a^t B_1(t-x)\, p(x,t-x)\, m(x,t)dx & \text{for } a \leqslant t \leqslant 2a \end{cases}$$

for $2 \leqslant k \leqslant N$, we have

$$B_k(t) = \begin{cases} B_{k-1}(t) & \text{for } 0 \leqslant t \leqslant (k-1)a \\ G(t) + \int_a^t B_{k-1}(t-x)\, p(x,t-x)\, m(x,t)dx & \text{for } (k-1)a \leqslant t \leqslant ka \end{cases}$$

and then

$$B_{N+1}(t) = \begin{cases} B_N(t) & \text{for } 0 \leqslant t \leqslant Na \\ G(t) + \int_a^t B_N(t-x)\, p(x,t-x)\, m(x,t)dx & \text{for } Na \leqslant t \leqslant b \\ \int_a^t B_N(t-x)\, p(x,t-x)\, m(x,t)dx & \text{for } b \leqslant t \leqslant (N+1)a. \end{cases}$$

Finally, for $m \geqslant 2$, we have:

$$B_{N+m}(t) = \begin{cases} B_{N+m-1}(t) & \\ \int_a^b B_{N+m-1}(t-x)\, p(x,t-x)\, m(x,t)dx & \text{for } (N+m-1)\, a \leqslant t \leqslant (N+m)\, a \end{cases}$$

These steps describe our second method of solution to the linear Voltera integral equation which represent the mathematical model. This method is not difficult to carry out if a, b, k(x,o), m(x,t) and p(x,t) are given explicitly.

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التنبوء بدرجة الحرارة الصغرى في منطقة بغداد

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باحث علمي

ملخص البحث

يتضمن البحث علاقة رياضية تجريبية (Emperical Formula) للتنبوء بدرجة الحرارة الصغرى في بغداد وبقية المنطقة الوسطى من القطر . وقد جرى استنتاج العلاقة بالاعتماد في الاساس على علاقات مشابهة يجرى استخدامها في مناطق اخرى من العالم الا ان خصوصية العلاقة الجديدة هي انطباقها على الظروف المحلية . وقد استنتجت العلاقة باستخدام بيانات عشرة سنوات لمدينة بغداد (الفترة 1960 - 1969) . واثبتت الاختبارات دقة جيدة في النتائج اذ ان معامل التعالق .(الارتباط) بين درجة الحرارة الصغرى التي تتنبأ بها المعادلة ودرجة الحرارة الصغرى الحقيقية هو 96% وان ميل الخط البياني الذي يمثل العلاقة بين الدرجتين هو 0.91% وبخطأ قياسي قدره $\pm 2C^{0}$. ويقترح البحث تغيير قيم بعض الثوابت في العلاقة لتكون قابلة للتطبيق في مناطق القطر الاخرى .

المقدمة

درجة الحرارة الصغرى هي أوطأ درجة حرارة يصل اليها الهواء السطحي اثناء التبريد الاشعاعي الليلي وتسجل بواسطة محرار النهاية الصغرى الذي يوضع داخل صندوق المحارير المعروف والذي يرتفع 1.5 مترا عن سطح الارض عادة .

ونظرا لاهمية درجة الحرارة الصغرى في تكوين الصقيع والضباب وما يترتب عليها من احتياطات ضرورية الاتخاذ لوقاية المزروعات وارشاد وسائط النقل المختلفة ... الخ فان اهتماما كبيرا قد وجه للتنبوء بدرجة الحرارة الصغرى .

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وبسبب صعوبة حل المعادلات النظرية التي تحكم الانتقال الحراري والتي يمكن بواسطتها تعيين درجة الحرارة الصغرى للهواء السطحي مثل المعادلة :

$$F_N + H + L + G = 0$$

حيث F_N هي صافي كثافة الفيض الاشعاعي للارض و H تمثل الانتقال الحراري بواسطة الدوامات الاضطرابية في الهواء و L الحرارة الكامنة للتبخر و G حرارة التوصيل داخل التربة من والى السطح ، يضاف الى ذلك ان التطبيق المباشر لمثل هذه المعادلات غير ممكن حتى عند اجراء بعض التبسيطات التي تنعكس على صحة نتائجها ، فقد التجأ الكثير من المختصين لتطبيق المعادلات التجريبية نظرا لسهولة تطبيقها ودقة نتائجها الى الحد الذي يفي بالغرض في كثير من الاحيان .

هناك معادلات تجريبية كثيرة استخدمت لهذا الغرض تستند الى الطرق الاحصائية في استنباطها وتعتمد في تطبيقها على عنصري درجة الحرارة والرطوبة بمقاييسهما المختلفة كدرجة الحرارة العظمى مثلا بالنسبة للعنصر الاول ودرجة الندى او المحرار الرطب او الرطوبة النسبية بالنسبة للعنصر الثاني مع الاخذ بنظر الاعتبار في الحسابات كمية الغيوم والرياح السطحية او الرياح في طبقات الجو العليا في وقت قياس درجات الحرارة سالفة الذكر او كمية الغيوم وسرعة الرياح المتوقعة خلال الليل . كما في الامثلة الواردة في الملحق رقم (١) . ولكن بالنظر للاختلافات المحلية لطبيعة المنطقة من حيث ارتفاعها وانخفاضها عن مستوى سطح البحر وللاختلافات الحادثة في طول النهار والليل والتي تحدد بدورها التسخين والتبريد وكذلك انواع الترب وطبيعة الغطاء النباتي ، كل ذلك جعل تطبيق هذا النوع من المعادلات ينحصر محليا ولا يصح تطبيقه في مناطق اخرى كما قد لايصح تطبيق البعض منها الا لفترة زمنية محددة من السنة .

واستنادا للترابط المبدئي بين درجات الحرارة الصغرى والعظمى ونقطة الندى بالاضافة الى كمية الغيوم وسرعة الرياح فقد افترض في هذا البحث وجود معادلة تجريبية تحكم هذه المتغيرات .

وباستخدام المعلومات الحقيقية اليومية لعشرة سنوات لتلك المتغيرات تم تحديد ذلك الترابط والتوصل الى معادلة رياضية تجريبية ثبتت صحة استخدامها للتنبوء بدرجة الحرارة الصغرى لمدينة بغداد والمنطقة الوسطى من القطر العراقي ، كما نعتقد انها ممكنة الاستخدام في مناطق القطر الاخرى عدا المنطقة الشمالية وقد تحتاج الى بعض التعديلات في قيم الثوابت K_2 و K_3 (سيرد تعريفها لاحقا) لتصبح قابلة للتطبيق في جميع مناطق القطر .

ان وجود الثابت الاضافي K_3 في هذه المعادلة ساعد في جعل هذه المعادلة قابلة للتطبيق لكافة ايام السنة مما يميز هذه المعادلة عن سواها خصوصا في مناطق خطوط العرض التي يكون فيها

الفرق كبيرا في درجة الحرارة الصغرى بين فصلي الصيف والشتاء . لقد طبقت هذه المعادلة لسنتين معلومتي النتائج وحسب معامل الارتباط (التعالق) بين درجة الحرارة الصغرى المتنبأ بها والصغرى الفعلية فوجد انه يساوي 96 % مما يشير الى مدى تقارب نتائج هذه المعادلة مع الواقع ويؤكد امتيازها بشكل قاطع على المعادلات المناظرة .

المعادلة والنتائج

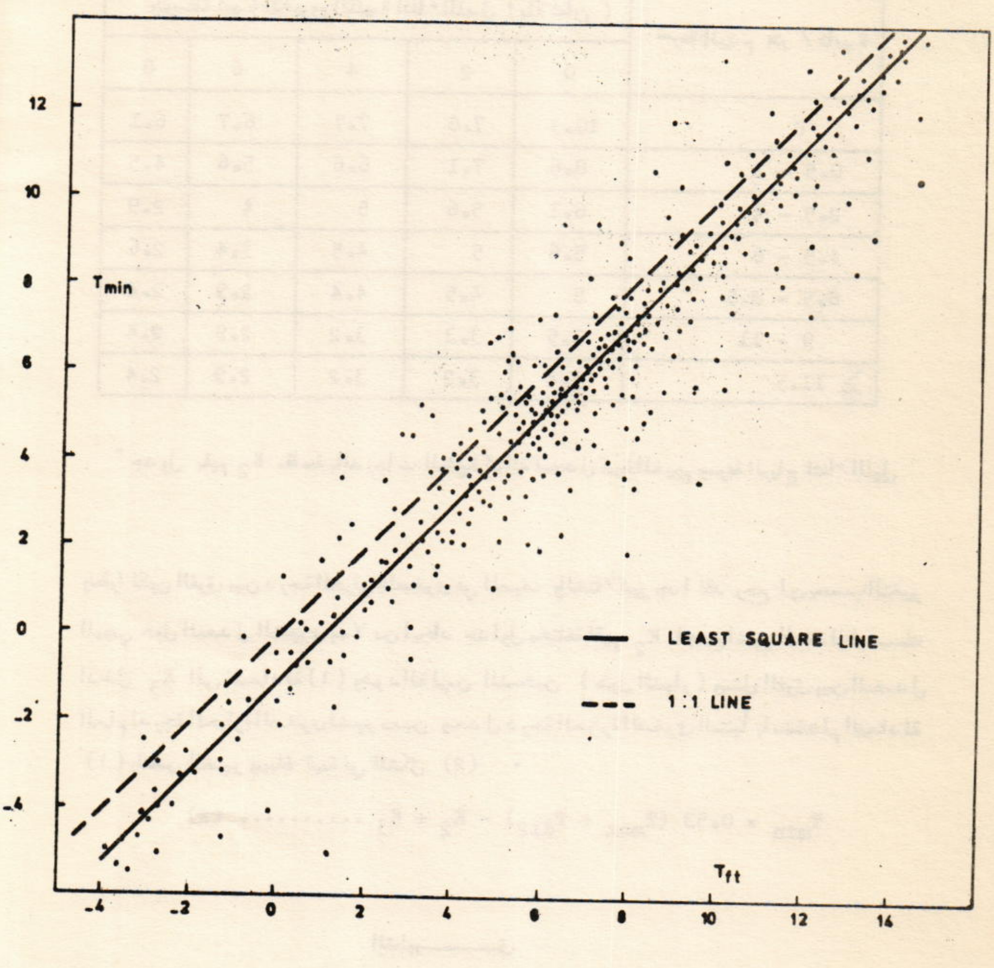
دلت الاختبارات التي جرت لتطبيق بعض المعادلات التجريبية المستخدمة في مناطق عديدة من العالم والتي تتوفر المعلومات المحلية الاولية اللازمة لتطبيقها في العراق على انها لاتعطي نتائج صحيحة لمناطق القطر المختلفة . يستثنى من ذلك معادلة مكنزى (ملحق رقم ١) حيث وجد انها تعطي نتائج مقاربة في شهري كانون الاول والثاني لمنطقة بغداد لذلك تم حساب تلك الفروقات البسيطة واعتمدت هذه المعادلة للمقارنة في قيم K_2 المحسوبة لشهر كانون الثاني .

نظرا لكون درجة الحرارة العظمى (Tmax) هي دالة للتسخين اليومي خلال النهار ودرجة حرارة الندى (Td_{12}) معبرة عن رطوبة الهواء السطحي فان درجة الحرارة الصغرى تتأثر بـ Tmax من جهة وبـ Td_{12} من الجهة الثانية . ورغم عدم توفر مبررات نظرية كافية لاعتبار هذين التأثيرين متكافئين الا ان العادة جرت على افتراض التكافوء (المصادر المذكورة في ملحق رقم ١) وقد اثبتت التجربة صحة هذا الافتراض الى حد كبير لذلك فقد اعتمد نفس المبدأ في وضع العلاقة الاولية الخاصة بهذا البحث وبالشكل التالي :

$$T\acute{m}in = K_1 \ (Tmax + Td_{12}) - K_2 \ .... \ (1)$$

حيث Tmax هي درجة الحرارة العظمى و Td_{12} درجة الندى عند الساعة الثانية عشر ظهرا حسب التوقيت العالمي و K_1 ثابت مطلق و K_2 دالة للغيوم والرياح السطحية .

وباستخدام القيم الحقيقية اليومية لكمية الغيوم وسرعة الرياح اثناء الليل بالاضافة الى قيم T_{max} و T_{d12} و T_{min} ولعشر سنوات (1960 - 1969) تم حساب قيم K_1 و K_2 بيانيا كما في الشكل (1) حيث وجد ان $K_1 = 0.53$ وقيم K_2 مبينة في الجدول ادناه :



12
10
8
6
4
2
0
-2
-4
T_{min}
LEAST SQUARE LINE
1:1 LINE
T_{ft}
-4
-2
0
2
4
6
8
10
12
14

الشكل 1

سرعة الريح متر / ثانية	متوسطة كمية الغيوم الكلية اثناء الليل (بالاثمان)				
	0	2	4	6	8
0	10.3	7.8	7.7	6.7	6.1
0.5 - 2	8.6	7.1	6.6	5.6	4.5
2.5 - 4	6.1	5.6	5	4	2.9
4.5 - 6	5.6	5	4.5	3.4	2.6
6.5 - 8.5	5	4.5	4.4	2.9	2.4
9 - 11	3.5	3.3	3.2	2.9	2.4
≥ 11.5	3.4	3.2	3.2	2.9	2.4

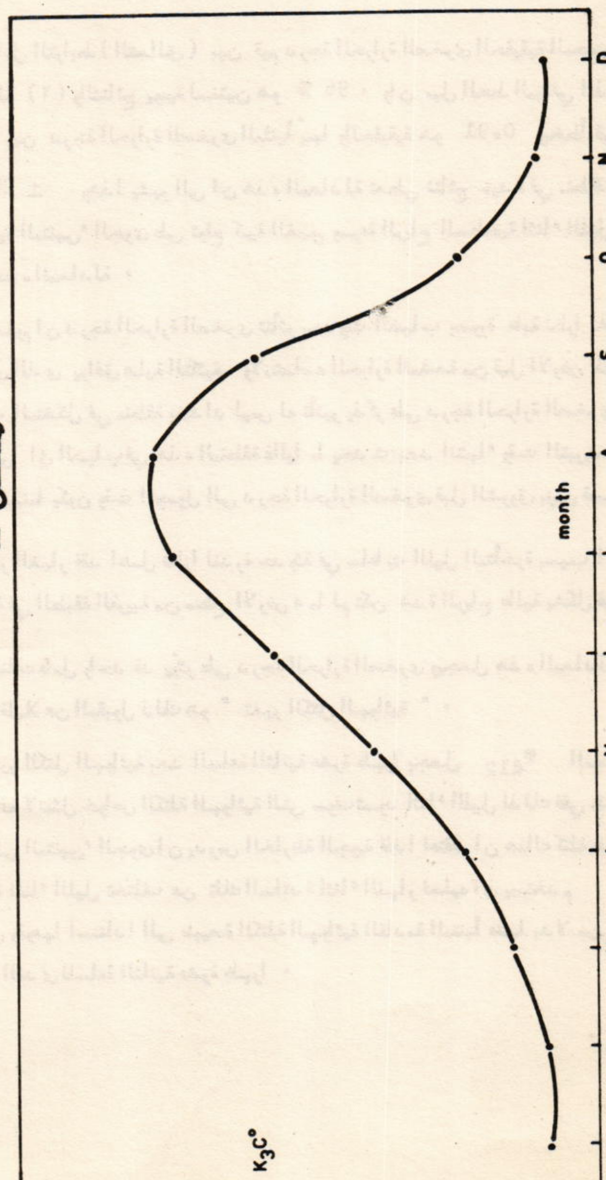
" جدول بقيم K_2 مقاسة بالدرجات المئوية كدالة لمعدل كمية الغيوم وسرعة الرياح اثناء الليل "

ونظرا لكون الفرق بين درجة الحرارة الصغرى في الصيف والشتاء كبير جدا فقد رجح ان يحسب التغير اليومي حول المعدل الشهري بدلا من ايجاد جداول مختلفة لقيم K_2 لجميع اشهر السنة لذلك ادخل K_3 الى المعادلة (١) وهو دالة لزمن التسخين (طول النهار) ويمثل الفرق بين المعدل العام لدرجة الحرارة الصغرى لشهر معين ومعدل درجة الحرارة الصغرى المتنبأ باستخدام المعادلة (١) ولنفس الشهر ومبينة قيمة في الشكل (2) .

$$T_{min} = 0.53\ (T_{max} + T_{d12}) - K_2 + K_3 \quad \ldots\ldots\ldots\ldots \quad (2)$$

التطبيـــــق

يمكن ان تستخدم المعادلة (٢) للتنبوء بدرجة الحرارة الصغرى في منطقة بغداد وذلك باخذ قيم T_{max} و T_{d12} والتنبوء بقيم تقريبية لكمية الغيوم وسرعة الرياح السطحية التي سوف تسود في الليل ومنها يستخرج K_2 من الجدول اما قيم K_3 فتؤخذ من الشكل (2) حسب التاريخ من السنة .



الشكل 2

المناقشـــــة

١ـ ان معامل الترابط (التعالق) بين قيم درجة الحرارة الصغرى الحقيقية المحسوبة باستخدام المعادلة (٢) والنتائج يومية لسنتين هو % 96 • وان ميل الخط البياني الذي يمثل العلاقة بين درجة الحرارة الصغرى المتنبأ بها والحقيقية هو 0.91 وخطأ قياسي مقداره $\pm 2C^{\circ}$ وهذا يشير الى ان هذه المعادلة تعطي نتائج جيدة في منطقة بغداد • كما ان قابلية المتنبيء الجوى على توقع كمية الغيوم وسرعة الرياح السطحية اثناء الليل تعزز من دقة نتائج هذه المعادلة •

٢ـ من المعلوم ان درجة الحرارة الصغرى تتأثر بحدوث الضباب بصورة عامة نظرا للانبعاثات الحرارى الذى يرافق عملية التكثيف ولامتصاصه الحرارة المشعة من قبل الارض ولكن وجد ان الضباب المتشكل في منطقة بغداد ليس له تأثير يذكر على درجة الحرارة الصغرى وهذا قد يرجع الى ان الضباب في هذه المنطقة غالبا ما يحدث بعد انتهاء وقت التبريد عند شروق الشمس بينما يكون وقت الوصول الى درجة الحرارة الصغرى قبل الشروق بزمن قصير •

٣ـ اما تأثير الغبار فقد اهمل نظرا لندرة حدوثة في ساعات الليل المتأخرة بسبب الاستقرارية العالية في الطبقة القريبة من سطح الارض ، ما لم تكن شدة الرياح عالية بشكل غير عادى •

٤ـ يبقى هناك عامل واحد قد يؤثر على درجة الحرارة الصغرى ويجعل هذه المعادلة تعطي نتائج تبتعد قليلا عن المقبول ذلك هو " تغير الكتل الهوائية " •

ان تغير الكتل الهوائية بعد الساعة الثانية عشرة ظهرا يجعل T_{d12} المستخدمة في المعادلة لاتمثل خواص الكتلة الهوائية التي سوف تسود اثناء الليل لذلك ففي مثل هذه الحالة يجب على المتنبيء الجوى ان يدرس الخارطة الجوية فاذا اعتقد ان هناك كتلة هوائية سوف تسود المنطقة اثناء الليل تختلف عن تلك السائدة اثناء النهار فعليه ان يستخدم T_{d12} التي تنبأ عن وقوعها استنادا الى طبيعة الكتلة الهوائية القادمة المتنبأ عنها بدلا من القيمة الحقيقية لدرجة الندى للساعة الثانية عشرة ظهرا •

شكــــــــــــر

نتقدم بشكرنا الجزيل الى هيئة الانواء الجوية العراقية وبالاخص السيد رئيس الهيئــة الاستاذ عبد الغني السلطان والسيد جعفر العاملـي رئيس قسم المنـاخ والســـيد عدنـان العاملـي على تعاونهم معنـا وتزويدهـم لنـا بكافـة البيانـات التـي يسـرت انجـاز هـذا العمــل .

ملحـــق رقـــم (١)

بعـــض المعـــادلات الاختباريـــة

— معادلة كرادوك وبريجاد (Craddock and Pritchard) [3]

$$T_{min} = 0.316\ T_{12} + 0.548\ T_{d12} - 1.24 + K$$

حيث K دالة للغيوم وسرعة الرياح الجيوستروفيكية .

— معادلة بويدن (Boyden) [1]

$$T_{min} = \frac{1}{2}(T_w + T_d) - K$$

حيث T_w و T_d هما درجتا حرارة المحرار الرطب ونقطة الندى عند وقت تسجيل درجة الحرارة العظمى و K دالة للغيوم الواطئة والرياح السطحية .

— معادلة مكنزى (Mckenzie) [3]

$$T_{min} = \frac{1}{2}(T_{max} + T_d) - K$$

حيث T_d عند وقت تسجيل T_{max} و K دالة للغيوم الواطئة والرياح السطحية .

— معادلة سميث J.W. (Smith) [4]

$$T_{min} = T_d + a + bR + cR^2$$

حيث a و b و c ثوابت و R الرطوبة النسبية .

— معادلة ساندرز (Saunders) [5]

$$T_r = \frac{1}{2}(T_{max} + T_d) - K\ ,\quad T_{min} = K_2 T_r$$

— معادلة سفيرف (Zverev) [6]

$$T_{min} = T_{13} - \frac{1}{2}(T_{13} - T_{d13}) - 6$$

$$T_{min} = T_{19} - \frac{1}{2}(T_{19} - T_{12}) - 4$$

(Luterstein and Chudnovski) (2) — معادلة لوترستين

$$T_{min} = T_o - E\,(P\,\delta_1 + R\,\delta_2)$$

حيث T_o درجة عند غروب الشمس وعلى ارتفاع مترين من سطح الارض .

P , R معاملان يعتمدان على خط العرض والفصل .

δ_1 , δ_2 دالتان لرطوبة الارض وسرعة الريح على ارتفاع عشرة امتار .

E الاشعاع المؤثر .

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مجلة علوم المستنصرية

١٩٨٠

رقم الايداع في المكتبة الوطنية ٢٧٨ لسنة ١٩٨٠

مجلة
علوم المستنصرية

المجلد ٥ العدد ٢ . ١٩٨٠

كلية العلوم — الجامعة المستنصرية — بغداد — العراق —

تعليمات للمؤلفين

١. تقدم ثلاث نسخ من البحث مطبوعة على الآلة الكاتبة وعلى ورق ابيض صقيل وتترك مسافة ٢,٥ سم على يسار كل صفحة .
٢. تقدم خلاصة باللغة العربية وأخرى باللغة الانكليزية وتطبع كل منهما على ورقة منفصلة
٣. يطبع عنوان البحث وكذلك اسم المؤلف (او المؤلفين) وعنوانه على ورقة منفصلة ويكتب اسم المؤلف كاملا كان يكتب (احمد م. علي) .
٤. تقدم الرسوم التوضيحية منفصلة عن مسودة البحث وترسم بالحبر الصيني الاسود على ورق شفاف وترفق ثلاث صور لكل رسم وتكتب عناوين الرسوم على نفس الورقة .
٥. تنظم الجداول باسلوب تجعلها مفهومة دون اللجوء الى النص وذلك باعطاء كل جدول وكل عمود وصفا واضحا .
٦. لايجوز اعطاء المعلومات ذاتها بالرسم وبالجدول في وقت واحد الا اذا اقتضت ضرورة النقاش ذلك .
٧. يشار الى المصدر برقم ضمن قوسين [ ] بعد الجملة مباشرة وتطبع كافة المصادر على ورقة منفصلة
٨. من المحبذ حيثما كان ممكنا ان يتسلسل البحث ليتضمن المقدمة ، طرق التجربة ، النتائج . المناقشة.

مجلة علوم المستنصرية

المجلد ٥ العدد ٢ كانون الاول ١٩٨٠

مطبعة اوفسيت باسم ☎ ٨٨٨٧٥٤٥