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Degeneracy Effects of Energy loss Straggling of Homo and Hetero Di-Cluster Ions

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ABSTRACT

The energy loss straggling is obtained from an exact quantum mechanical evaluation, which takes into account the degeneracy of the target plasma, and later it is compared with common classical and degeneracy approximation as a function of incident Homo (H-H, He-He) and Hetero (He-H) di-cluster energy in Kev with different kinds of plasma target. For homonuclear di-clusters (H-H) and (He-He) decreasing temperature, the exact calculation approaches the high degeneracy limit, but the differences are still significant. However, as the temperature rises, the exact result approaches the classical limit. Finally, the energy loss straggling increases with the increasing atomic number of the projectiles (He-He). Our research focuses on targets in the weakly coupled electron gas limit, where we can use the random phase approximation (RPA). This kind of plasma has not been widely researched, considering the fact that it is essential for inertial confinement fusion (ICF).

KEYWORDS: Straggling; plasma degeneracy; Dielectric function; exact calculation; Fermi level.

الخلاصــة

تم الحصول على التطوح فى فقدان الطاقة كدالة لطاقة سقوط عنقود ثنائى الذرة المتجانسة (H-H, He-He) والمختلفة (He-H) بالكيلو الكترون فولت من حساب الميكانيك الكمى التام والتى أخذت بنظر الاعتبار انحلال البلازما, وتم مقارنة النتائج مع التقريب الكلاسيكى والانحلالى لمختلف أنواع البلازما. بالنسبة إلى العنقود الثنائى الذرة المتجانس (H-H, He-He) وعند زيادة درجة الحرارة البلازما فإن الحل التام يقترب من الحل الذى يخص الانحلال التام للبلازما ولكن لايزال هناك بعض الفروقات . مع ذلك كلما ترتفع درجة حرارة البلازما فان الحل التام يقترب من الحل كلاسيكى , أخيرا تطوح فقدان الطاقة يزداد مع زيادة العدد الذرى للقذيفة الساقطة. تم التركيز على البلازما التى تكون لها الالكترونات ضعيفة الاقتران والتى من الممكن فى هذه الحالة استخدام تقريب الطور العشوائي (RPA) من المهم دراسة هذا النوع من البلازما على نطاق واسع من أجل فهم الاندماج بالقصور الذاتي (ICF).

INTRODUCTION

Energy loss and energy loss straggling are very important parameters to characterize cluster ion penetration processes in matter. These energy loss parameters are also relevant in depth profiling, radiation damage, surface analytical techniques, and other applications [1]. We will employ the random phase approximation (RPA), which consists of considering the effect of the particle as a perturbation, so that the energy loss is proportional to the square of the particle charge simplified to a treatment of the properties of the medium only. In the previous literatures, several different measurements of ion stopping power in a homogeneous electron-gas process have been suggested, Arista and Brandt [2] and Bert and Deutsch [3] have been considered the calculation of the energy loss straggling in quantum mechanical plasma of arbitrary degeneracy but without considering damping due to these collisions. The mermin dielectric function has been successfully applied to solid (dense degenerate electron gas) and for classical plasmas (non-degenerate electron gas) by M. D. Barriga-Carasco [4, 5]. Finally, M. Mery, *et.al.* [6] studied the electronic energy loss of

$H^{+¿¿}$ and proton (fragments) from the dissociated of $H_2^{+¿¿}$ ions interacting with ultra-thin amorphous silicon films in order to create atomic structures of nanoscopic dimensions with new and interesting physical properties.

Degeneracy plasmas are of particular importance because of their significant uses in scientific technology and astrophysics. The quantum effects become very important when plasma density is sufficiently increased [7]. This involves degeneracy effects that are important at the temperature of plasma $T \ll T_F = \frac{E_F}{k_B}$, where Fermi temperature $E_F = \left(\frac{\hbar^2}{2m}\right)\left(3\pi^2 n_0\right)^{\frac{2}{3}}$. Particle dispersive effects appear to be significant for short-scale lengths. (Compared with the de-Broglie length characteristic) when $\frac{\hbar\omega_p}{k_B T} \approx 1$. The average distance between particles is $r_s = \left(\frac{3}{4\pi n}\right)^{\frac{1}{3}}$, which is Wigner-Seitz, and the mean distance of the closest approach is $r_c = \left(\frac{2e^2}{m\upsilon^2}\right)$.Through balancing the thermal energy of a particle into one dimension against the repulsive electrostatic potential of a binary pair. In other words [8]

$$12m \quad 2=e2rc \tag{1}$$

If the significance ratio r_s/r_c is small, then the kinetic energies of charged particles are minimal relative to the potential energy interaction. Such plasmas are referred to as strong couples. When the ratio is high, however, strong electrostatic interactions between individual particles are uncommon, and relatively are, events. i. e (when the ratio $\frac{r_s}{r_c}$ is high the electrostatic interaction between the particles is rare and occurs relatively). All of the other particles within its Debye electrostatically influence a typical particle. However, this interaction very rarely results in a significant difference in potential. Such plasmas are termed weakly coupled.

Let us define plasma parameter [9].

$$\Lambda = \frac{4\pi}{3} n \lambda_D^3 = \left(\frac{4\pi\varepsilon_0^{\frac{3}{2}}}{3e^3}\right)\frac{T^{\frac{3}{2}}}{n^{\frac{1}{2}}} \tag{2}$$

Where λ_D is given by:

$$\lambda_D = \sqrt{\frac{k_B T_e}{4\pi e^2 n_e}} \tag{3}$$

In the case that $\Lambda \ll 1$, corresponding to strongly coupled plasma, the Debye sphere is sparsely filled, while for $\lambda_D \gg 1$.The Debye sphere is heavily populated, corresponding to plasma with weak coupling. It could be seen from Eq. (2) that strongly coupled plasma tends to be cold and dense. On the other hand, weakly coupled plasma tends to be hot and dilute plasma [10].

If the ion beam collides with the plasma target, not all of the beam's ions slowdown in the same manner, as the electron energy loss is a stochastic mechanism. It depends on many factors. For instance, the electron density of the target may not be uniform. It is therefore useful to characterize the energy loss straggling that explains the statistical variations in the energy loss of the ion [11]. In order to be consistent with the normal concept of energy loss $S = \frac{-\Delta E}{\Delta l}$, as the magnitude of the average energy loss ΔE per unit of path length Δl, we describe the straggling of energy loss Ω^2 as the variation in energy loss per unit of path length [12].

$$\Omega^2 = \frac{\langle(\Delta E - \langle\Delta E\rangle)^2\rangle}{\Delta l} = \frac{\langle(\Delta E)^2\rangle - \langle\Delta E\rangle^2}{\Delta l} \tag{4}$$

Where $\langle\ldots\rangle$ denotes mean values.

The aim of the paper is to calculations the energy loss straggling for Homo (H-H, He-He) and Hetero (He-He) nuclear di-cluster ions from an exact quantum mechanical estimation, that takes into account the degeneration of the target plasma. Later, the results were compared to traditional classical and degeneracy approximations.

THEORY

The energy loss straggling rate in the dielectric formalism is given by [13],

$$\Omega^2 = \frac{2Z^2}{\pi v^2}\int_0^\infty \frac{dk}{k}\int_0^{kv} d\omega\omega^2\left[2N(w)+1\right]\Im\left(\frac{-1}{\epsilon(k,\omega)}\right) \tag{5}$$

Where w and k denote energy and momentum transfer, respectively, and Z denotes ion charge.

$N(\omega)=[\exp(\beta\omega)-1]^{-1}$ Is the Planck function and $\beta=\frac{1}{k_B T}$. The Planck function and the dielectric function all contain temperature dependency. $\epsilon(k,\omega)$. [2]

The dielectric function is the basis of the dielectric formalism $\epsilon(k,\omega)$ of the target system. The real and imaginary parts of the dielectric function can be separated in the Random Phase Approximation (RPA) setting [14].

$$\epsilon(k,\omega)=\epsilon_r(k,\omega)+i\,\epsilon_i(k,\omega) \quad (6)$$

Where

$$\epsilon_r(k,\omega)=1+\frac{1}{4Z^3\pi k_F}\left[g(u+z)-g(u-z)\right] \quad (7)$$

With

$$g(x)=\int_0^{\infty}\frac{y\,dy}{\exp(Dy^2-\beta\mu)}\ln\ln\left|\frac{x+y}{x-y}\right| \quad (8)$$

And

$$\epsilon_i(k,\omega)=\frac{1}{8Z^3K_F}\theta\ln\ln\left[\frac{1+\exp\left[\beta\mu-D(u-z)^2\right.}{1+\exp\left[\beta\mu-D(u+z)^2\right.}\right] \quad (9)$$

Where $u=\frac{\omega}{Kv_F}$, and $z=\frac{K}{2K_F}$ are the common dimensionless variables and $\beta=1/k_B T$ [15, 16].

The Fermi velocity in (a.u) is $v_F=k_F=\sqrt{2E_F}$, the degeneracy parameter is $D=E_F/k_B T$ and μ is the plasma's chemical potential.

We are going to evaluate special cases of degeneracy, $D=E_F/k_B$, [17]

(i) plasma with high degenerate

$$D=\frac{E_F}{k_B T}\gg 1$$

At the high degeneracy limit, D>>1, Eq. (8) reduces to,

$$g(x)\quad x+\frac{1}{2}(1-x^2)\ln\ln\left|\frac{1+x}{1-x}\right| \quad (10)$$

And

$$\epsilon_i(k,\omega)=\{\frac{1}{8z^3k_F}\frac{\omega}{E_F},\frac{1}{8z^3k_F}\left[-1(u-z)^2\right],0,(u \quad (11)$$

To give rise to the linear dielectric function [18]. Also, when $k_B T\ll\omega$, then $N(\omega)\rightarrow 0$, so the mathematical description for the energy loss straggling simplifies to,

$$\Omega^2=\frac{2Z^2}{mv^2}\int_0^{\infty}\frac{dk}{k}\int_0^{kv}d\omega\omega^2\Im\left(\frac{-1}{\epsilon(k,\omega)}\right) \quad (12)$$

(ii) Non-Degenerate plasma,

$$D=\frac{E_F}{k_B T}<<1$$

When $k_B T\gg w$, which means that $\frac{\omega}{k_B T}\ll 1$, then the quantity:

$$2N+1=\frac{2}{\exp\exp\left(\frac{\omega}{k_B T}\right)-1}+1\approx\frac{2}{1+\left(\frac{\omega}{k_B T}-1\right)}+1=\frac{2k_B T}{\omega}$$

, because $\frac{\omega}{k_B T}\ll 1$,

Tends to $\frac{2k_B T}{w}$. Then the straggling integral Eq. (5), becomes,

$$\Omega^2=\frac{2Z_1^{\,2}}{\pi v^2}\int_0^{\infty}\frac{dk}{k}\int_0^{kv}d\omega\,\omega^2\left(\frac{2k_B T}{\omega}\right)\left(\frac{-1}{\varepsilon(k,\omega)}\right), \quad (13)$$
$$¿\frac{2Z_1^{\,2}}{\pi v^2}(2k_B T)\int_0^{\infty}\frac{dk}{k}\int_0^{kv}d\omega\,\omega\left(\frac{-1}{\varepsilon(k,\omega)}\right)$$

Eq. (13) is identical to the stopping power equation,

$$S=\frac{2Z^2}{mv^2}\int_0^{\infty}\frac{dk}{k}\int_0^{kv}d\omega\omega Im\left(\frac{-1}{\epsilon(k,\omega)}\right) \quad (14)$$

Therefore, the relation between Eqns. (13 and 14) is given as follows:

$$\Omega^2=2k_B TS \quad (15)$$

As $k_B T\gg w$, only low projectile velocities are true for the last approximation. $v\leq 0.15v_{th}$ Where v_{th} is the target electrons' thermal velocity $v_{th}=\sqrt{k_B T}$. Eq. (15) gives the results for classical plasmas [19].

The other point of view: the ratio of equation integrals given in Eqns. (13 and 14) is given as follows [12].

$$\frac{\Omega^2}{S}=\frac{\int_0^{\infty}\frac{dk}{k}\int_0^{kv}\omega^2 d\omega\left[\frac{-1}{\varepsilon(k,\omega)}\right]}{\int_0^{\infty}\frac{dk}{k}\int_0^{kv}\omega d\omega\left[\frac{-1}{\varepsilon(k,\omega)}\right]}\approx\langle E\rangle \quad (16)$$

Statistically the average energy $\langle E\rangle=2k_B T$

RESULTS

Figures. (1, 2, and 3) represent the normalized energy straggling Ω^2/Ω_B^2 as a function of the incident for homo (H-H, He-He) and Hetero (He-H) nuclear di-cluster ions in Kev for multiple plasma targets (densities and temperatures), where $\Omega_B^2 = 4\pi n_e Z^2$ is the Bohr formula.

The exact calculations are achieved from Eqns. (7,9), Considering the degeneracy of the plasma electron D, then contrast to the results of energy straggling achieved with high degeneration limits, $k_B T \leq E_F$, and high temperature $k_B T \geq E_F$. If the degeneracy of the target is big, then the high degeneracy level is approaching for exact straggling. If the degeneracy is minimal, however, the exact outcome reaches the classical level. This can be a rough approach for testing if any approximation, quantum or classical, a particular plasma is well adapted for the energy straggling estimation.

DISCUSSION AND CONCLUSION

Figures 1, 2, and 3 represent the energy straggling as a function of incident homonuclear di-cluster ((H-H), (He-He)) and heteronuclear di-cluster (He-H) energy in KeV . Good agreement is achieved with available data of Barriga (2008) [12] for homonuclear di. $D = 0.01633, 0.1633)$ and $n_e = 10^{22}\,cm^{-3}$ compared with Fig (1c) with (D=1.6331), the quantum form cases closer to classical and exact cases, and diverges when D-cluster (H-H). The classical approach to the exact formula at $T = (100, 10)\,eV$ (i. e. decreases, while at $T = (100,10)\,eV$, $(i.e.\,D = 0.0758, 0.758)$ and density $n_e = 10^{23}\,cm^{-3}$, the quantum model disagrees with exact and classical as shown in Figure 1d, e. Figures 2 and 3 display the same as in Figure 1 But for (He-He) Helium- Helium di-cluster ions and Helium-Hydrogen (He-H) di-cluster ions.

When the density of electron gas increases, the Fermi energy level E_F increases as well, which causes degeneracy D, increases and gives the quantum model to be close to the exact and the classical model.

For homonuclear di-cluster (H-H) and (He-He) decreasing temperature, the exact computation oncoming to the high degeneracy level, but the variation is still considerable. However, as the temperature rises, the exact outcome is approaching the classical limit. Finally, the energy loss straggling is increasing with the increasing of atomic number of the projectiles (He-He).

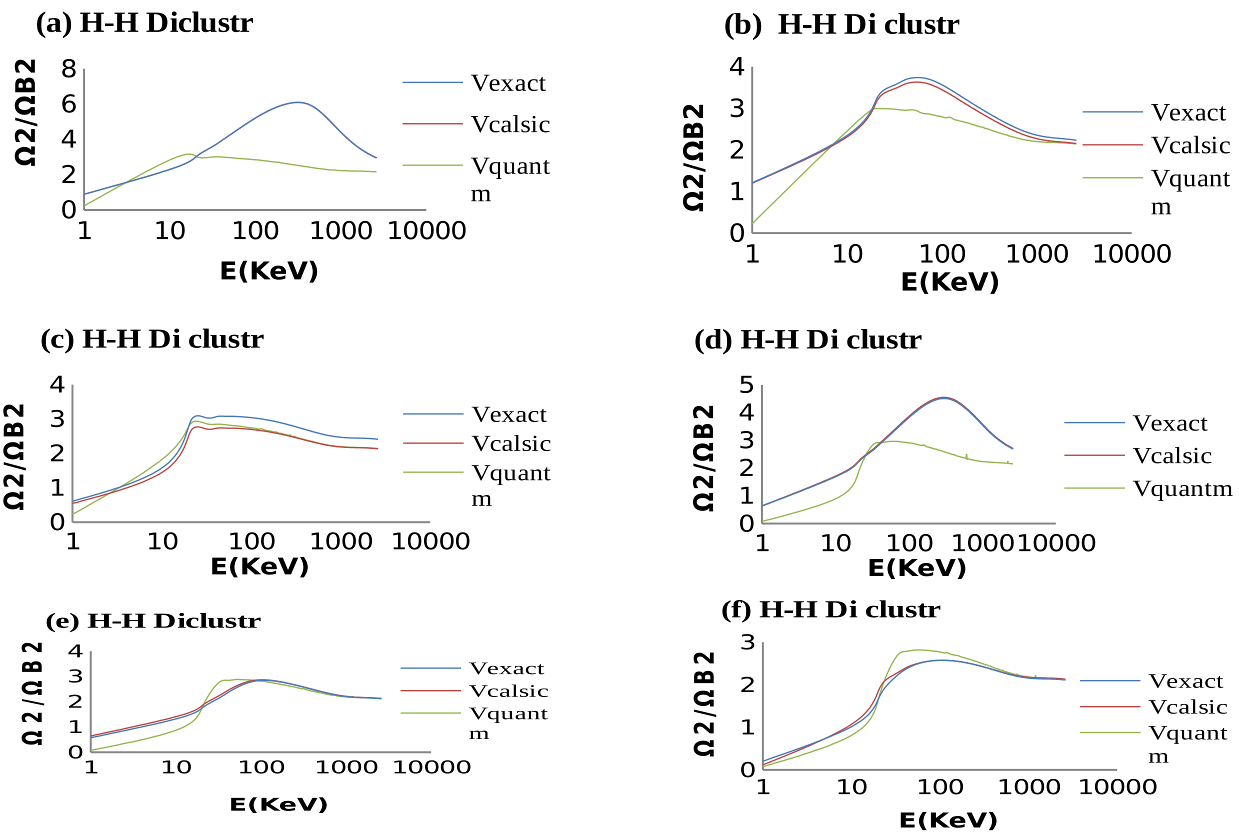

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Figure 1. RPA H-H di-cluster energy straggling as a function of its energy but with various forms of plasma target (a) $n_e = 10^{22}\,cm^{-3}$ and T=100 eV, (b) $n_e = 10^{22}\,cm^{-3}$ and T=10 eV, (c) $n_e = 10^{22}\,cm^{-3}$ and T=1eV, (d) $n_e = 10^{23}\,cm^{-3}$ and T= 100 eV, (e) $n_e = 10^{23}\,cm^{-3}$ and T= 10 eV and (f) $n_e = 10^{23}\,cm^{-3}$ and T=1eV

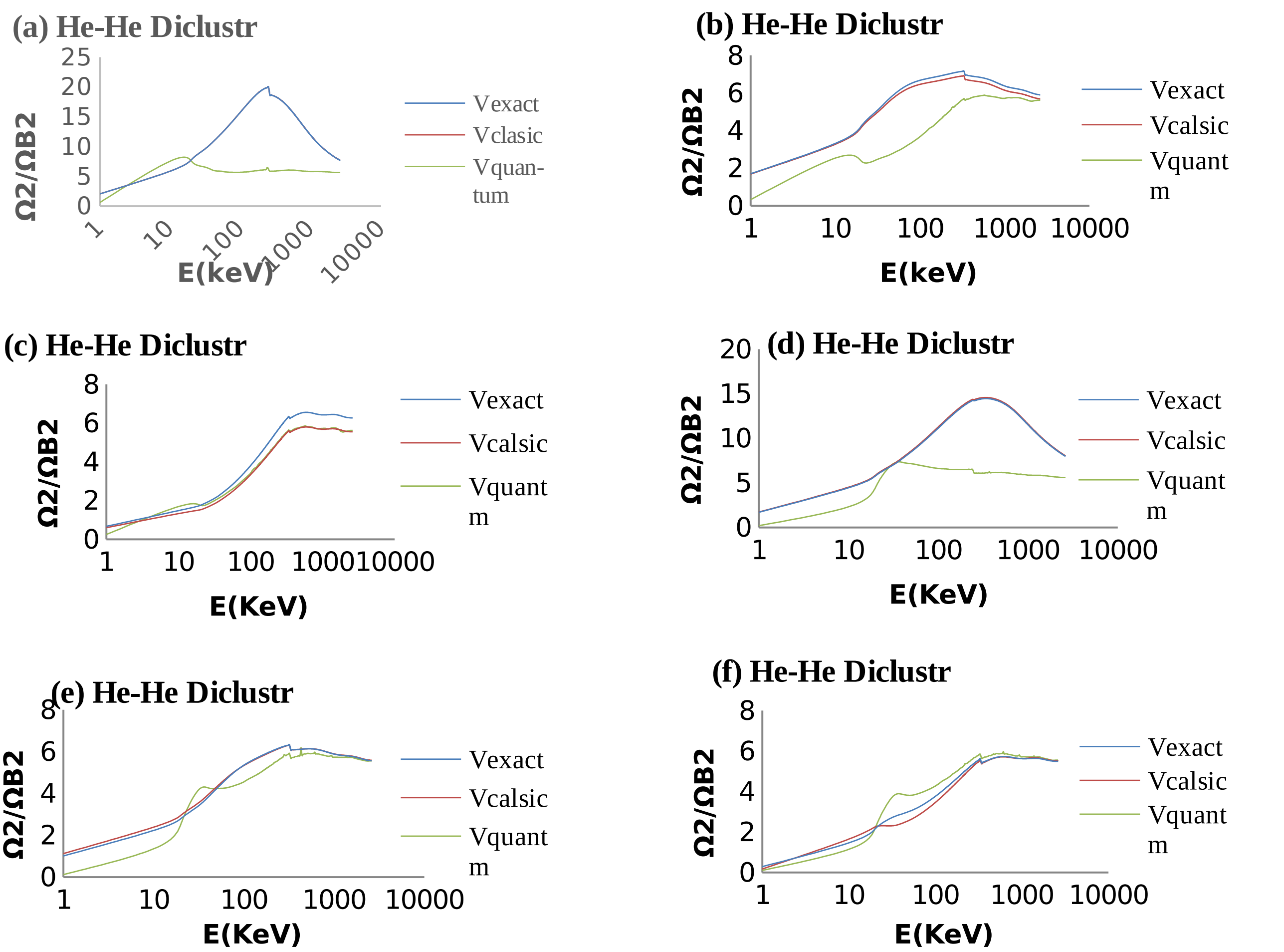
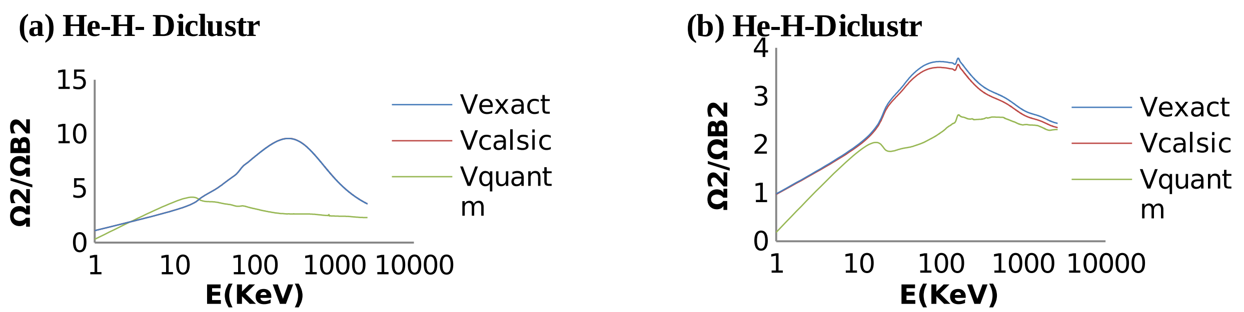


Figure 2. RPA He-He di-cluster energy straggling as a function of its energy but with various forms of plasma target (a) ne=$10^{22}\,cm^{-3}$ and T=100 eV, (b) ne=$10^{22}\,cm^{-3}$ and T=10 eV, (c) ne= $10^{22}\,cm^{-3}$ and T=1eV , (d) ne= $10^{23}\,cm^{-3}$ and T= 1 eV, (e) ne= $10^{23}\,cm^{-3}$ and T= 10 eV and (f) ne= $10^{23}\,cm^{-3}$ and T=100 eV.



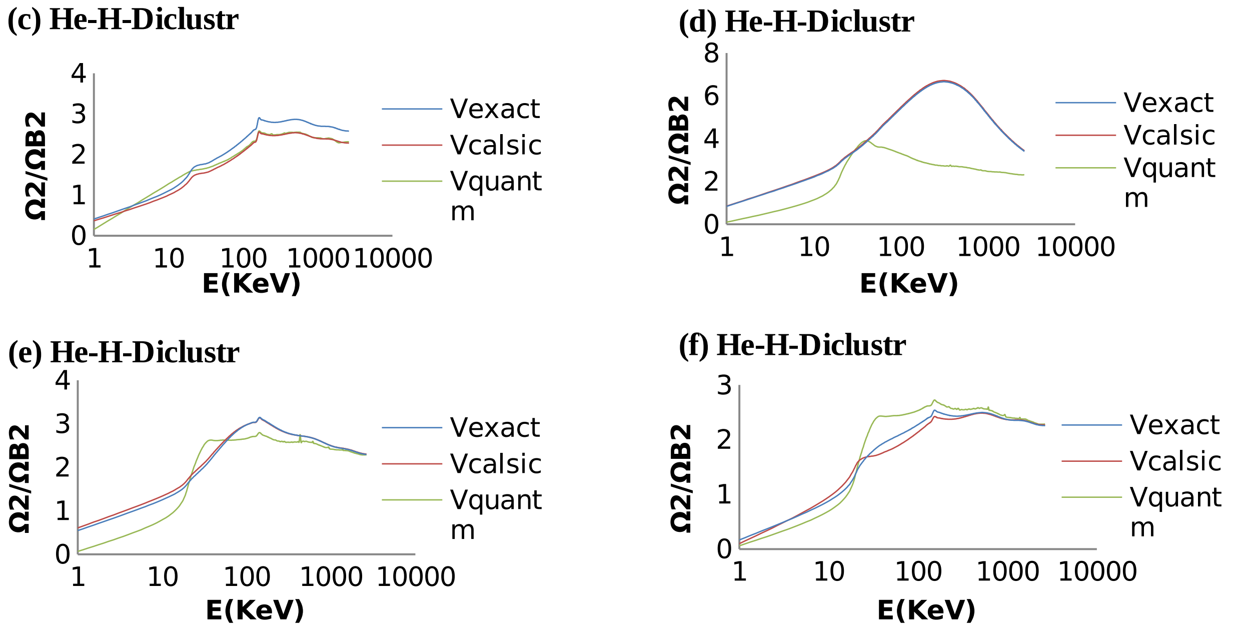


Figure 3. RPA He-H di-cluster energy straggling as a function of its energy but with various forms of plasma target (a) n_e= $10^{22}\ cm^{-3}$ and T=100 eV, (b) n_e=$10^{22}\ cm^{-3}$ and T=10 eV, (c) n_e=$10^{22}\ cm^{-3}$ and T=1eV, (d) n_e= $10^{23}\ cm^{-3}$ and T= 100 eV, (e) n_e= $10^{23}\ cm^{-3}$ and T= 10 eV and (f) n_e= $10^{23}\ cm^{-3}$ and T=1eV

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