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Characterization of Homogeneous System of Difference Equations

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ABSTRACT

In this paper we introduced new definitions of the system of homogenous difference equations of order two; namely homogenous and semi homogenous system, where we focused on finding the equivalents for these definitions of order one as well as of order greater than one for the system of difference equations of the second order and given some examples. We also a given formula to find the power of the matrix that we used in this research.

KEYWORDS: Difference equation; homogenous system; semi-homogenous; difference system.

الخلاصة

قدمنا في هذه البحث تعاريف جديدة لنظام المعادلات الفرقية المتجانسة من الرتبة اثنين؛ اسميناهما نظام متجانس وشبه متجانس، حيث ركزنا على إيجاد المكافئات لهذه التعاريف من الرتبة واحد وكذلك الرتبة الأكبر من واحد لنظام المعادلات الفرقية من الرتبة الثانية واعطينا بعض ألامثلة. كما قدمنا أيضًا صيغة لإيجاد القوة للمصفوفة، التي استفدنا منها في بحثنا هذا.

INTRODUCTION

Difference equations is very important branch in mathematics, it runs parallel to differential equations and is no less important on it. The general difference equation is defined as follows [1].

$$x(n+t) = f(x(n)) \qquad \text{.................}(1)$$

Difference equation can be defined as: an equation showing the relationship between successive values of a sequence and the differences between them. it often rearranged as a recursive formula, hence the output of system can be calculated from the input signal and it's past outputs. For example

$a(t) + 7a(t-1) + 2a(t-2) = b(t) - 4b(t-1)$.

This relation expresses itself in the general form to difference equation (1)

but this study, takes $t = 1$.

We will adopt the notation

$$f^2(x_0) = f(x_0) = f(f(x_0)), f^3(x_0) = f(f(f(x_0))), \ldots\ldots, etc.$$

Where $f(x_0)$ is called the first iterate of x_0 under f;

$f^2(x_0)$ is called the second iterate of $x_0$under f;

In general, $f^n(x_0)$ is the n^{th} iteration of x_0 under$f$. Thus

$$x(n+1) = f^{n+1}(x_0) = f[f^n(x_0)] = f(x(n)).$$

Letting $x(n) = f^n(x)$.

Differential equations are included in many fields, it found in medicine, economics[7], physics[6][7], chemistry ... etc. and have many applications, for example, the question developed by the world that led to the Hanoi horoscope can be found in [6].

The concept of homogeneity as found in differential equations is also found in difference equations. There are two concepts of homogeneity, depending on the order of the equation, for homogeneity in the equations of the first order is known by the following equation

$$x(n+1) = a(n)X(n) + b(n) \qquad \text{.............}(2)$$

where $b(n) = 0$, otherwise equation (2) called nonhomogeneous [2].

While for the order that is larges or equal to two, the homogeneity intended by

$x(n+t) + a_1(n)x(n+k-1) + \cdots +$
$a_k(n)x(n) = g(n)$ …………………(3)

that the right side of equation (3) is equal to zero[3].

In differential equations there is another concept of homogeneity of the first-order equations, which is known as the following

$f(\alpha x, \alpha y) = \alpha^m f(x,y)$ ……………..(4)

where $m > 0$ [3].

These concepts apply to differential and difference systems

$X(n+1) = \acute{A}(c)X(n)$ …………………(5)

as well

Based on the concept of homogeneity (4) Asadi in [4], generalized the concept of homogeneity on the systems of difference equations and called it generalization of homogeneity and defined it as follows:

$F\left(A(c)X(n)\right) = (A(c))^m F(x(n))$ …….(6)

where $X = (x_1, x_2, \dots, x_n)^T$,
$F = (f_1, f_2, \dots, f_n)$,$f_i$ is continuous and differential function defined on $D \subseteq R$, $m > 0$ and for any matrix $A(c)$ such that

$c \in (1-p, 1+p)$,$p > 0$ ……………..(7)

But he did not delve into it, just giving a definition and some prelude to the definition.

In this research without using condition (7) we call the definition (6) the homogeneity of the homogeneous system.

A new concept called semi homogenous of homogenous system of difference equations if there exists a non-zero, nonidentity real matrix A such that the equation (6) is hold.

This paper focused on finding the equivalents for these definitions of order 1 as well as of order greater than one for the system of difference equations of the second order (i.e. the matrix has dimensions 2) and given some examples.

REREPRESENTATION OF A^n

Lemma 2-1 [3]:

We can representation of An in the form.

$A^n = \sum_{i=1}^{h} v_i\,(n)\,N(i-1)$

Where A is a real Matrix, $N(n) = \prod_{i=1}^{n}(A - \alpha_i I)$, $v_1(n) = \alpha_1^n$ and $v_i(n) = \sum_{j=0}^{n-1} \alpha_i^{n-1-j} v_{i-1}(j)$, $i = 2,3,\dots,k$.

Theorem 2-2:

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of real numbers with eigenvalues $\alpha_1$and α_2 ($\alpha_1 \neq \alpha_2$then we can find A^n by the following formula $A^n =$

$$\begin{pmatrix} \alpha_1^n + \frac{[(\alpha_1)^n - (\alpha_2)^n](a_{11} - \alpha_1)}{(\alpha_1 - \alpha_2)} & \frac{[(\alpha_1)^n - (\alpha_2)^n](a_{12})}{(\alpha_1 - \alpha_2)} \\ \frac{[(\alpha_1)^n - (\alpha_2)^n](a21)}{(\alpha_1 - \alpha_2)} & \alpha_1^n + \frac{[(\alpha_1)^n - (\alpha_2)^n](a_{22} - \alpha_1)}{(\alpha_1 - \alpha_2)} \end{pmatrix}$$

Proof:

By lemma 2-1 we have

$N(0) = I$, $N(1) = (A - \alpha_1 I)$

$$= \begin{pmatrix} a_{11} - \alpha_1 & a_{12} \\ a21 & a_{22} - \alpha_1 \end{pmatrix}$$

$v_1(n) = \alpha_1^n$, $v_2(n) = \alpha_2^{n-1-1}\ v_1(n)$

$A^n = v_1(n)N(0) + v_2(n)N(1)$

$$= \begin{pmatrix} \alpha_1^n + \frac{\alpha_2^{n-1}\left(\frac{\alpha_1}{\alpha_2}\right)^{n-1}(a_{11} - \alpha_1)}{\frac{\alpha_1}{\alpha_2} - 1} & \frac{\alpha_2^{n-1}\left(\frac{\alpha_1}{\alpha_2}\right)^{n-1} a_{12}}{(\frac{\lambda 1}{\lambda 2} - 1)} \\ \frac{\alpha_2^{n-1}\left(\frac{\alpha_1}{\alpha_2}\right)^{n-1} a_{21}}{\frac{\alpha_1}{\alpha_2} - 1} & \alpha_1^n + \frac{\alpha_2^{n-1}\left(\frac{\alpha_1}{\alpha_2}\right)^{n-1}(a_{22} - \alpha_1)}{\frac{\alpha_1}{\alpha_2} - 1} \end{pmatrix}$$

$$A^n = \begin{pmatrix} \alpha_1^n + \frac{(\alpha_1^n - \alpha_2^n)(a_{11} - \alpha_1)}{(\alpha_1 - \alpha_2)} & \frac{(\alpha_1^n - \alpha_2^n)(a_{12})}{(\alpha_1 - \alpha_2)} \\ \frac{(\alpha_1^n - \alpha_2^n)(a_{21})}{(\alpha_1 - \alpha_2)} & \alpha_1^n + \frac{(\alpha_1^n - \alpha_2^n)(a_{22} - \alpha_1)}{(\alpha_1 - \alpha_2)} \end{pmatrix} \dots\dots\dots (*)$$

Notation: The aim of the above formula (*) is to easily calculate the power of the matrix if this power is a large number.

Example2.3:

Let $A = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$ we can find A^n by the above formula

The eigenvalues of A is $\alpha_1 = 1$, $\alpha_2 = 2$

By theorem 2-2

$$A^n = \begin{pmatrix} \alpha_1^n + \frac{(\alpha_1^n - \alpha_2^n)(a_{11} - \alpha_1)}{(\alpha_1 - \alpha_2)} & \frac{(\alpha_1^n - \alpha_2^n)(a_{12})}{(\alpha_1 - \alpha_2)} \\ \frac{(\alpha_1^n - \alpha_2^n)(a_{21})}{(\alpha_1 - \alpha_2)} & \alpha_1^n + \frac{(\alpha_1^n - \alpha_2^n)(a_{22} - \alpha_1)}{(\alpha_1 - \alpha_2)} \end{pmatrix}$$

$$= \begin{pmatrix} (1)^n + \frac{(2)^{n-1}\left[\left(\frac{1}{2}\right)^n - 1\right](1-1)}{\left(\frac{1}{2} - 1\right)} & \frac{(2)^{n-1}\left[\left(\frac{1}{2}\right)^n - 1\right](0)}{\left(\frac{1}{2} - 1\right)} \\ \frac{(2)^{n-1}\left[\left(\frac{1}{2}\right)^n - 1\right](1)}{\left(\frac{1}{2} - 1\right)} & (1)^n + \frac{(2)^{n-1}\left[\left(\frac{1}{2}\right)^n - 1\right](2-1)}{\left(\frac{1}{2} - 1\right)} \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & 0 \\ 2^n - 1 & 2^n \end{pmatrix}$$

,n=10, then

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$$A^{10} = \begin{pmatrix} 1 & 0 \\ 2^{10} - 1 & 2^{10} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1023 & 1024 \end{pmatrix}$$

CHARACTERIZATION OF HOMOGENEITY OF HOMOGENOUS SYSTEM

Definition 3-1:

The system (5) is called semi homogenous of homogenous system of difference equations if there exist a non-zero, nonidentity real matrix A such that the equation (6) is hold.

Consider the difference system of order two

$F(X(n)) = \acute{A}X(n)$ ……. (8)

where F is continuous function define on $D \subseteq R^2$ (that is $F = (f, g)$) and $F(X(n)) = X(n+1))$ can be written as:

$\begin{pmatrix} x(n+1) \\ y(n+1) \end{pmatrix} = \begin{pmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{pmatrix} \begin{pmatrix} x(n) \\ y(n) \end{pmatrix}$ ……….. (9)

Now we will find the necessary and sufficient conditions to the system (8) to be semi homogenous of order one.

MAIN RESULT ONE

Theorem 3-2:

The necessary and sufficient conditions to the homogenous system of difference equations (8) to be homogenous of order one is

$a' = d',\ b' = c' = 0$. Where $\acute{A} = \begin{pmatrix} \acute{a} & \acute{b} \\ \acute{c} & \acute{d} \end{pmatrix}$.

Proof:

Necessary condition:

Since F is homogenous of degree one, then for any matrix $A_{2\times2}$, we have $F(AX(n)) = AF(X(n))$, that is

$\begin{pmatrix} f(ax+by) \\ g(cx+dy) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} f(x) \\ g(y) \end{pmatrix}$,

therefore

$$\begin{pmatrix} a(\acute{a}x + \acute{b}y) + by \\ \acute{c}x + \acute{d}(cx+dy) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} f(x) \\ g(y) \end{pmatrix}$$

that is

$$\begin{pmatrix} \acute{a}ax + (\acute{a}b + \acute{b})y \\ (\acute{c} + \acute{d}c)x + d\acute{d}y \end{pmatrix} = \begin{pmatrix} (a\acute{a} + b\acute{c})x + (a\acute{b} + b\acute{d})y \\ (c\acute{a} + d\acute{c})x + (c\acute{b} + d\acute{d})y \end{pmatrix}$$

and we have

$b\acute{c} = 0, \quad \acute{a}b + \acute{b} = a\acute{b} + b\acute{d}, \quad \acute{c} + \acute{d}c = c\acute{a} + d\acute{c}, c\acute{b} = 0$

If $b \neq 0, c \neq 0$, then $\acute{c} = \acute{b} = 0$, and $a' = d'$.

Now if $b = 0$ or $c = 0$ no loss of generally, let $b = 0$, then we have

$$\begin{pmatrix} \acute{a}ax + \acute{b}y \\ (\acute{c} + \acute{d}c)x + d\acute{d}y \end{pmatrix} = \begin{pmatrix} a\acute{a}x + a\acute{b}y \\ (c\acute{a} + d\acute{c})x + (c\acute{b} + d\acute{d})y \end{pmatrix}$$

and $\acute{c} = \acute{b} = 0$, $a' = d'$.

Sufficient condition:

F is homogenous of degree one if for any matrix $A_{2\times2}$ we have $F(AX(n)) = AF(X(n))$.

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a matrix, the $F(AX(n)) = F\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} X(n)\right) = \begin{pmatrix} f \\ g \end{pmatrix}\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} f(ax+by) \\ g(cx+dy) \end{pmatrix}$n, now since $d' = a'$, $b' = c' = 0$, we have

$$\begin{aligned} f(ax+by) &= a'(ax+by) + b'y \\ &= a\acute{a}x + a\acute{b}y + b\acute{a}y \\ &= a(\acute{a}x + \acute{b}y) + b(\acute{c}x + \acute{d}y) \\ &= af(x) + bg(y) \text{ and} \end{aligned}$$

$$\begin{aligned} g(cx+dy) &= c'x + d'(cx+dy) \\ &= c\acute{c}x + c\acute{a}x + d\acute{c}x + d\acute{d}y \\ &= c(\acute{a}x + \acute{b}y) + d(\acute{c}x + \acute{d}y) \\ &= cf(x) + dg(y). \end{aligned}$$

That is $F(AX(n)) = \begin{pmatrix} af(x) + bg(y) \\ cf(x) + dg(y) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} f(x) \\ g(y) \end{pmatrix} = AF(X(n))$.

Hence F is homogenous of order one.

Example 3-3:

Consider the system of difference equations

$$x(n+1) = 3x(n)$$
$$y(n+1) = 3y(n)$$

then since $a' = d',\ b' = c' = 0$ then by theorem(3.2) this system is homogeneous of degree 1 .

To check that we use definition of homogeneous:

Take $A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$ (for special) or any matrix of real numbers.

We must prove that $F\left(\begin{pmatrix}1 & 1\\ -2 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}\right) = \begin{pmatrix}f(x+y)\\ g(-2x+4y)\end{pmatrix} = \begin{pmatrix}1 & 1\\ -2 & 4\end{pmatrix}\begin{pmatrix}f(x)\\ g(y)\end{pmatrix}$.

The left hand $(x+y) = 3(x+y) = 3x+3y$, $g(-2x+4y) = 3(-2x+4y) = -6x+12y$, that is

$$F\left(\begin{pmatrix}1 & 1\\ -2 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}\right) = \begin{pmatrix}3x+3y\\ -6x+12y\end{pmatrix}.$$

The right hand $\begin{pmatrix}1 & 1\\ -2 & 4\end{pmatrix}\begin{pmatrix}3x\\ 3y\end{pmatrix} = \begin{pmatrix}3x+3y\\ -6x+12y\end{pmatrix}$.

So F is homogeneous of order 1.

Now we study under any assumptions homogenous system of difference equations become homogenous of order greater than one.

THE MAIN RESULT TWO

Theorem 3-4:

The necessary and sufficient conditions to a homogenous system of difference equations (8) to be semi homogenous of order n where $n \geq 2$ is the matrix $\acute{A} = \begin{pmatrix}\acute{a} & \acute{b}\\ \acute{c} & \acute{d}\end{pmatrix}$ be a solution of the following algebraic system

$$\left.\begin{array}{r}(a-A)\, a' - Bc' = 0\\ ba' + (1-A)\, b' - Bd' = 0\\ C\acute{a} + (D-1)\acute{c} - c\acute{d} = 0\\ C\acute{b} + (D-d)\acute{d} = 0\end{array}\right\}\ldots\ldots\ldots\ldots(10)$$

Where $A = \begin{pmatrix}a & b\\ c & d\end{pmatrix}, A^n = \begin{pmatrix}A & B\\ C & D\end{pmatrix} = \begin{pmatrix}\alpha_1^n + \frac{(\alpha_1^n-\alpha_2^n)(a_{11}-\alpha_1)}{(\alpha_1-\alpha_2)} & \frac{(\alpha_1^n-\alpha_2^n)(a_{12})}{(\alpha_1-\alpha_2)}\\ \frac{(\alpha_1^n-\alpha_2^n)(a_{21})}{(\alpha_1-\alpha_2)} & \alpha_1^n + \frac{(\alpha_1^n-\alpha_2^n)(a_{22}-\alpha_1)}{(\alpha_1-\alpha_2)}\end{pmatrix}$,

$\alpha_1 \neq \alpha_2$

Proof:

Necessary condition:

A solution of the algebraic system (10) is not unique since there is many solutions of the system (10); that is, there is many homogenous system of order n depending on $\acute{A}$.

Now to proof for order n; that is, to prove $F(AX) = A^nF(X)$ for some A,

$$F(AX) = \begin{pmatrix}f(ax+by\\ g(cx+dy)\end{pmatrix} = \begin{pmatrix}a\acute{a}x + (b\acute{a}+\acute{b})y\\ (\acute{c}+c\acute{d})x + d\acute{d}y\end{pmatrix}$$

$$A^nF(X) = \begin{pmatrix}(A\acute{a}+B\acute{c})x + (A\acute{b}+B\acute{d})y\\ (C\acute{a}+D\acute{c})x + (C\acute{b}+D\acute{d})y\end{pmatrix}.$$

Assume that, $\acute{a} = \acute{d} = 1$, then from system (4) we get $\acute{c} = \frac{a-A}{B}$, and

$Ad - AD = Cb - CB + d - D$

$CB - DA = a - A - Da + cB$, hence

$$A^nF(X) = \begin{pmatrix}(A\acute{a}+B\acute{c})x + (A\acute{b}+B\acute{d})y\\ (C\acute{a}+D\acute{c})x + (C\acute{b}+D\acute{d})y\end{pmatrix}$$

$$= \begin{pmatrix}\left(A + B\left(\frac{a-A}{B}\right)\right)x + \left(A\frac{d-D}{C} + B\right)y\\ \left(C + D\left(\frac{a-A}{B}\right)\right)x + \left(C\frac{d-D}{C} + D\right)y\end{pmatrix}$$

$$= \begin{pmatrix}ax + \frac{Ad-AD+CB}{C}y\\ \left(C + \frac{Da-DA}{B}\right)x + dy\end{pmatrix}$$

$$= \begin{pmatrix}ax + \frac{Cb-CB+d-D+CB}{C}y\\ \left(\frac{CB+Da-DA}{B}\right)x + dy\end{pmatrix}$$

$$= \begin{pmatrix}ax + \left(b + \frac{d-D}{C}\right)y\\ \left(\frac{a-A}{B} + \frac{cB-Da+Da}{B}\right)x + dy\end{pmatrix}$$

$$= \begin{pmatrix}ax + (b+\acute{b})y\\ (\acute{c}+c)x + dy\end{pmatrix} = \begin{pmatrix}f(ax+by\\ g(cx+dy)\end{pmatrix}$$

$$= F(AX)$$

Hence F is semi homogenous of order n.

Sufficient condition:

Let the system (8) be semi homogenous of order n where $n \geq 2$ that is $F(AX) = A^nF(X)$ for some ($n \in Z^+, and\ A \neq 0, A \neq I$), hence

$$A(\acute{a}x + \acute{b}y) + B(\acute{c}x + \acute{d}y) = (A\acute{a}+B\acute{c})x + (A\acute{b}+B\acute{d})y$$
$$= a\acute{a}x + (b\acute{a}+\acute{b})y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(11)$$

$$C(\acute{a}x + \acute{b}y) + D(\acute{c}x + \acute{d}y) = (C\acute{a}+D\acute{c})x + (C\acute{b}+D\acute{d})y$$
$$= (\acute{c}+\acute{d}c)x + (d\acute{d})y \ldots\ldots\ldots\ldots\ldots..\ldots(12)$$

From (11) and (12) we have

$$(a-A)\, a' - Bc' = 0$$
$$ba' + (1-A)\, b' - Bd' = 0$$
$$C\acute{a} + (D-1)\acute{c} - c\acute{d} = 0$$
$$C\acute{b} + (D-d)\acute{d} = 0.$$

To illustrate the Theorem 2.4, the following example introduced.

Example 3-5:

Let $F(X(n)) = \acute{A}X(n)$ be a system of difference equations

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where $\acute{A}=\begin{pmatrix}\frac{2}{3} & 1\\ 1 & \frac{2}{3}\end{pmatrix}$ then, there exist $A=\begin{pmatrix}-1 & 2\\ 2 & -1\end{pmatrix}$ such that
$F\big(AX(n)\big)=A^2F(X(n))$.

Solve:
The eigenvalues of A are $\alpha_1=1$, $\alpha_2=-3$.

$$A^n=\begin{pmatrix}\alpha_1^n+\frac{(\alpha_1^n-\alpha_2^n)(a_{11}-\alpha_1)}{(\alpha_1-\alpha_2)} & \frac{(\alpha_1^n-\alpha_2^n)(a_{12})}{(\alpha_1-\alpha_2)}\\ \frac{(\alpha_1^n-\alpha_2^n)(a_{21})}{(\alpha_1-\alpha_2)} & \alpha_1^n+\frac{(\alpha_1^n-\alpha_2^n)(a_{22}-\alpha_1)}{(\alpha_1-\alpha_2)}\end{pmatrix}$$

$$A^2=\begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}$$

$$f(x)=a'x+b'y=\frac{2}{3}x+y$$

$$g(y)=c'x+d'y=x+\frac{2}{3}y$$

$$F\begin{pmatrix}ax+by\\ cx+dy\end{pmatrix}=A^2\begin{pmatrix}f(x)\\ g(y)\end{pmatrix}$$

$$F\left(\begin{pmatrix}-1 & 2\\ 2 & -1\end{pmatrix}\begin{matrix}x\\ y\end{matrix}\right)=\begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}\begin{pmatrix}\frac{2}{3}x+y\\ x+\frac{2}{3}y\end{pmatrix}$$

The left hand is $\begin{pmatrix}f(-x+2y)\\ g(2x-y)\end{pmatrix}=\begin{pmatrix}\frac{-2}{3}x+\frac{7}{3}y\\ \frac{7}{3}x-\frac{2}{3}y\end{pmatrix}$

The right hand is

$$\begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}\begin{pmatrix}\frac{2}{3}x+y\\ x+\frac{2}{3}y\end{pmatrix}=\begin{pmatrix}\frac{-2}{3}x+\frac{7}{3}y\\ \frac{7}{3}x-\frac{2}{3}y\end{pmatrix},$$

Hence $F$is semi homogenous of order 2

Remark 3-6:
We can solve example (3-5) by using theorem (3-4) that is the matrix $\acute{A}$ is satisfy the system (10)

$$(a-A)\,a'-Bc'=0$$
$$ba'+(1-A)\,b'-Bd'=0$$
$$C\acute{a}+(D-1)\acute{c}-c\acute{d}=0$$
$$C\acute{b}+(D-d)\acute{d}=0$$

Definition 3-7:
The system (5) is called a self-semi homogeneous if $A=\acute{A}$.
Remark 3-8- A system is self-semi homogeneous of order m iff
$F\big(AX(n)\big)=A^{m+1}X(n))$.

Remark 3-9:
Every homogeneous system is self-semi homogeneous and every self-semi homogeneous is semi homogeneous, the converse in general is not true.

Remark 3-10:
Example 3.5 explains remark (3-9). It is semi homogeneous not homogeneous and not self-semi homogeneous.

CONCLUSIONS
The necessary and sufficient conditions to a homogenous system of difference equations (8) to be semi homogenous of order n where $n\geq 2$ is the matrix $\acute{A}=\begin{pmatrix}\acute{a} & \acute{b}\\ \acute{c} & \acute{d}\end{pmatrix}$ be a solution of the following algebraic system

$$\left.\begin{matrix}(a-A)\,a'-Bc'=0\\ ba'+(1-A)\,b'-Bd'=0\\ C\acute{a}+(D-1)\acute{c}-c\acute{d}=0\\ C\acute{b}+(D-d)\acute{d}=0\end{matrix}\right\}$$

Where
$A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}, A^n=\begin{pmatrix}A & B\\ C & D\end{pmatrix}$

$$=\begin{pmatrix}\alpha_1^n+\frac{(\alpha_1^n-\alpha_2^n)(a_{11}-\alpha_1)}{(\alpha_1-\alpha_2)} & \frac{(\alpha_1^n-\alpha_2^n)(a_{12})}{(\alpha_1-\alpha_2)}\\ \frac{(\alpha_1^n-\alpha_2^n)(a_{21})}{(\alpha_1-\alpha_2)} & \alpha_1^n+\frac{(\alpha_1^n-\alpha_2^n)(a_{22}-\alpha_1)}{(\alpha_1-\alpha_2)}\end{pmatrix},$$

$\alpha_1\neq\alpha_2$
The necessary and sufficient conditions to the homogenous system of difference equations (8) to be semi homogenous of order one is
$a'=d',\ b'=c'=0.$ Where $\acute{A}=\begin{pmatrix}\acute{a} & \acute{b}\\ \acute{c} & \acute{d}\end{pmatrix}$.

FUTURE WORK
1-Define (semi) homogeneous of non-homogeneous system of difference equations.
2- Find the necessary and sufficient conditions to the (non) homogenous system of difference equations of order greater than two to be a semi-homogenous.

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