Al-Mustansiriyah Journal of Science
ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 31, Issue 4, 2020 DOI: http://doi.org/10.23851/mjs.v31i4.913

Research Article Open Access

On Some Results in Fuzzy Length Space

Raghad I. Sabri*, Mayada Nazar, Mohammed Ali

Department of Applied Sciences, University of Technology, Baghdad, Iraq.

*Contact email: 100247@uotechnology.edu.iq

Article Info

Received
25/09/2020

Accepted
26/10/2020

Published
20/12/2020

ABSTRACT

In this paper, depending on the notion of fuzzy length space we define the Cartesian product of two fuzzy length spaces. we proved that the Cartesian product of two fuzzy length spaces is a fuzzy length space. More accurately, the Cartesian product of two complete fuzzy length spaces is proved to be a complete fuzzy length space. Furthermore, the definitions of sequentially fuzzy compact fuzzy length space, countably fuzzy compact fuzzy length space, locally fuzzy compact fuzzy length space are introduced, and theorems related to them are proved.

KEYWORDS: Fuzzy Length Space, Fuzzy point, Sequentially fuzzy compact fuzzy length space, Countably fuzzy compact fuzzy length space, Locally fuzzy compact fuzzy length space.

الخلاصة

في هذا البحث ، بناءً على فكرة فضاء الطول الضبابي ، عرفنا الضرب الديكارتي لفضائيين ضبابيين الطول. لقد أثبتنا أن الضرب الديكارتي لفضائيين ضبابيين الطول هوفضاء طول ضبابي. وبدقة أكثر ، ثبت أن الضرب الديكارتي لفضاءات طول ضبابية كاملة هو فضاء طول ضبابي كامل. من ناحية اخرى ، تم تقديم تعريفات فضاء الطول الضبابي المتراص ضبابياً بالتتابع ، فضاء الطول الضبابي المتراص ضبابياً بشكل محسوب ، فضاء الطول الضبابي المتراص ضبابياً محليا ، وتم إثبات النظريات المتعلقة بها.

INTRODUCTION

The publications in the theory of fuzzy set which was firstly introduced by Zadeh [1] show the intention of the researchers to generalize the classical notion of a set and a proposition to accommodate fuzziness. Zadeh indicates and writes in his research that, the concept of fuzzy sets is more general than ordinary sets, as the concept of fuzzy sets provides an appropriate starting point for building a conceptual framework that, in many respects, parallels the framework used in ordinary sets, indicating that fuzzy sets have a wider scope of application. In 1984, Katsaras [2] studied some properties of fuzzy normed and semi-normed spaces. Many authors like Felbin [3], Jian-Zhong and Xiao Xing-huaZhu [4] gave the concept of fuzzy normed spaces in various approaches.

On the other hand, the notions of fuzzy normed space have been studied from various points of view. In particular, Xiao and Zhu [5] introduced a simplified definition of fuzzy normed linear space. They discussed some various structures of a fuzzy normed linear space. In 2009 Sadeqi and Kia [6] studied fuzzy normed space and its topological properties. Another approach for fuzzy normed spaces was considered in [7-11].

The first aim goal of this paper is to study the Cartesian product of two fuzzy length spaces and investigate some important properties of this subject. The second goal is to introduced the definition of sequentially, countably, locally fuzzy compact fuzzy length space and proved basic theorems related to it. The remainder of this paper is organized as follows: In section 2, the main important properties of fuzzy length space on a fuzzy set are covered concisely. In section 3 we introduce the definition of the Cartesian product of two fuzzy length space and prove that the Cartesian product of two complete fuzzy length spaces must be complete fuzzy length space. Finally, in section 4 we presented the concept of sequentially, countably, locally fuzzy compact fuzzy length space, and the main theorems related to it will be proved.

PRELIMINARIES

Definition 2.1 [12]**:**
A binary operation $\diamond:[0,1]^2\rightarrow[0,1]$ is called t-norm if $\forall p,q,t,s\in[0,1]$
the following conditions hold:
(1) $p\diamond q = q\diamond p$
(2) $p\diamond 1 = p$
(3) $(p\diamond q)\diamond t = p\diamond(q\diamond t)$,
(4) If $p\le q$ and $t\le s$ then $p\diamond t \le q\diamond s$.

Remark 2.2 [12]**:**
For each $p>q$, there is t with $p\diamond t \ge q$ and for each s, there is σ with $s\diamond s\ge\sigma$ where $p,q,t,s,\sigma\in [0, 1]$.

Definition 2.3 [13]**:**
Let U be a universal set. A fuzzy point p in U is a fuzzy set with a single element and is denoted by a_α or (a,α).

Definition 2.4 [14]**:**
Suppose that $\breve{\mathcal{A}}$ is a fuzzy set in a linear space $\mathcal{L}$. Let $\breve{\mathcal{F}}$ be a fuzzy set from $\breve{\mathcal{A}}$ to [0,1] and $\diamond$ be a t-norm with:
$(\breve{\mathcal{F}}1)$ $\breve{\mathcal{F}}(a_\alpha) > 0$ for all $a_\alpha \in \breve{\mathcal{A}}$
$(\breve{\mathcal{F}}2)$ $\breve{\mathcal{F}}(a_\alpha) = 0$ if and only if for $a_\alpha = 0$
$(\breve{\mathcal{F}}3)$ $\breve{\mathcal{F}}(ca,\alpha) = \breve{\mathcal{F}}\left(a,\frac{\alpha}{|c|}\right)$ where $c \in \mathbb{K}, c \neq 0$
$(\breve{\mathcal{F}}4)$ $\breve{\mathcal{F}}(a_\alpha + b_\beta) \ge \breve{\mathcal{F}}(a_\alpha) \diamond \breve{\mathcal{F}}(b_\beta)$ for all $a_\alpha, b_\beta \in \breve{\mathcal{A}}$
$(\breve{\mathcal{F}}5)$ $\breve{\mathcal{F}}$ is a continuous fuzzy set for all $a_\alpha, b_\beta \in \breve{\mathcal{A}}$ and $\alpha, \beta \in [0,1]$.
Then $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is called a fuzzy length space (briefly, FL-space) on the fuzzy set $\breve{\mathcal{A}}$.

Definition 2.5 [14]**:**
Suppose that $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ be an FL-space and assume that $a_\alpha \in \breve{\mathcal{A}}$,where $\alpha \in [0,1]$. Given $0 < \epsilon < 1$ then, $\breve{\mathcal{B}}(a_\alpha,\epsilon)=\{b_\beta \in \breve{\mathcal{A}}: \breve{\mathcal{F}}(b_\beta - a_\alpha) > 1-\epsilon\}$ is called the fuzzy open fuzzy ball with a center $a_\alpha \in \breve{\mathcal{A}}$ and radius ϵ.

Definition 2.6 [14]**:**
In an FL-space $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$, a fuzzy sequence $\{(a_n,\alpha_n)\}$ is said to be
(i)Fuzzy converges to a fuzzy point $a_\alpha \in \breve{\mathcal{A}}$ if for each $0 < \gamma < 1$, $\exists$ N with $\breve{\mathcal{F}}((a_n,\alpha_n) - (a,\alpha)) > 1-\gamma$, for all $n \ge N$.
(ii) Fuzzy Cauchy if for any $0 < \gamma < 1$, $\exists$ N with $\breve{\mathcal{F}}((a_n,\alpha_n) - (a_m,\alpha_m) > 1-\gamma$, for all $n, m \ge N$.

Definition 2.7 [15]**:**
Let $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ be an FL-space and $\tilde{\mathcal{C}} \subseteq \breve{\mathcal{A}}$. Let $\widehat{G}$ be a family of fuzzy open fuzzy sets in $\breve{\mathcal{A}}$ that has the property $\tilde{\mathcal{C}} \subseteq \bigcup_{C\in\widehat{G}} C$, in other words, for any $a_\alpha \in \tilde{\mathcal{C}}$ there exists $C \in \widehat{G}$ such that $a_\alpha \in C$, then $\widehat{G}$ is said to be a fuzzy open fuzzy covering or fuzzy open fuzzy cover of $\tilde{\mathcal{C}}$. A finite subfamily of $\widehat{G}$ that itself represents a fuzzy cover is said to be a finite sub-covering or a finite subcover of $\tilde{\mathcal{C}}$.

Definition2.8 [15]**:**
Let $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ be an FL-space if there exists a finite sub-covering for every fuzzy open fuzzy covering $\widehat{G}$ of $\breve{\mathcal{A}}$, this means there exists a finite subfamily $\{G1, G2, G3, \ldots, Gn\} \subseteq \widehat{G}$ such that $\breve{\mathcal{A}} \subseteq \bigcup_{i=1}^{n} G_i$ then $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ is called a fuzzy compact space.

CARTESIAN PRODUCT OF TWO FUZZY LENGTH SPACE

The Cartesian product of two fuzzy length spaces will be introduced in this section. Then we prove that the Cartesian product of two fuzzy length spaces is also fuzzy length space. In the end, we prove that the Cartesian product of two complete fuzzy length spaces is a complete fuzzy length space.
For any two sets $\mathcal{A}$ and $\mathcal{B}$, the Cartesian product $\mathcal{A}\times\mathcal{B}$ is defined by $\mathcal{A}\times\mathcal{B} = \{(a,b): a \in \mathcal{A}, b \in \mathcal{B}\}$. The Cartesian product of two fuzzy length spaces introduced as follows:

Definition 3.1:
Suppose that $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ and $(\breve{\mathcal{M}}, \breve{\mathcal{F}}_{\breve{\mathcal{M}}}, \diamond)$ be two FL- spaces. The Cartesian product of $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ and $(\breve{\mathcal{M}}, \breve{\mathcal{F}}_{\breve{\mathcal{M}}}, \diamond)$ is the product space $(\breve{\mathcal{A}}\times\breve{\mathcal{M}}, \breve{\mathcal{F}}, \diamond)$ where $\breve{\mathcal{A}}\times\breve{\mathcal{M}}$ is the Cartesian product of the fuzzy sets $\breve{\mathcal{A}}$ and $\breve{\mathcal{M}}$ and $\breve{\mathcal{F}}$ is a mapping from $\breve{\mathcal{A}}\times\breve{\mathcal{M}}$ into [0,1] given by :
$\breve{\mathcal{F}}((a_\alpha, b_\beta)) = \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha)\odot\breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta)$, for all $(a_\alpha, b_\beta) \in \breve{\mathcal{A}}\times\breve{\mathcal{M}}$.

Theorem 3.2:
If $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ and $(\breve{\mathcal{M}}, \breve{\mathcal{F}}_{\breve{\mathcal{M}}}, \diamond)$ are two FL- spaces under the same continuous triangular norm. Then $(\breve{\mathcal{A}}\times\breve{\mathcal{M}}, \breve{\mathcal{F}}, \diamond)$ is an FL- space.

ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 31, Issue 4, 2020 DOI: http://doi.org/10.23851/mjs.v31i4.913

Proof:

For each $(a_\alpha, b_\beta), (u_\gamma, v_\delta)$ in $\breve{\mathcal{A}} \times \breve{\mathcal{M}}$, we have $(\breve{\mathcal{F}}1)$ since $\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) > 0$ and $\breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta) > 0$ we have $\breve{\mathcal{F}}((a_\alpha, b_\beta)) = \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta)$ hence $\breve{\mathcal{F}}((a_\alpha, b_\beta)) > 0$ for all $(a_\alpha, b_\beta) \in \breve{\mathcal{A}} \times \breve{\mathcal{M}}$.

$(\breve{\mathcal{F}}2)$ $\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) = 0$ if and only if $a_\alpha = 0$ and $\breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta) = 0$ if and only if $b_\beta = 0$. Together $\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta) = 0$ if and only if $(a_\alpha, b_\beta) = (0,0)$. Hence $\breve{\mathcal{F}}((a_\alpha, b_\beta)) = 0$ if and only if $(a_\alpha, b_\beta) = (0,0)$.

$(\breve{\mathcal{F}}3)$ For each $a_\alpha \in \breve{\mathcal{A}}$, $\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(ca, \alpha) = \breve{\mathcal{F}}_{\breve{\mathcal{A}}}\left(a, \frac{\alpha}{|c|}\right)$ where $c \in \mathbb{K}, c \neq 0$ and $\breve{\mathcal{F}}_{\breve{\mathcal{M}}}(cb, \beta) = \breve{\mathcal{F}}_{\breve{\mathcal{M}}}\left(b, \frac{\beta}{|c|}\right)$.

Now we have

$$\breve{\mathcal{F}}((c(a_\alpha, b_\beta)) = \breve{\mathcal{F}}((ca, \alpha), (cb, \beta))$$
$$= \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(ca, \alpha) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(cb, \beta)$$
$$= \breve{\mathcal{F}}_{\breve{\mathcal{A}}}\left(a, \frac{\alpha}{|c|}\right) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}\left(b, \frac{\beta}{|c|}\right)$$
$$= \breve{\mathcal{F}}\left(((a_\alpha, b_\beta), \frac{\gamma}{|c|}\right)$$

where

$\gamma = \min\{\alpha, \beta\}, \alpha, \beta \in [0,1]$.

$(\breve{\mathcal{F}}4)$ $\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha + u_\gamma) \geq \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(u_\gamma)$. Also $\breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta + v_\delta) \geq \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(v_\delta)$. Now

$$\breve{\mathcal{F}}\left((a_\alpha, b_\beta) + (u_\gamma, v_\delta)\right) \geq \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha + u_\gamma) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta + v_\delta)$$
$$\geq [\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(u_\gamma)] \diamond [\breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(v_\delta)]$$
$$\geq [\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta)] \diamond [\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(u_\gamma) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(v_\delta)]$$
$$\geq \breve{\mathcal{F}}((a_\alpha, b_\beta)) \diamond \breve{\mathcal{F}}((u_\gamma, v_\delta)).$$

$(\breve{\mathcal{F}}5)$ Since $\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha)$ is continuous for all $a_\alpha \in \breve{\mathcal{A}}$ and $\breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta)$ is continuous for all $b_\beta \in \breve{\mathcal{M}}$. Hence $\breve{\mathcal{F}}((a_\alpha, b_\beta)) = \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta)$ is continuous for each $(a_\alpha, b_\beta) \in \breve{\mathcal{A}} \times \breve{\mathcal{M}}$ where $\alpha, \beta \in [0,1]$.

Hence $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \diamond)$ is an FL- space.

Let $\odot$ and $\diamond$ are continuous t-norm then $\odot$ is said to be stronger than $\diamond$ if for each p,q,t,s $\in [0,1]$. Then $(p \diamond q) \odot (t \diamond s) \geq (p \odot t) \diamond (q \odot s)$.

Theorem 3.3:

Suppose that $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ and $(\breve{\mathcal{M}}, \breve{\mathcal{F}}_{\breve{\mathcal{M}}}, \diamond)$ be two FL- spaces under t-norm $\diamond$. If $\odot$ is a continuous triangular norm stronger than $\diamond$ then $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$ is an FL- space.

Proof:

Assume that $(a_\alpha, b_\beta), (u_\gamma, v_\delta) \in \breve{\mathcal{A}} \times \breve{\mathcal{M}}$. Then

$$\breve{\mathcal{F}}\left((a_\alpha, b_\beta) + (u_\gamma, v_\delta)\right) = \breve{\mathcal{F}}((a_\alpha + u_\gamma), (b_\beta + v_\delta))$$
$$= \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha + u_\gamma) \odot \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta + v_\delta)$$
$$\geq [\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(u_\gamma)] \odot [\breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta) \diamond \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(v_\delta)]$$
$$\geq [\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) \odot \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta)] \diamond [\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(u_\gamma) \odot \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(v_\delta)]$$
$$\geq \breve{\mathcal{F}}\left((a_\alpha, b_\beta)\right) \diamond \breve{\mathcal{F}}\left((u_\gamma, v_\delta)\right).$$

$(\breve{\mathcal{F}}5)$ Since $\breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha)$ is continuous for all $a_\alpha \in \breve{\mathcal{A}}$ and $\breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta)$ is continuous for all $b_\beta \in \breve{\mathcal{M}}$.

Hence $\breve{\mathcal{F}}((a_\alpha, b_\beta)) = \breve{\mathcal{F}}_{\breve{\mathcal{A}}}(a_\alpha) \odot \breve{\mathcal{F}}_{\breve{\mathcal{M}}}(b_\beta)$ is continuous for each $(a_\alpha, b_\beta) \in \breve{\mathcal{A}} \times \breve{\mathcal{M}}$ where $\alpha, \beta \in [0,1]$. Hence $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$ is an FL- space.

The sequence $\{(a_n, \alpha_n)\}$ in an FL- space $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is called fuzzy converges to a fuzzy point $a_\alpha \in \breve{\mathcal{A}}$ if $\lim_{n\to\infty} \breve{\mathcal{F}}((a_n, \alpha_n) - a_\alpha) = 1$[15]. The next proposition discuss the convergence of a sequence in an FL- space $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$.

Proposition 3.4:

Let $\{(a_n, \alpha_n)\}$ be a sequence in an FL- space $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ converges to $a_\alpha \in \breve{\mathcal{A}}$ and $\{(b_n, \beta_n)\}$ is a sequence in an FL- space $(\breve{\mathcal{M}}, \breve{\mathcal{F}}_{\breve{\mathcal{M}}}, \diamond)$ converge to $b_\beta \in \breve{\mathcal{M}}$. If the continuous triangular norm $\odot$ is stronger than $\diamond$ then $\{((a_n, \alpha_n), (b_n, \beta_n))\}$ is a sequence in $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$ converge to (a_α, b_β) in $\breve{\mathcal{A}} \times \breve{\mathcal{M}}$, where $\breve{\mathcal{F}} = \breve{\mathcal{F}}_{\breve{\mathcal{A}}} \odot \breve{\mathcal{F}}_{\breve{\mathcal{M}}}$.

Proof:

From the previous Theorem 3.3, $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$ is FL- space. Now for each $(a_\alpha, b_\beta), (u_\gamma, v_\delta) \in \breve{\mathcal{A}} \times \breve{\mathcal{M}}$, $\lim_{n\to\infty} \breve{\mathcal{F}}[((a_n, \alpha_n), (b_n, \beta_n)) - (a_\alpha, b_\beta)] = \lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{A}}}[(a_n, \alpha_n) - a_\alpha] \odot \lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{M}}}[(b_n, \beta_n) - b_\beta] = 1$

Hence $\{((a_n, \alpha_n), (b_n, \beta_n))\}$ converge to (a_α, b_β).

Proposition 3.5:

Let $\{(a_n, \alpha_n)\}$ be a fuzzy Cauchy sequence in an FL- space $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ and $\{(b_n, \beta_n)\}$ is a fuzzy

Cauchy sequence in an FL- space $(\breve{\mathcal{M}}, \breve{\mathcal{F}}_{\breve{\mathcal{M}}}, \diamond)$. If $\odot$ stronger continuous triangular norm than $\diamond$ then $\{((a_n, \alpha_n), (b_n, \beta_n))\}$ is fuzzy Cauchy sequence in an FL- space $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$ where $\breve{\mathcal{F}} = \breve{\mathcal{F}}_{\breve{\mathcal{A}}} \odot \breve{\mathcal{F}}_{\breve{\mathcal{M}}}$.

Proof:

From the previous Theorem 3.3, $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$ is an FL- space. Now for each $(a_\alpha, b_\beta) \in \breve{\mathcal{A}} \times \breve{\mathcal{M}}$,

$\lim_{n\to\infty} \breve{\mathcal{F}}[((a_n, \alpha_n), (b_n, \beta_n)) - ((a_m, \alpha_m), (b_m, \beta_m))] = \lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{A}}}[(a_n, \alpha_n) - (a_m, \alpha_m)] \odot \lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{M}}}[(b_n, \beta_n) - (b_m, \beta_m)]$

Since $\{(a_n, \alpha_n)\}$ be a fuzzy Cauchy sequence in $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ then $\lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{A}}}[(a_n, \alpha_n) - (a_m, \alpha_m)] = 1$. Also, since $\{(b_n, \beta_n)\}$ is a fuzzy Cauchy sequence in $(\breve{\mathcal{M}}, \breve{\mathcal{F}}_{\breve{\mathcal{M}}}, \diamond)$, then $\lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{M}}}[(b_n, \beta_n) - (b_m, \beta_m)] = 1$.

We get $\lim_{n\to\infty} \breve{\mathcal{F}}[((a_n, \alpha_n), (b_n, \beta_n)) - ((a_m, \alpha_m), (b_m, \beta_m))] = 1$

Hence $\{((a_n, \alpha_n), (b_n, \beta_n))\}$ is fuzzy Cauchy sequence in an FL- space $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$.

It is well known that in an FL- space $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ every fuzzy convergent sequence is fuzzy Cauchy[15]. Besides**,** FL- space $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ is called fuzzy complete if each fuzzy Cauchy sequence in $\breve{\mathcal{A}}$ fuzzy converges to a fuzzy limit point in $\breve{\mathcal{A}}$. The product of two complete FL- space spaces is proved to be complete FL- space in the following theorem.

Theorem 3.6 :

Let $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ and $(\breve{\mathcal{M}}, \breve{\mathcal{F}}_{\breve{\mathcal{M}}}, \diamond)$ are two complete FL- spaces. If $\odot$ is a continuous triangular norm stronger than the continuous triangular norm $\diamond$ then $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$ is a complete FL- space.

Proof:

$(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$ is an FL- space by Theorem 3.3. Let $\{((a_n, \alpha_n), (b_n, \beta_n))\}$ be a fuzzy Cauchy sequence in $\breve{\mathcal{A}} \times \breve{\mathcal{M}}$, that is for each $(a_\alpha, b_\beta) \in \breve{\mathcal{A}} \times \breve{\mathcal{M}}$,

$\lim_{n\to\infty} \breve{\mathcal{F}}[((a_n, \alpha_n), (b_n, \beta_n)) - ((a_m, \alpha_m), (b_m, \beta_m))] = \lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{A}}}[(a_n, \alpha_n) - (a_m, \alpha_m)] \odot \lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{M}}}[(b_n, \beta_n) - (b_m, \beta_m)]$

Hence $\lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{A}}}[(a_n, \alpha_n) - (a_m, \alpha_m)] = 1$ and $\lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{M}}}[(b_n, \beta_n) - (b_m, \beta_m)] = 1$. Therefore $\{(a_n, \alpha_n)\}$ is a fuzzy Cauchy sequence in $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ and $\{(b_n, \beta_n)\}$ is a fuzzy Cauchy sequence in $(\breve{\mathcal{M}}, \breve{\mathcal{F}}_{\breve{\mathcal{M}}}, \diamond)$. But $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ and $(\breve{\mathcal{M}}, \breve{\mathcal{F}}_{\breve{\mathcal{M}}}, \diamond)$ are complete FL- space, then there is a fuzzy point $a_\alpha \in \breve{\mathcal{A}}$ and $b_\beta \in \breve{\mathcal{M}}$ such that $\lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{A}}}[(a_n, \alpha_n) - a_\alpha] = 1$ and $\lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{M}}}[(b_n, \beta_n) - b_\beta] = 1$. Hence

$\lim_{n\to\infty} \breve{\mathcal{F}}[((a_n, \alpha_n), (b_n, \beta_n)) - (a_\alpha, b_\beta)] = \lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{A}}}[(a_n, \alpha_n) - a_\alpha] \odot \lim_{n\to\infty} \breve{\mathcal{F}}_{\breve{\mathcal{M}}}[(b_n, \beta_n) - b_\beta] = 1$.

Thus $\{((a_n, \alpha_n), (b_n, \beta_n))\}$ fuzzy converges to (a_α, b_β) in $\breve{\mathcal{A}} \times \breve{\mathcal{M}}$. This shows $(\breve{\mathcal{A}} \times \breve{\mathcal{M}}, \breve{\mathcal{F}}, \odot)$ is a complete an FL- space.

LOCALLY AND SEQUENTIALLY COMPACT FUZZY LENGTH SPACE

This section is devoted to presenting the concept of the sequentially, countably, and locally fuzzy compact fuzzy length space and clarifying the relationship between these concepts. First, we introduce the concept of sequentially fuzzy compact fuzzy length space as follows.

Definition 4.1:

An FL- space $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ is called sequentially fuzzy compact FL- space if each sequence in $\breve{\mathcal{A}}$ has a convergent subsequence.

Theorem 4.2:

Let $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ be an FL- space. Then the following statements are equivalent:

(1) $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ is a sequentially fuzzy compact FL- space.

(2) Every infinite subset has a fuzzy limit point.

Proof:

Let $\tilde{\mathcal{C}}$ be an infinite subset of $\breve{\mathcal{A}}$ and let $\{(a_n, \alpha_n)\}$ be a sequence in $\tilde{\mathcal{C}}$. Since $(\breve{\mathcal{A}}, \breve{\mathcal{F}}_{\breve{\mathcal{A}}}, \diamond)$ is sequentially fuzzy compact the sequence contains a convergent subsequence and the limit is a fuzzy limit point of $\tilde{\mathcal{C}}$.

Now consider $\{(a_n, \alpha_n)\}$ be a sequence in $\breve{\mathcal{A}}$. If the set $\{(a_1, \alpha_1), (a_2, \alpha_2), \ldots,\}$ is finite, then one of the fuzzy points, say $(a_{i_\circ}, \alpha_{i_\circ})$, satisfies $(a_{i_\circ}, \alpha_{i_\circ}) = (a_j, \alpha_j)$, $j \in N$. Hence the sequence $\{(a_{i_\circ}, \alpha_{i_\circ})\}(s_{i_\circ})$ is a subsequence of $\{(a_n, \alpha_n)\}$ fuzzy converges to $(a_{i_\circ}, \alpha_{i_\circ})$. Suppose that the set $\{(a_1, \alpha_1), (a_2, \alpha_2), \ldots,\}$ is infinite. Then by

ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 31, Issue 4, 2020 DOI: http://doi.org/10.23851/mjs.v31i4.913

assumption, it has at least one fuzzy limit point $a_\alpha \in \check{\mathcal{A}}$. Let $n_1 \in N$ with $\check{\mathcal{F}}_{\check{\mathcal{A}}}[(a_{n_1}, \alpha_{n_1}) - a_\alpha] > 0$. Let $n_{k+1} \in N$ with $n_{k+1} > n_k$ and $\check{\mathcal{F}}_{\check{\mathcal{A}}}((a_{n_{k+1}}, \alpha_{n_{k+1}}) - a_\alpha) > 1 - \frac{1}{k+1}$. Then the sequence $\{(a_{n_k}, \alpha_{n_k})\}$ fuzzy converges to a_α.

In an FL- space $(\check{\mathcal{A}}, \check{\mathcal{F}}_{\check{\mathcal{A}}}, \diamond)$ a subset $\tilde{\mathcal{C}}$ is said to be totally fuzzy bounded (or fuzzy pre-compact) if for each $0 < \epsilon < 1$, there exist fuzzy points $\omega_1, \omega_2, \omega_3, \dots, \omega_n$ such that the fuzzy set $\{(b_1, \beta_1), (b_2, \beta_2), \dots, (b_n, \beta_n)\} \subset \tilde{\mathcal{C}}$ whenever a_α in $\check{\mathcal{A}}$, $\check{\mathcal{F}}(a_\alpha - (b_i, \beta_i)) > (1 - \epsilon)$ for some $(b_i, \beta_i) \in \{(b_1, \beta_1), (b_2, \beta_2), \dots, (b_n, \beta_n)\}$. This fuzzy set of fuzzy points $\{\{(b_1, \beta_1), (b_2, \beta_2), \dots, (b_n, \beta_n)\}$ is called the ϵ-fuzzy net[15]. The following proposition shows the equivalent relationship between fuzzy compact and sequentially fuzzy compact FL- space.

Proposition 4.3:
Let $(\check{\mathcal{A}}, \check{\mathcal{F}}_{\check{\mathcal{A}}}, \diamond)$ be an FL- space. Then the following statements are equivalent:
(1)$(\check{\mathcal{A}}, \check{\mathcal{F}}_{\check{\mathcal{A}}}, \diamond)$ is a fuzzy compact.
(2) $(\check{\mathcal{A}}, \check{\mathcal{F}}_{\check{\mathcal{A}}}, \diamond)$ is sequentially fuzzy compact
Proof:
Let $\check{\mathcal{A}}$ be fuzzy compact then by Theorem 2.2.13[15] $\check{\mathcal{A}}$, is fuzzy totally bounded and complete. Suppose that $\{(a_n, \alpha_n)\}$ be any sequence in $\check{\mathcal{A}}$. Since $\check{\mathcal{A}}$ is totally fuzzy bounded using Theorem 2.2.9[15], we have the sequence $\{(a_n, \alpha_n)\}$ contains a fuzzy Cauchy subsequence say $\{(a_{n_k}, \alpha_{n_k})\}$. So $\{(a_{n_k}, \alpha_{n_k})\}$ fuzzy converges to $a_\alpha \in \check{\mathcal{A}}$ since $\check{\mathcal{A}}$ is fuzzy complete. Hence every sequence $\{(a_n, \alpha_n)\}$in $\check{\mathcal{A}}$ contains a fuzzy convergent subsequence $\{(a_{n_k}, \alpha_{n_k})\}$.

Now suppose that $\check{\mathcal{A}}$ sequentially fuzzy compact, that means every sequence $\{(a_n, \alpha_n)\}$ in $\check{\mathcal{A}}$ has a fuzzy convergent subsequence $\{(a_{n_k}, \alpha_{n_k})\}$. Now by using Theorem 2.2.9[15], we have $\check{\mathcal{A}}$ is totally fuzzy bounded. It remains to show that $\check{\mathcal{A}}$ is fuzzy complete. let $\{(a_n, \alpha_n)\}$ be a fuzzy Cauchy sequence in $\check{\mathcal{A}}$. By assumption $\{(a_n, \alpha_n)\}$ has a subsequence $\{(a_{n_k}, \alpha_{n_k})\}$ that fuzzy converges to a fuzzy point $a_\alpha \in \check{\mathcal{A}}$. We shall show that $\{(a_n, \alpha_n)\}$ fuzzy converges to a_α. Let $0 < \gamma < 1$ be given by Remark (2.2), there is $0 < \mu < 1$ such that $(1-\mu) \circledcirc (1-\mu) > (1-\gamma)$. Now $\{(a_{n_k}, \alpha_{n_k})\}$ fuzzy converges to a_α, that is, there exist N1 such that $\check{\mathcal{F}}_{\check{\mathcal{A}}}\left((a_{n_k}, \alpha_{n_k}) - a_\alpha\right) > (1-\mu)$ for all $n_k \geq N1$. Since the sequence $\{(a_n, \alpha_n)\}$ is fuzzy Cauchy, there exist N2 such that $\check{\mathcal{F}}_{\check{\mathcal{A}}}\left((a_n, \alpha_n) - (a_m, \alpha_m)\right) > (1-\mu)$ for all $m, n \geq N2$. Let $N = \min\{N1, N2\}$ then $\check{\mathcal{F}}_{\check{\mathcal{A}}}[(a_n, \alpha_n) - a_\alpha] \geq \check{\mathcal{F}}_{\check{\mathcal{A}}}\left((a_n, \alpha_n) - (a_{n_k}, \alpha_{n_k})\right) \diamond \check{\mathcal{F}}_{\check{\mathcal{A}}}\left((a_{n_k}, \alpha_{n_k}) - a_\alpha\right) > (1-\mu) \diamond (1-\mu) > (1-\gamma)$
For all $n \geq N$. This completes the proof.
Now in the following definition, we introduce the concept of countably fuzzy compact FL- space.

Definition 4.4:
An FL- space $(\check{\mathcal{A}}, \check{\mathcal{F}}, \diamond)$ is said to be countably fuzzy compact if every open countable cover has a finite subcover.
In the following theorem, a fuzzy compact FL-space $(\check{\mathcal{A}}, \check{\mathcal{F}}, \diamond)$ is proved to be a countably fuzzy compact.

Theorem 4.5:
A fuzzy compact FL- space $(\check{\mathcal{A}}, \check{\mathcal{F}}, \diamond)$ is countably fuzzy compact.
Proof:
Let $\widehat{G}$ be a countably fuzzy open fuzzy cover for $\check{\mathcal{A}}$, and since $\check{\mathcal{A}}$, is fuzzy compact, then $\widehat{G}$ has finite subcover which covering $\check{\mathcal{A}}$. Hence $(\check{\mathcal{A}}, \check{\mathcal{F}}, \diamond)$ is countably fuzzy compact FL- space.

Theorem 4.6:
Suppose that $(\check{\mathcal{A}}, \check{\mathcal{F}}, \diamond)$ be an FL- space. If $(\check{\mathcal{A}}, \check{\mathcal{F}}, \diamond)$ is countably fuzzy compact, then $(\check{\mathcal{A}}, \check{\mathcal{F}}, \diamond)$ is fuzzy totally bounded.
Proof:
Consider $\widehat{G}_i = \breve{\mathcal{B}}((a_i, \alpha_i), \epsilon)$ be a countable fuzzy open fuzzy cover for $\check{\mathcal{A}}$, with center(a_i, α_i) and radius ϵ. Since $\check{\mathcal{A}}$ is countably fuzzy compact, then there exist a finite subcover of $\check{\mathcal{A}}$, that is means for each $a_\alpha \in \check{\mathcal{A}}, a_\alpha \in \bigcup_{i=1}^{n} \widehat{G}_i$, this means for each $a_\alpha \in \check{\mathcal{A}}$ there is $j = \{1, 2, 3, \dots, n\}$ such that $a_\alpha \in \bigcup_{i=1}^{n} \widehat{G}_j = \breve{\mathcal{B}}((a_j, \alpha_j), \epsilon)$. Then the set $\{(a_1, \alpha_1), (a_2, \alpha_2), \dots, (a_n, \alpha_n)\}$ is form an ϵ-fuzzy net for $\check{\mathcal{A}}$, i.e., $(\check{\mathcal{A}}, \check{\mathcal{F}}, \diamond)$ is fuzzy totally bounded.

Theorem 4.7:
Suppose that $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ be a sequentially fuzzy compact FL- space. Then $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is a countably fuzzy compact.
Proof:
Since $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is sequentially fuzzy compact FL- space, and by using Proposition 4.3 and Theorem 4.5, then $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is a countably fuzzy compact FL- space.

In an FL- space $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$, a fuzzy set $\tilde{\mathcal{C}}$ is a fuzzy neighborhood of a fuzzy point a_α if there exists an fuzzy open fuzzy ball $\breve{\mathcal{B}}(a_\alpha, \epsilon)$ with centre $a_\alpha \in \breve{\mathcal{A}}$ and radius ϵ, such that $\breve{\mathcal{B}}(a_\alpha, \epsilon) = \{b_\beta \in \breve{\mathcal{A}}: \breve{\mathcal{F}}(b_\beta - a_\alpha) > 1 - \epsilon\}$ is contained in $\tilde{\mathcal{C}}$.

Definition 4.8:
A FL- space $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is called locally fuzzy compact FL- space if each $a_\alpha \in \breve{\mathcal{A}}$ has a compact fuzzy neighborhood.

Theorem 4.9:
Every fuzzy compact FL- space $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is a locally fuzzy compact FL- space**.**
Proof:
Suppose that $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ be a fuzzy compact FL- space. Then by Definition 2.8 every fuzzy open fuzzy cover $\widehat{G}_i$ has finite subcover, that is mean $\mathcal{S} \subseteq \cup_{i=1}^{n} \widehat{G}_i$. Now for each $a_\alpha \in \breve{\mathcal{A}}$ implies $\breve{\mathcal{B}}(a_\alpha, \epsilon) \subseteq \cup_{i=1}^{n} \widehat{G}_i$, that is mean each $a_\alpha \in \breve{\mathcal{A}}$ has a fuzzy compact $\breve{\mathcal{B}}(a_\alpha, \epsilon)$. Hence $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is a locally fuzzy compact FL- space.

Proposition 4.10:
If $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is sequentially fuzzy compact FL- space, then $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is a locally fuzzy compact FL- space**.**
Proof:
Since $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is sequentially fuzzy compact FL- space, then by Proposition 4.3 $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is fuzzy compact and by Theorem 4.9 we obtain $(\breve{\mathcal{A}}, \breve{\mathcal{F}}, \diamond)$ is a locally fuzzy compact FL- space.

CONCLUSIONS
The concept of the Cartesian product of two fuzzy length spaces is presented. It was proved that the Cartesian product of two complete fuzzy normed spaces is complete fuzzy normed space. Also, we have introduced the concept of the sequentially, countably, and locally fuzzy compact fuzzy length space with a discussion and study of the basic theorems of these concepts.

REFERENCES

[1] L.Zadeh, Fuzzy sets, Inf. Control, (1965), p. 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

[2] A.Katsaras, Fuzzy topological vector spaces II, Fuzzy Sets and Systems, vol. 12,(1984),p. 143-154. https://doi.org/10.1016/0165-0114(84)90034-4

[3] C. Felbin, Finite-dimensional fuzzy normed linear spaces, Fuzzy Sets and Systems, vol. 48, (1992),p.239-248. https://doi.org/10.1016/0165-0114(92)90338-5

[4] X. Jianzhong and Z.Xinghua, Fuzzy normed space of operators and its completeness, Fuzzy Sets and Systems, (2003), P.389-399. https://doi.org/10.1016/S0165-0114(02)00274-9

[5] X. Jianzhong and Z.Xinghua, On linearly topological structure and property of fuzzy normed linear space, Fuzzy Sets and Systems, (2002),p.153–161. https://doi.org/10.1016/S0165-0114(00)00136-6

[6] I.Sadeqi and F. Kia, Fuzzy normed linear space and its topological structure, Chaos Solitions and Fractals, vol.5, (2009), p.2576-2589. https://doi.org/10.1016/j.chaos.2007.10.051

[7] D.Bayaz, D.Fataneh and R.Asghar, Some Properties of Fuzzy Normed Linear Spaces and Fuzzy Riesz Bases, MATHEMATICA, No2, (2019),p.111–128. https://doi.org/10.24193/mathcluj.2019.2.02

[8] R. Kočinac, V. Khan, K. Alshlool and H. Altaf, On some topological properties of intuitionistic 2-fuzzy n-normed linear spaces, Hacet. J. Math. Stat, vol. 49, (2020),p.208–220. https://doi.org/10.15672/hujms.546973

[9] R. KoIinac, Some Topological Properties of Fuzzy Antinormed Linear Spaces, Hindawi Journal of Mathematics, (2018), Article ID 9758415, p.6. https://doi.org/10.1155/2018/9758415

[10] T.Phurba and B.Tarapada, Some fixed point results in fuzzy cone normed linear space, Tamang and Bag Journal of the Egyptian Mathematical Society, (2019),p.27-46. https://doi.org/10.1186/s42787-019-0045-6

[11] V. Govindana and S. Murthyb ,Solution And Hyers-Ulam Stability Of n-Dimensional Non-Quadratic Functional Equation In Fuzzy Normed Space Using Direct Method, Fuzzy Sets and Systems,vol. 16,(2019),p. 384–391. https://doi.org/10.1016/j.matpr.2019.05.105

ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 31, Issue 4, 2020 DOI: http://doi.org/10.23851/mjs.v31i4.913

[12] A.George and P.Veeramani, On Some Results in Fuzzy Metric Spaces, Fuzzy Sets and Systems, vol. 64,(1994), p. 395-399. https://doi.org/10.1016/0165-0114(94)90162-7

[13] G.Gebru and B. Krishna Reddy, Fuzzy Set Field and Fuzzy Metric, Hindawi Publishing Corporation, Advances in Fuzzy Systems,(2014), p.1-8. https://doi.org/10.1155/2014/968405

[14] R.Jehad and I.Jaafar, The Fuzzy Length of Fuzzy Bounded Operator, Iraqi Journal of Science, vol. 58,(2017), p. 284-291. https://doi.org/10.24996/ijs.2017.58.3A.11

[15] I.Jaafar, Properties of Fuzzy Length on Fuzzy Set, M.Sc. Thesis, University of Technology, Iraq,2016.