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Weight Distribution of Some Codewords of 3-ary Linear Code over GF(27)

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ABSTRACT

This paper is devoted to introduce the structure of the p-ary linear codes C(n,q) of points and lines of PG(n,q),q = p^h, when p=3. The linear code C(2,27) is given with its generator matrix. Also, some of weight distributions are calculated. Finally, the generator matrix to the dual code of C(2,27) has been founded.

KEYWORDS: Finite Projective Space, Incidence Matrix, Linear Code, Weight Distribution.

الخلاصة

هذا البحث مكرس لتقديم بنية ال p-ary للترميزات الخطية C(n,q) من نقاط وخطوط ال PG(n,q) حيث $q=p^h$ عندما p=3. تم إعطاء الترميز الخطي C (2,27) مع المصفوفة المولدة الخاصة به ، وتم أيضًا حساب بعض توزيع الوزن لها. اخيرا, المصفوفة المولدة للترميز المزدوج لي C(2,27) قد تم ايجاده.

INTRODUCTION

Let $GF(q), q = p^h$, p prime, $h \geq 1$ be Galois field, and let F_q^n be the n-dimensional vector space over $GF(q)$.

A *linear code* of length n and dimension k over $GF(q)$ is a k-dimensional subspace C of F_q^n and denoted by $C[n,k]$. For any two elements of F_q^n, $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_n)$, the *Hamming distance* $d(x,y)$ between them is defined by $d(x,y) = |\{i : 1 \leq i \leq n, x_i \neq y_i\}|$. For $x \in F_q^n$, the *Hamming weight* $w(x)$ is defined as number of non-zero coordinate of x; that is, $w(x) = d(x,0)$. If the minimum weight of nonzero elements in $C[n,k]$ is d denoted by $d(C)$, then the linear code C denoted by $C[n,k,d]$. The elements of $C[n,k,d]$ are called *codewords*. A $k \times n$ matrix G, whose rows form a linearly independent basis for $C[n,k]$ is called a *generator matrix* for C. If $C[n,k]$ has minimum distance d, then it can detect $d-1$ errors and correct $e = \lfloor (d-1)/2 \rfloor$ errors, where $\lfloor m \rfloor$ denotes the integer part of m. The *support* of a codeword x, denoted by $supp(x)$, is the set of all non-zero positions of x. Here, A_i denotes the number of codewords with weight i, $0 \leq i \leq n$. The *weight distribution* of $C[n,k]$ is defined as the sequence $A_0, A_1, \dots, A_n$. Clearly, the weight distribution can give the minimum distance of the code. For details and properties of linear codes, see [1-4].

The *n-dimensional projective space over the finite field* $GF(q)$, denoted by $PG(n,q)$, is the set consisting of the equivalence classes $[X]$ of non-zero vectors X of the $(n+1)$-dimensional vector space F_q^{n+1}; $[X] = \{Y : Y = tX \text{ for some } t \in GF(q) - 0\}$. The elements of $PG(n,q)$ are called *points* and the point denoted $P(X)$ is the equivalence class of the vector X. The number of points in $PG(n,q)$ is $\theta(n,q) = \frac{q^{n+1}-1}{q-1}$. For furthers about finite projective geometry, see [5].

Definition 1.1 [1,2]**:** The incidence matrix $IM^* = (a_{ij})$ of points and k-dimensional projective subspaces in the projective space $PG(n,q), q = p^h, p$ prime, $h \geq 1$, is defined as the matrix whose rows are indexed by the k-

spaces of $PG(n,q)$ and whose columns are indexed by the points of $PG(n,q)$, and with entry

$$a_{ij} = \begin{cases} 0 & \text{if point } j \text{ belongs to } k-\text{space } i, \\ 1 & \text{otherwise.} \end{cases}$$

Clearly, dimension of IM^* is $\theta(n,q) \times \theta(n,q)$.

Definition 1.2 [1,3]**:** The p-ary linear code of points and k-dimensional projective subspaces of $PG(n,q), q = p^h$, p prime, $h \geq 1$, $1 \leq k \leq n-1$, is the code generated by the rows of the incidence matrix IM^* and is denoted by $C_k = C_k(n,q)$. In the particular case that $k=1$ and $n=2$, we denote the p-ary linear code of points and lines of a projective plane $PG(2,q)$, by $C(2,q)$.

The minimum weight of $C(2,q)$ is $q+1$ which proved in [8] by giving the general case for that. Therefore, $e = \left\lfloor \frac{d-1}{2} \right\rfloor = \left\lfloor \frac{q+1-1}{2} \right\rfloor = \left\lfloor \frac{q}{2} \right\rfloor$. Clearly from Definition 1.1 that length of $C(2,q)$ is $\theta(2,q)$.

The most important property of the rows of incidence matrix is that each i-th row is just circulate to the $(i-1)$-th row except the last row.

In 1960s, *E Prange* and *L D Rudolph* perceived that projective planes could be used to construct error-correcting codes by using the incidence matrix. In the 1970's, the code of points and t-dimensional projective spaces was studied, but apart from the determination of the codewords of minimum weight and weight distributions, not much was known. Recently, calculating weight distributions of linear codes become an important research topic in coding theory since a few weights can be applied to secret sharing, association schemes, combinatorial designs, authentication codes and strongly regular graphs, see [6,7]. Some researchers spend their effort to calculate weight distributions and studied linear codes with a few weight, for example see [8-11].

In this paper, the most details of the 3-ary linear code of points and lines of a projective plane $PG(2,27)$, $C(2,27)$. Because of the large number of codewords of $C(2,27)$ which is 3^{217}, the weight distributions have been not computed completely. The number of codewords, κ, that entered to account of weight distribution are 13723544; that is, $3^{14} \leq \kappa \leq 3^{15}$. The calculations are done with the mathematical programming language GAP [12].

The Linear Code $C(2,27)$

The incidence matrix $IM^* = (a_{ij})$ of points and lines in the projective plane $PG(2,27)$, is a matrix of dimension 757×757, where $\theta(2,27) = 757$. The matrix IM^* is written by filling each row by identify with the corresponding support of the certain row so, each row of the matrix IM^*, r_i, are written identifying to the $support(r_i)$ as below.

$$IM^* = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_{756} \\ r_{757} \end{pmatrix}, \text{where}$$

r_1 =1, 2, 4, 10, 28, 82, 149, 168, 212, 244,309, 356, 386, 399, 438, 445,461,502, 506, 555, 576, 624, 634, 659, 674, 725, 730, 747;

r_2 =2, 3, 5, 11, 29, 83, 150, 169, 213, 245, 310, 357,387, 400, 439, 446, 462,503, 507, 556, 577,625, 635, 660, 675, 726, 731, 748;

$\vdots$

r_{756} =2, 8, 26, 80,147, 166,210, 242, 307, 354, 384, 397, 436, 443,459,500,504,553,574, 622, 632, 657, 672, 723, 728, 745, 756, 757;

r_{757} =1, 3, 9, 27, 81, 148, 167, 211, 243, 308, 355, 385, 398, 437, 444, 460,501, 505, 554, 575, 623, 633, 658, 673, 724, 729, 746, 757.

The number of linearly independent rows of IM^* which generated $C(2,27)$ is 217; that is, the generator matrix of $C(2,27)$, let denoted it by Ψ, is of dimension 217×757. This matrix was computed by executed an algorithm on GAP mathematical language program. The details of the matrix Ψ are written below. Here, n_{r_i} denote the order of the row r_i and $=_s$ denote the size of $support(r_i)$.

Table 1. Details of the generator matrix Ψ of $C(2,27)$

n_{r_i}	$=_s$	n_{r_i}	$=_s$	n_{r_i}	$=_s$
1	28	101	421	201	345
2	28	102	421	202	345
3	28	103	421	203	370
4	28	104	439	204	370
5	28	105	445	205	373
6	28	106	424	206	372
7	28	107	448	207	361
8	28	108	430	208	373
9	28	109	463	209	367
10	28	110	433	210	343
11	28	111	436	211	367
12	187	112	412	212	379
13	187	113	423	213	379
14	225	114	436	214	403
15	145	115	445	215	357
16	235	116	447	216	364

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17	274	117	447	217	376
18	232	118	421		
19	232	119	430		
20	232	120	427		
21	364	121	427		
22	303	122	436		
23	340	123	409		
24	340	124	409		
25	331	125	423		
26	331	126	423		
27	376	127	430		
28	322	128	409		
29	313	129	417		
30	313	130	424		
31	391	131	418		
32	322	132	403		
33	388	133	403		
34	403	134	430		
35	369	135	415		
36	382	136	415		
37	435	137	424		
38	355	138	396		
39	403	139	391		
40	403	140	406		
41	426	141	400		
42	415	142	409		
43	427	143	409		
44	420	144	409		
45	420	145	403		
46	420	146	403		
47	451	147	408		
48	471	148	409		
49	471	149	417		
50	463	150	426		
51	460	151	409		
52	438	152	409		
53	454	153	412		
54	430	154	406		
55	457	155	414		
56	445	156	400		
57	460	157	400		
58	489	158	412		
59	478	159	385		
60	459	160	420		
61	478	161	390		
62	478	162	390		
63	478	163	390		
64	465	164	376		
65	493	165	381		
66	469	166	381		
67	460	167	406		
68	454	168	406		
69	460	169	406		
70	463	170	378		
71	463	171	382		
72	463	172	376		
73	463	173	385		
74	489	174	397		
75	471	175	406		
76	463	176	400		
77	442	177	400		
78	451	178	388		
79	478	179	355		
80	448	180	355		
81	442	181	385		
82	442	182	384		
83	433	183	391		
84	433	184	391		
85	462	185	373		
86	462	186	397		
87	451	187	379		
88	445	188	379		
89	426	189	385		
90	430	190	379		
91	442	191	388		
92	442	192	397		
93	462	193	373		
94	462	194	352		
95	445	195	352		
96	427	196	361		
97	448	197	379		
98	448	198	379		
99	438	199	379		
100	438	200	364		

From Table 1, the following numerical information are deduced and given in Table 3. Here, $n_{=_s}$ denote the number of rows identifying to $=_s$.

Table 2. Numerical information of the generator matrix Ψ of $C(2,27)$

No.	$=_s$	$n_{=_s}$	***No.***	$=_s$	$n_{=_s}$
1	28	11	39	406	6
2	145	1	40	408	1
3	187	2	41	409	9
4	225	1	42	412	3
5	232	3	43	414	1
6	235	1	44	415	3
7	274	1	45	417	2
8	303	1	46	418	1
9	313	2	47	420	4
10	322	2	48	421	4
11	331	2	49	423	3
12	340	2	50	424	3
13	343	1	51	426	3
14	345	2	52	427	4
15	352	2	53	430	6
16	355	3	54	433	3
17	357	1	55	435	1
18	361	2	56	436	3
19	364	3	57	438	3
20	367	2	58	439	1
21	369	1	59	442	5
22	370	2	60	445	5
23	372	1	61	447	2

24	373	4	62	448	4
25	376	4	63	451	3
26	378	1	64	454	2
27	379	8	65	457	1
28	381	2	66	459	1
29	382	2	67	460	4
30	384	1	68	462	4
31	385	4	69	463	7
32	388	3	70	465	1
33	390	3	71	469	1
34	391	4	72	471	3
35	396	1	73	478	5
36	397	3	74	489	2
37	400	5	75	493	1
38	403	8			

The first row r_1 and last row r_{217} of Ψ are given below.

r_1 = [2, 2, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0, 0, 0,
0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0, 0, 0,0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0,0, 0, 0, 0, 0,0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0,
0, 0, 0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0, 0, 0, 0, 0, 0, 0, 0,
0, 0,0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 2, 0, 0, 0, 0, 0, 0, 0,0, 0, 0, 0, 0, 0, 0,0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2,
0, 0, 0, 0, 0,0, 2, 0, 0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0,0, 0, 0, 0,0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0,0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0,
0, 0, 0, 0, 0,0, 0, 0, 0, 0, 0, 0, 0,0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0,0, 0, 0, 2, 0, 0, 0, 0, 0, 0,0, 0, 0, 2, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],

r_{217} =[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0,0,0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0,0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 2, 0,
2,2,2, 2,1, 2, 1, 2, 1, 1, 2, 0, 0, 0, 2, 1, 1, 0, 1, 1, 1,
1, 0, 0, 1, 2, 0, 1, 1, 2, 2, 0, 0, 1, 0, 0, 0, 1, 1,0, 0,
1,1, 2, 1, 1, 1, 0, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
2, 2, 0, 0, 0, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 0, 2, 0, 1,
0, 2, 0, 1, 1, 2, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 2, 0, 0, 2,
2, 2, 1, 0, 0, 0, 0, 1, 0, 2, 0, 1, 0,1, 1, 1, 2, 0, 0, 2,
2, 1, 0, 0, 0, 2, 1, 1, 1, 1, 1, 2, 1, 2, 2, 0, 1, 2, 0, 2,
1, 1, 0, 1, 2, 2, 1, 0, 1, 0, 2, 1, 2, 1, 2,2, 2, 0, 1, 0,
1, 1, 2, 2, 0, 0, 0, 1, 0, 2, 1, 1, 2, 2, 2, 1, 0, 1, 0, 1,
0, 1, 1, 1, 0, 2, 1,1, 0, 0, 1, 1, 2, 2, 2, 2, 0, 2, 2, 0,
1, 0, 1, 1, 0, 0, 0, 0, 0, 2, 1, 2, 1, 0, 1, 0, 2, 1, 0, 0,
0, 0, 2, 0,0, 1, 0, 2, 0, 2, 2,0, 0, 2, 2, 0, 1, 0, 0, 1, 0,
2, 1, 2, 2, 1, 0, 1, 0, 2, 1, 2, 2, 2, 0, 2, 2, 1, 1, 1, 2,
2, 0, 2, 0, 1, 1, 1, 2, 1, 1, 0, 1, 2, 0, 2, 2, 1, 2, 2, 1,
1, 0, 2, 0, 2, 1, 2, 0, 1, 0, 1, 1, 1, 2, 1, 1, 0, 0, 0, 2,
1, 2, 2, 2, 1,2, 1, 0, 2, 0, 0, 0, 0, 0, 2, 2, 0, 2, 1, 2,
2, 0, 1, 1, 0, 1, 1, 0, 2, 1, 1, 0, 2,1, 0, 2, 1, 0, 1, 0,
0, 2, 2,2, 1, 2, 0, 1, 0, 2, 1, 0, 0, 1, 0, 0, 2, 2, 2, 2,
2, 1, 1, 2, 1, 2, , 12, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 2, 0,
0,1, 0, 1, 2, 1, 1, 0, 2, 0, 1, 0, 0, 0, 0, 1, 0, 2, 1, 2,
2, 2, 2, 0, 2, 1, 0,1, 1, 0, 0, 0, 2, 0, 0, 1, 0, 2, 0,1, 0,
1, 1, 1, 2, 0, 0, 1, 0, 2, 0, 2, 1, 2, 1, 0, 1, 0, 1, 1, 2,
1, 0, 0, 0,1, 1, 1, 1, 2, 0, 1, 2, 2, 2, 2, 0,2, 2, 1, 2, 2,
0, 2, 2, 0, 2, 0, 1, 0, 1, 0, 2, 2, 0, 1, 2, 2, 1, 2, 2, 1,
2, 2, 1, 1, 2, 1, 0, 0, 2, 1, 2, 1,1, 1, 2, 0, 2, 1, 2, 0,
2, 0, 1, 2, 2, 1, 1, 0, 2, 0, 2, 1, 2, 1, 1, 2, 2, 1,1, 2,
2, 2, 1, 0, 1, 1, 1, 0, 2, 1, 0, 1, 2].

From the above details, the following theorem is deduced.

Theorem 2.1: The code $C(2,27)$ is $C[757,217,28]$ with $e = 13$ and the weight distributions A_i where $28 \leq i \leq 493$, of some codewords are given in Table 3.

Table 3. Weight distributions of $C(2,27)$

No.	$\boldsymbol{i}$	$\leq \boldsymbol{A_i}$	**No.**	$\boldsymbol{i}$	$\leq \boldsymbol{A_i}$
1	0	1	56	382	572040
2	28	11	57	384	208915
3	145	1	58	385	411688

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4	187	2	59	387	141123
5	225	1	60	388	276142
6	232	3	61	390	88094
7	235	1	62	391	171354
8	274	1	63	393	49373
9	303	1	64	394	97457
10	313	2	65	396	26695
11	315	1	66	397	51446
12	316	6	67	399	13392
13	318	11	68	400	24926
14	319	36	69	402	5978
15	321	34	70	403	11621
16	322	132	71	405	2513
17	324	156	72	406	4687
18	325	277	73	408	912
19	327	571	74	409	1696
20	328	771	75	411	314
21	330	1131	76	412	646
22	331	2326	77	414	88
23	333	2597	78	415	188
24	334	5234	79	417	44
25	336	6286	80	418	46
26	337	11643	81	420	9
27	339	12941	82	421	21
28	340	24909	83	423	2
29	342	25913	84	424	6
30	343	49282	85	426	3
31	345	46016	86	427	4
32	346	90768	87	430	6
33	348	80597	88	433	3
34	349	156121	89	435	1
35	351	129584	90	436	3
36	352	253985	91	438	3
37	354	190027	92	439	1
38	355	377754	93	442	5
39	357	263616	94	445	5
40	358	528767	95	447	2
41	360	342185	96	448	4
42	361	689293	97	451	3
43	363	413522	98	454	2
44	364	835372	99	457	1
45	366	467226	100	459	1
46	367	941145	101	460	4
47	369	492408	102	462	4
48	370	987442	103	463	7
49	372	476411	104	465	1
50	373	963073	105	469	1
51	375	433169	106	471	3
52	376	875170	107	478	5
53	378	366524	108	489	2
54	379	731494	109	493	1
55	381	286103			

A dual code of a $C[n,k,d]$ over F_q, denoted by $C^{\perp}$, is defined by

$$C^{\perp}=\left\{(x_1,\dots,x_n)\in F_q^n:\sum_{i=1}^{n}x_ic_i=0\right\}$$

For all $c=(c_1,\dots,c_n)\in C$.

It is known that $C^{\perp}$ has length n and dimension $n-k$, see [4]. The relation between the generator matrices of code and it dual code is as follows:

$G=[I_kA_{k\times n}]$ is generator matrix of a code $C[n,k]$ iff $H=[-A^TI_{n\times(n-k)}]$ is a generator matrix of $C^{\perp}$, see [4].

Therefore, the following code is founded.

Corollary 2.2: The dual code of $C(2,27)$ is $C[757,540,140]$ with $e=69$.

Proof: The first row s_1 and last row s_{217} of the generator matrix of $C^{\perp}$ are as below:

s_1 =[2, 2, 2, 1, 1, 1, 2, 0, 0, 0, 1, 1, 0, 2, 2, 2, 0, 0, 2, 1, 0, 2, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 2, 2, 0, 0, 1, 0, 1, 1, 2, 1, 2, 0, 2, 2, 0, 2, 1, 2, 0, 2, 2, 2, 2, 1, 2, 1, 1, 1, 0, 1, 2, 1, 0, 0, 1, 0, 2, 1, 2, 0, 2, 0, 2, 0, 0, 0, 1, 2, 2, 1, 2, 0, 2, 0, 0, 2, 1, 2, 2, 0, 2, 1, 0, 2, 0, 2, 1, 2, 1, 0, 2, 2, 0, 1, 0, 0, 0, 2, 1, 2, 0, 1, 0, 1, 0, 0, 0, 1, 1, 2, 0, 2, 0, 0, 0, 1, 1, 1, 1, 0, 1, 2, 0, 1, 2, 1, 1, 1, 2, 1, 2, 2, 0, 1, 1, 2, 2, 1, 0, 0, 1, 0, 1, 2, 0, 1, 1, 0, 2, 0, 1, 0, 0, 0, 1, 2, 1, 2, 0, 1, 2, 2, 1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 2, 2, 0, 0, 2, 0, 0, 2, 2, 1, 1, 2, 2, 2, 2, 2, 0, 0, 1, 0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0,

0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0, 0, 0, 0, 0, 0, 0],
s_{217} =[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2,
2, 2, 1, 1,1, 2, 0, 0, 0, 1, 1, 0, 2, 2, 2, 0, 0, 2, 1, 0,
2, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 2, 2, 0, 0,
1, 0, 1, 1, 2, 1, 2, 0, 2, 2, 0, 2, 1, 2, 0, 2, 2, 2, 2, 1,
2, 1, 1, 1, 0, 1, 2, 1, 0, 0, 1, 0, 2, 1, 2, 0, 2, 0, 2, 0,
0, 0, 1, 2, 2, 1, 2, 0, 2, 0, 0, 2, 1, 2, 2, 0, 2, 1, 0, 2,
0, 2, 1, 2, 1, 0, 2, 2, 0, 1, 0, 0, 0, 2, 1, 2, 0, 1, 0, 1,
0, 0, 0, 1, 1, 2, 0, 2, 0, 0, 0, 1, 1, 1, 1, 0, 1, 2, 0, 1,
2, 1, 1, 1, 2, 1, 2, 2, 0, 1, 1, 2, 2, 1, 0, 0, 1, 0, 1, 2,
0, 1, 1, 0, 2, 0, 1, 0, 0, 0, 1, 2, 1, 2, 0, 1, 2, 2, 1, 1,
0, 0, 1, 0, 1, 1, 1, 1, 2, 2, 0, 0, 2, 0, 0, 2, 2, 1, 1, 2,
2, 2, 2, 2, 0, 0, 1, 0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0,0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0].

CONCLUSIONS

From points and lines of 2-dimensional projective space over the finite field $GF(27)$, the code $C(2,27)$ of length 757, dimension 217 and minimum weight 28 has been founded using incident matrix over $p = 3$. The generator matrix of the code has been given by gave details of its rows using support function. From this code, another code, which is the dual code, has been computed. Also, some of weight distributions of $C(2,27)$ have been computed.

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