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Design and Analysis of Novel Six-Dimensional Hyper Chaotic System

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ABSTRACT

The chaotic system has been widely studied. A new six-dimension hyper chaotic system is introduced in this paper. We used a new chaotic system based on a six-dimension for the purpose of increasing chaos in the system, where the new system has eleven positive parameters, complicated chaotic dynamics behaviors and gives an analysis of the new systems. The basic characteristics and dynamic behavior of this system are investigated with a presence of chaotic attractor, Dissipativity, symmetry, equilibrium points, Lyapunov Exponents, Kaplan-Yorke dimension, waveform analysis and sensitivity toward initial conditions. The results of the analysis exhibit that the new system contains three unstable equilibrium points and the six Lyapunov exponents. Maxim non-negative Lyapunov Exponent (MLE) is obtained as 4.72625, and Kaplan-Yorke are obtained as 3.92566, and the new system characteristics with, unstable, high complexity, and unpredictability, the new system dynamics is simulated utilizing MATHEMATICA program. The phase portraits and the qualitative properties of the new hyper chaotic system have been described at the detail.

KEYWORDS: Chaotic, Hyper, Dynamical system, Unstable, Waveform analysis, Sensitivity.

الخلاصة

تمت دراسة النظام الفوضوي على نطاق واسع. تم تقديم نظام فوضوي جديد من ستة أبعاد في هذه الورقة. استخدمنا نظامًا فوضويًا جديدًا مبنيًا على ستة أبعاد لغرض زيادة الفوضى في النظام، حيث يحتوي النظام الجديد على احدى عشر متغيرًا إيجابيًا وسلوكيات ديناميكية فوضوية معقدة ويقدم تحليلاً للأنظمة الجديدة .يتم التحقيق في الخصائص الأساسية والسلوك الديناميكي لهذا النظام من خلال وجود جاذب فوضوي، وتبديد، وتماثل، ونقاط توازن، و Lyapunov Exponents، وبُعد Kaplan-Yorke، وتحليل شكل الموجة وحساسية تجاه الظروف الأولية. تظهر نتائج التحليل أن النظام الجديد يحتوي على ثلاث نقاط توازن غير مستقرة وستة أسسLyapunov يتم الحصول على مكسيم Lyapunov Exponent (MLE) غير السالب كـ 4.72625 وتم الحصول على Kaplan-Yorke كـ 3.92566، وخصائص النظام الجديد غير المستقرة ، والتعقيد العالي، وعدم القدرة على التنبؤ، يتم محاكاة ديناميكيات النظام الجديد باستخدام برنامج MATHEMATICA تم وصف صور المرحلة والخصائص النوعية للنظام الجديد شديد الفوضى بالتفصيل.

INTRODUCTION

Chaos is a characteristic of a complicated dynamic deterministic system with completely unpredictable behavior without perfect knowledge. In several areas, including sociology, economy, and biology, chaotic dynamic systems are being applied theories. Although disorderly activity in a wide variety of active subjects has become more evident over the past century. However, the theory of chaos was so incredible because chaos observed in practically trivial structures. The Tent Map is an example of a simple-equation system with a highly complex behavior [1]. The processes correspond in chaotic systems to the ergodicity of the mixing effects and the high sensitivity to small variations in initial conditions or control parameters. Therefore, the chaos phenomenon can be a prospective pseudo-random source in data encryption schemes. Like their classical counterparts, chaos-based cryptographic systems classified into two groups: stream ciphers and block ciphers. Because stream encryption schedules are typically higher, they used in real-time applications efficiently. A pseudo-random sequence is a common way of constructing a stream module. Many encryption schemes that use chaos-based pseudo-random number generators (PRNG) have been recently proposed [2].

A chaotic system is a non-linear deterministic dynamic system that reveals pseudo randomness behavior. The Chaotic sequence created in a simple way through the using of different equations [3].

Related Works

In **[2]**, A novel technique used to construct chaotic encryption schemes that are based on chaos with greater space in this study using adaptive symmetry chaotic maps. Using multi-parametric bifurcation analysis, it compares pseudo-random generators based on the conventional Zaslavsky map and the modern adaptive Zaslavsky cloud map and examines the map's parameter spaces. Study specifically shows that the adaptive Zaslavsky map produces random pseudo-random sequences, has a weak correlation and a larger parameter space. The method of increasing chaotic sequence time dependent on symmetry coefficient variability provided. The speed analysis shows that the proposed encryption algorithm is highly encrypted, consistent with the best solutions in a region. The results obtained can improve cryptography based on chaos and motivate further studies with favorable statistics on chaotic maps and synthesis of novel discrete models properties.

In [3] a new algorithm for the image encryption/decryption scheme depended on a novel six-dimensional hyper-chaotic system to achieve a high level of security, the chaotic sequence generated from system employ for permutation and diffusion the original image to create an encrypted image. The performance of the algorithm has been analyzed through analyzes statistical such as Histogram Analysis, Correlation Coefficient Analysis, Information Entropy Analysis, Key Space Analysis, Key Sensitivity Analysis, Number of Pixels Change Rate (NPCR), Unified Average Changing Intensity (UACI), Peak Signal to Noise Ratio, The experimental results show that the algorithm has good encryption performance, large key space equals to and the high sensitivity for small changes in secret key which makes the algorithm immune to Brute force attacks, and it can resist the statistical attacks, therefore, the presented encryption algorithm depends on a novel hyperchaotic system is more secure against the statistical and differential attacks.

In [4], a new and active algorithm for the chaotic scheme centered on the Sudoku matrix, where a new five-dimensional chaotic scheme is created to create the random key into encrypting the color picture. The encryption system includes a scrambled image by using the chaotic sequence generated, then perform an XOR operation between the Sudoku Matrix and scramble, and then perform the scrambling process again, and then perform XOR between a chaotic key and the original image after previous processes. Results showed that the characteristics of the proposed system are highly resistant to the various attacks.

In [5], A new encryption algorithm based on the chaotic multi-scroll method was proposed in this paper. The new multi-scroll method is more complex than other classic discrete systems, making it more suitable for the construction of an encrypted algorithm. For the Arnold cat transition, the connection between the neighboring pixels completely removed. The results of the experiments show that the encrypted image has a substantially higher encryption effect than the other encryption algorithms. This offered data support and theoretical guidance for studying related areas of cryptography such as random key generations and authentication of web apps.

In [6], this article proposes an encryption system based on an improved system of chaos that hides key information. The first thing to do is improve the chaotic CML diagram, and join the chaotic state from zero. The person authentication data separated into key and non-key information; and transmitted through various forms of communication. In order to improve the resistance to a selected plaintext attack, the RSA algorithm used to encrypt the size of key details. Scrambling performed with a sorting method using a CML program. The test results and security review indicate that the new algorithm has a wide key space, is keys and plaint text sensitive, has a strong cryptographic impact, and is resistant to common methods of attack.

Chaotic system types

Chaotic Separated into two categories: Discreet time chaotic and Continuous time chaotic. The map called Tent Chaotic is an example of a discreet category in time, while the Lorenz

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Chaotic Map shows a continuous category of time.

Table 1. illustrates types of chaotic system [7].

continue	Chen Celikovsky system	Hyper-chaotic Chen system	Hyper-Lorenz chaotic	Hyper-Lu chaotic	Hyper-Rössler chaotic	Lorenz system	Rikitake chaotic attractor	Hyperchaotic attractor
discrete	2D Lorenz system	Arnold's cat map	Duffing map	Hénon map	Hyper Logistic map	Tent map	Tinkerbell map	Circle map

Another classification that classifies the chaotic into One-Dimensional and Multi-Dimensional map. An example of the One-Dimensional is Logistic chaotic, while Arnold cat chaotic is an example of the Two-Dimensional.

Chaotic system dynamic analysis

Hyper chaotic Rossller Model the first modern hyperchaotic model. Many hyper systems subsequently advanced and the applications of this type lately been developed. Several other hyperchaotic systems implemented in the bygone era, such as Chen's hyperchaotic system, Lu's hyperchaotic system, Lorenz's hyperchaotic system, Chua's hyperchaotic chain, and Nikolov's hyperchaotic system [8].

The hyper-chaotic has more complicated and intense construction than a chaotic system for the reason that numerous ingrained advantages, the hyper-chaotic system has been widely utilized in numerous domains such as information science, electronics, mathematics, physics, communication as given. The building of different hyper-chaotic and their study are beneficial to reconnoitering the nature of hyper chaos. They are extremely interesting to create different hyper-chaotic with the most hyper-chaotic nature and complicated dynamics [9].

The research has ripened at the trend of styling upper dimensional hyper-chaotic and a current concentration is on designing 5-D systems with best characteristics and dynamics [10,11].

Fundamental features and complicated dynamics of the novel system inspected. For more details, see [10,11]. The fundamental features described briefly as follow:

A. Dissipativity.

B. Symmetry.

C. Equilibria.

D. Lyapunove Exponent's and dimensional.

E. Waveform analysis.

Proposed novel six-dimensional chaotic system

The novel six-dimensional autonomous system is obtained in system (1) as follows:

$$
\begin{aligned}
\frac{dx_1}{dt} &= -a\,x_1 + b\,x_2 - x_5 + x_6\,Sin(x_4) \\
\frac{dx_2}{dt} &= -c\,x_2 + d\,x_1 - e\,x_1\,x_3 - x_1\,Sin(x_5) \\
\frac{dx_3}{dt} &= -f\,x_3 + x_1\,x_2 + x_4\,Sin(x_1) \\
\frac{dx_4}{dt} &= -x_4 - x_2\,x_3 - g\,x_1\,Sin(x_6) \\
\frac{dx_5}{dt} &= -x_5 - h\,x_3 + i\,x_3\,Sin(x_2) \\
\frac{dx_6}{dt} &= -j\,x_6 - k\,x_3x_4 + x_2Sin(x_3)
\end{aligned} \qquad (1)
$$

Where x1, x2, x3, x4, x5, x6 and t ∈ $\Re$ called the states of system and a,b,c,d,e,f,g,h,i,j, and k are positive parameters of the system. The 6-Dimentional System (1) exhibits a chaotic attractor, when the system parameter values are chosen as:

a=10.2,b=12,c=5.1,d=30,e=2.5,f=2,g=5,h=0.5,i=10,j=17,and k=4.

We take the initial conditions as:

x1(0)=0.5, x2(0)=2, x3(0)=1.5, x4(0)=6, x5(0)=0.4 and x6(0)=1. The strange attractors of system (1) in 3-D view are shown in figure 1 to figure 8 and the strange attractors in 2-D view are shown in figure 9 to figure 16.

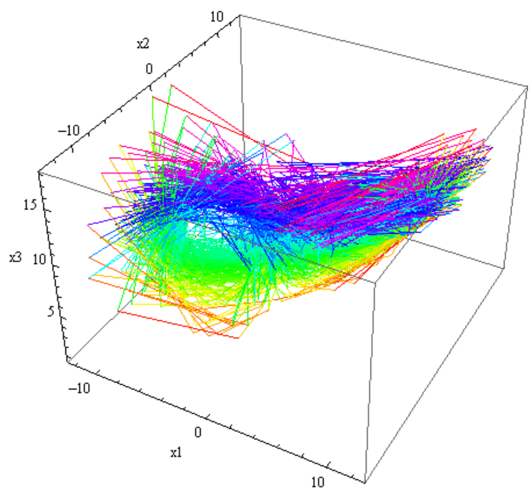

Figure 1. Chaotic attractors, three- dimensional view (x1-x2-x3).

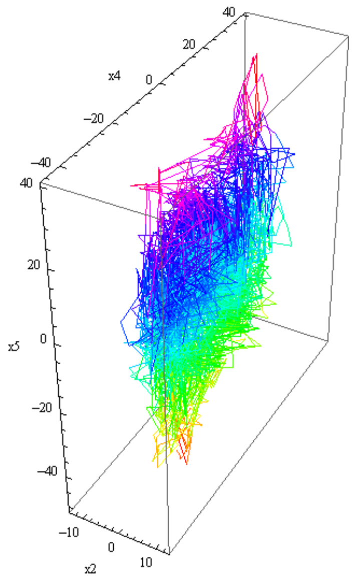

Figure 7. Chaotic attractors, three- dimensional view (x5-x4-x2).

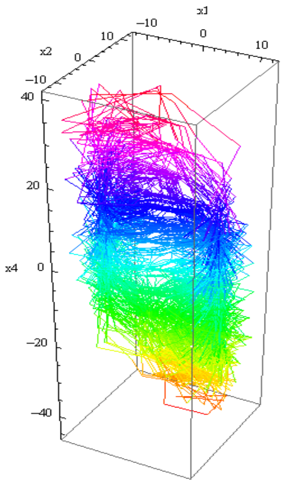

Figure 2. Chaotic attractors, three- dimensional view (x1-x2-x4).

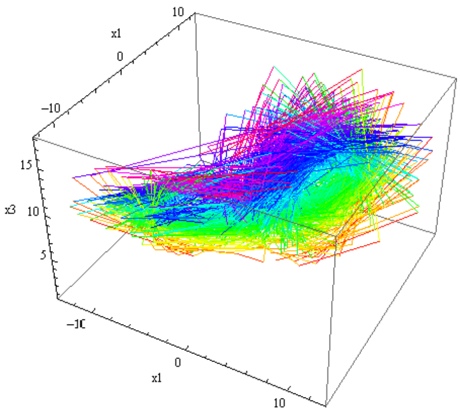

Figure 3. Chaotic attractors, three- dimensional view (x3-x2-x1).

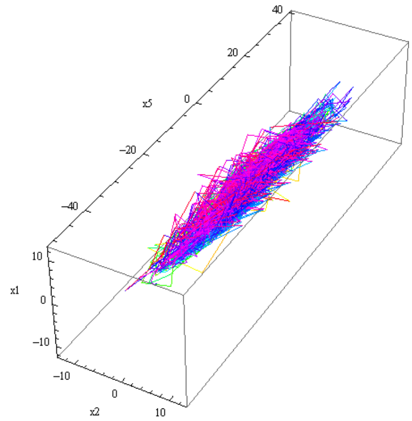

Figure 4. Chaotic attractors, three- dimensional view (x1-x5-x2).

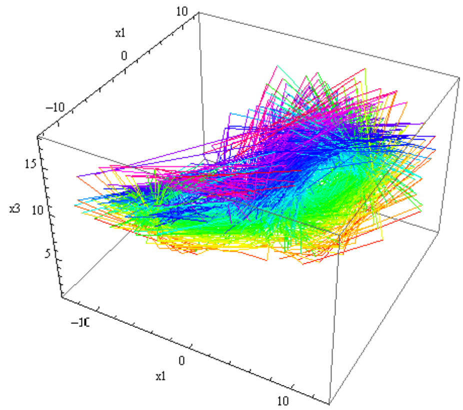

Figure 5. Chaotic attractors, three-dimensional view (x3-x2-x1).

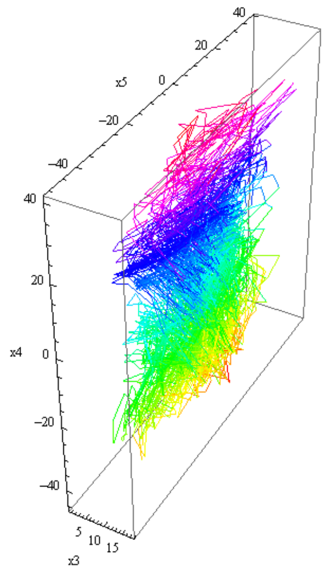

Figure 6. Chaotic attractors, three-dimensional view (x4-x5-x3).

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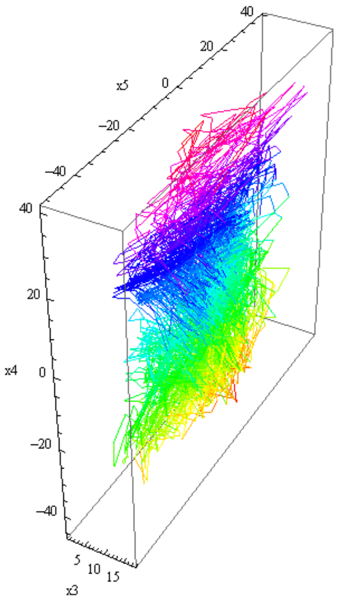

Figure 8. Chaotic attractors, three-dimensional view (x3-x4-x5).

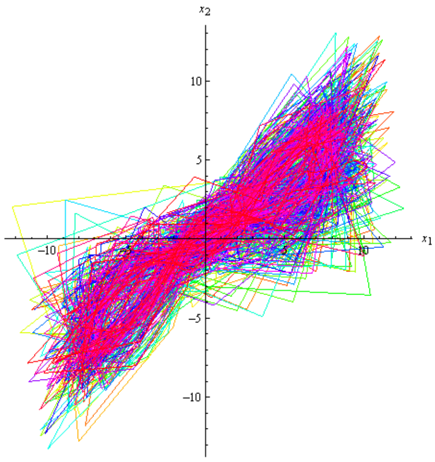

Figure 9. Chaotic attractors, phase plane (x2 –x1).

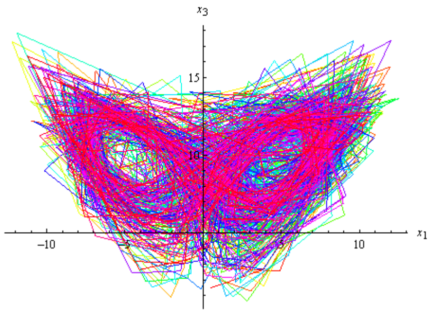

Figure 10. Chaotic attractors, phase plane (x3 –x1).

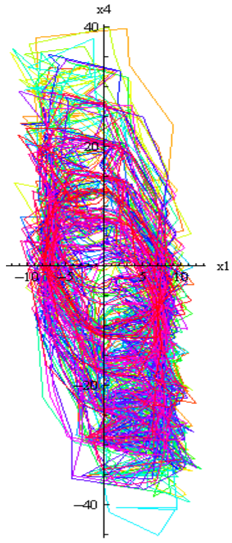

Figure 11. Chaotic attractors, phase plane (x4 –x1).

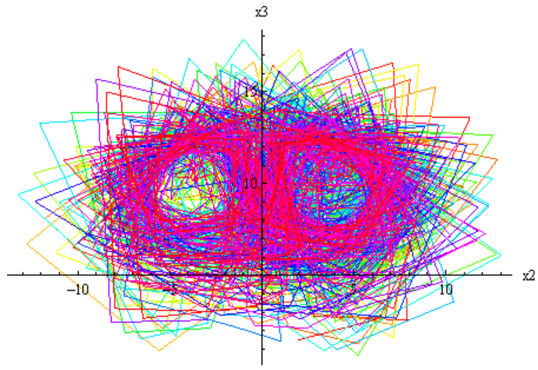

Figure 12. Chaotic attractors, phase plane (x3 –x2).

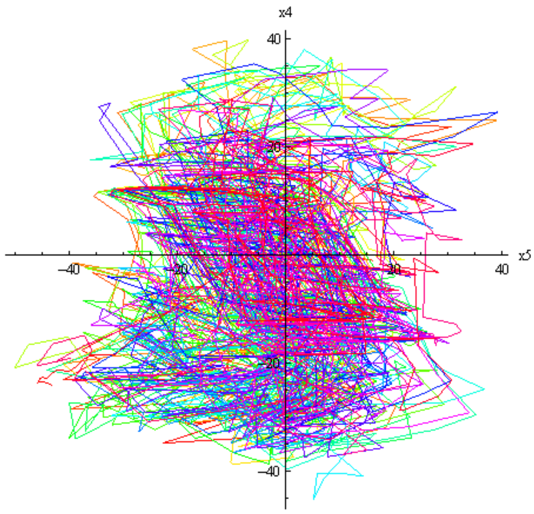

Figure 13. Chaotic attractors, phase plane (x4 –x5).

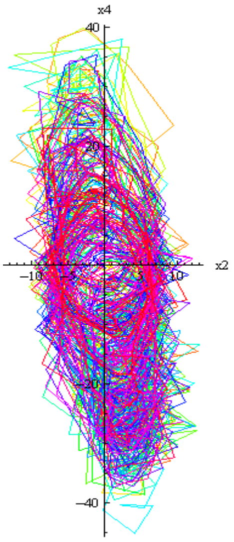

Figure 14. Chaotic attractors, phase plane (x4 –x2).

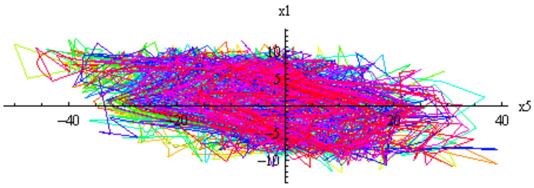

Figure 15. Chaotic attractors, phase plane (x1 –x5).

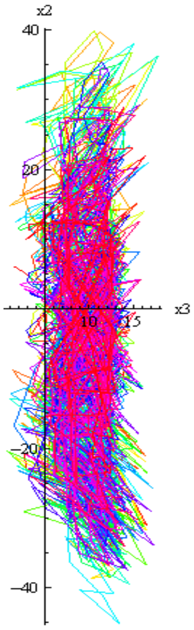

Figure 16. Chaotic attractors, phase plane (x2 –x3).

Dynamic analysis of proposed chaotic system

The essential and complex dynamic behavior of the new system are investigated in this section and it has the following basic characteristics**:**

A. Dissipativity

The system (1) can be expressed in vector notation as:

f = The divergence of the vector field f on R^6 is given by

$$\nabla \cdot f = \frac{\partial f_1}{\partial x1} + \frac{\partial f_2}{\partial x2} + \frac{\partial f_3}{\partial x3} + \frac{\partial f_4}{\partial x4} + \frac{\partial f_5}{\partial x5} + \frac{\partial f_6}{\partial x6} \quad (2)$$

We note that $\nabla \cdot f$ measures the rate at which volumes change under the flow Φ_t of *f*.

Let *D* be a region in R^6 with a smooth boundary and let *D(t)*= $\Phi_t(t)$,the image of *D* under Φ_t ,the time *t* of the flow of *f*. let *V(t)* be the volume of *D(t)*. By Liouville's theorem, we get:

$$\frac{dV}{dt} = \int_{D(t)} (\nabla f)\, dx1dx2dx3dx4dx5dx6 \quad (3)$$

For the system (1), we find that

$$\nabla \cdot f = \frac{\partial f_1}{\partial x1} + \frac{\partial f_2}{\partial x2} + \frac{\partial f_3}{\partial x3} + \frac{\partial f_4}{\partial x4} + \frac{\partial f_5}{\partial x5} + \frac{\partial f_6}{\partial x6} = (4)$$

-a-c-f-2-j<0 because *a*, *c* and *j* are positive constants.

Substituting (3) into (4) and simplifying, we get

$$\frac{dV}{dt} = (-a-c-f-2-j) \int_{D(t)} dx1dx2dx3dx4dx5dx6 \quad (5)$$

$$= (-a-c-f-2-j)\, V(t) = e^{-38.3}\, V(t)$$

Solving the first order linear differential equation, we obtain the unique solution $V(t) = V(0)\, e^{(-a-c-f-2-j)\,t}$

$$= V(0)\, e^{-38.3t} \quad (6)$$

Equation (4.11) shows that any volume V(t) must shrink exponentially fast to zero with time. Thus, the dynamical system described by system (1) is a dissipative system. As in system (1) is a dissipative system, all obits of the system (1) are eventually confined to a specific of R6 that has zero volume. Therefore, system asymptotic motion settles on a system attractor.

B. Symmetry and invariability

When the coordinate (x1,x2,x3,x4,x5,x6) is transformed into (-x1,-x2,x3,-x4,-x5,-x6) , the novel system is invariant and has the symmetry about the x3-axis. In order to demonstrate the conclusion, let

x1= -x1, x2= -x2, x3=x3, x4= -x4, x5=-x5 and x6= -x6. **(7)**

According to Eq. (4), the result obtained as follows:

It is easy to see that the proposed system is invariant under the coordinate's transformation (x1, x2, x3, x4, x5, x6) → (-x1, -x2, x3, -x4, -x5, -

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x6) which persists for all values of the system parameters. Thus, proposed system has rotation symmetry about the x3-axis any non-trivial trajectory of the system must have a twin trajectory. It is also clear to see that the x3-axis is invariant under system flow. In other words, this six-dimensional system could also have symmetric pairs of coexisting attractors like limit cycles or strange attractors.

C. Equilibrium Point

When the system parameter values chosen as:

a=10.2, b=12, c=5.1, d=30, e=2.5, f=2, g=5, h=0.5, i=10, j=17, and k=4.

The equilibrium point becomes:

E0{x1=0, x2=0, x3=0, x4=0, x4=0, x6=0},

The Jacobian matrix of system (3.1), let obvious why this second application is so vital as shown in Figure (1). For equilibrium point:

E_0\{x1=0 ,x2=0 ,x3=0 , x4=0 ,x5=0,x6=0\},and

a=10.2,b=12,c=5.1,d=30,e=2.5,
f=2,g=5,h=0.5,i=10,j=17,and k=4.

The Jacobian matrix has the following result:

To gain its eigenvalues, let |λI-J0|=0, Then the eigenvalues that corresponding to equilibrium E0(0,0,0,0,0,0) are respectively obtained as follows:

λ_1 =-26.7943, λ_2 =-17, $\lambda_3 =$ 11.4943, λ_4 = -2, λ_5 = -1 and λ_6 = -1.

Therefore, the equilibrium E0 (0,0,0,0,0,0) is a saddle point. So, and the hyper chaotic system is unstable at the point $E0$.

D. Lyapunov Exponents and Lyapunov Dimensions

According to the non-linear dynamic theory, the exponent of Lyapunov measures a quantitative test approach to sensitive dependency on the initial conditions. It is the average divergence rate of two adjacent trajectories (or convergence rate). In addition, the ten Lyapunov exponents of proposed system (1) by following parameters:

a=10.2, b=12, c=5.1, d=30, e=2.5, f=2, g=5, h=0.5, i=10, j=17and k=4.

Obtained as follows:

L1= 4.72625, L2= 1.06765, L3= -1.26405, L4= -4.89365, L5= -14.9575and L6=-21.0403.

It can be shown that the largest exponent in Lyapunov is positive, which indicates a chaotic system. Because the L1 and L2 are, a positive exponent of Lyapunov and the other four exponents are negative in Lyapunov. The system is therefore hyper-chaotic. The fractal dimension is a typical feature of the Kaplan-York dimension by Lyapunov exponents calculated from chaos, and DKY can expressed as:

$$\boldsymbol{D_{KY} = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i} \qquad \textbf{(8)}$$

Where j the first j Lyapunov exponent is nonnegative, namely, j is the maximum value of i value that meets both $\sum_{i=1}^{j} L_i > 0$ and $\sum_{i=1}^{j+1} L_i < 0$ at the same time. Li is in descending order of the sequence according to the sequence of Lyapunov exponents. DKY is the upper bound of the dimension of the system information. For the system in this work, by observing the values of ten Lyapunov exponents in the above, we determine that the value of j is nine , and then the Kaplan-Yorke dimension can be expressed from the above due to L1+L2+ L3 >0, and L1+L2+L3+L4+ L5+L6 <0, the Lyapunov dimension of the novel chaotic system is:

$$D_{KY} = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i$$

$$\boldsymbol{D_{KY} = 3 + \frac{1}{|L_{j+1}|}\sum_{i=1}^{3} L_i} \qquad \textbf{(9)}$$

$$= 3 + \frac{L_1+L_2+L_3}{L_4}$$

$$= 3 + (4.72625 + 1.06765 + -1.26405)/$$
4.89365 = 3.92566.

E. Sensitivity to initial conditions

The most distinguishing characteristic of a chaotic system definitely is its long-term unpredictability. This is because initial conditions are resistant to solutions. Eventually, two distinct initial conditions must be wide apart, no matter how close. Therefore, with some limitations of accuracy in an initial scenario, there will be a future period when predictions of accurate system status will not be feasible. Figures (17-22) conditions. The initial system values are set to:

x1(0)=0.5 , x2(0)=2, x3(0)=1.5, x4(0)=6, x5(0)=0.4 and x6(0)=1. for the solid line and x1(0)=0.5 , x2(0)=2, x3(0)=1.5, x4(0)=6, x5(0)=0.4 and x6(0)=1.00000000000001 for the dashed line.

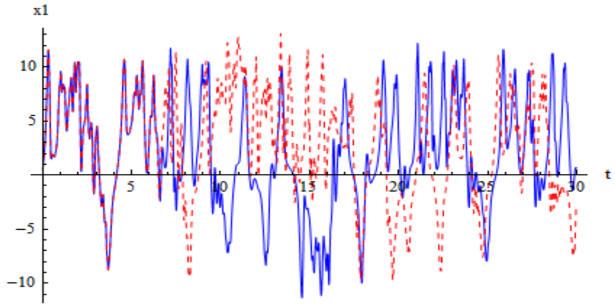

Figure 17. Sensitivity tests of the novel system x1(t).

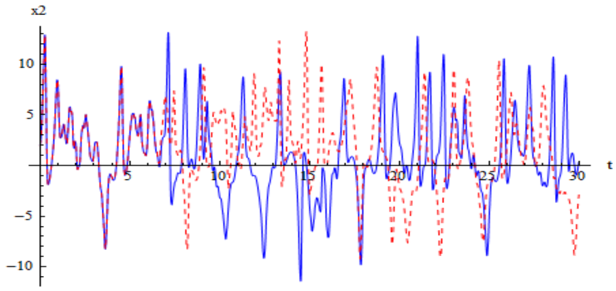

Figure 18. Sensitivity tests of the novel system x2(t).

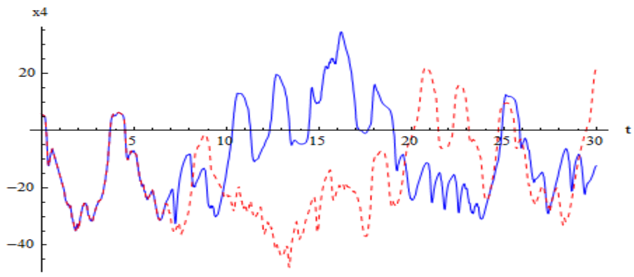

Figure 19. Sensitivity tests of the novel system x3(t).

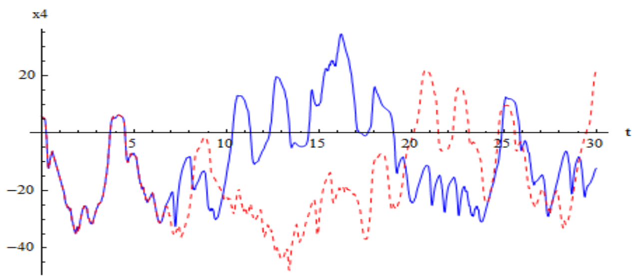

Figure 20. Sensitivity tests of the novel system x4(t).

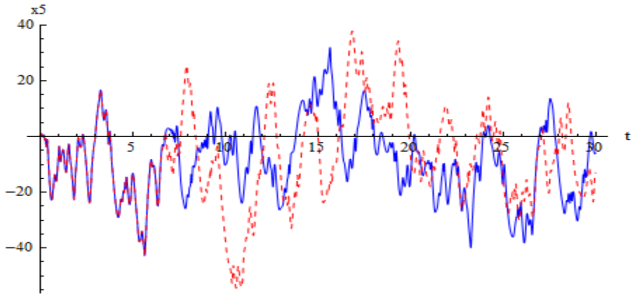

Figure 21. Sensitivity tests of the novel system x5(t).

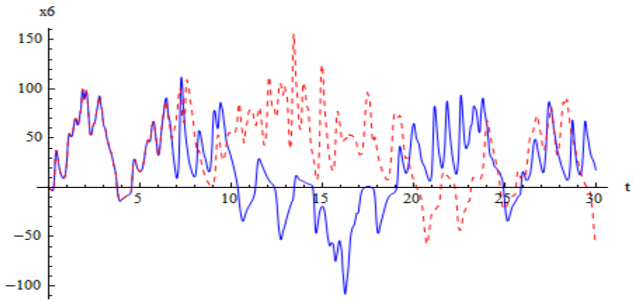

Figure 22. Sensitivity tests of the novel system x6(t).

Clearly, that the waveform of system (1) is non-periodic, has a better sensitivity to the initial conditions, and is called a sensitive dependence on the initial conditions.

Waveform analysis

As is well known, the waveform of a chaotic system should be aperiodic. In order to demonstrate that the proposed system is a chaotic system. Figures 23-28 shows the time versus the states plot obtained from the MATHEMATICA simulation.

The waveforms of (x1(t), x2(t), x3(t), x4(t), x5(t), x6(t)) in time domain are shown in Figures 23-28. The waveforms of (x1(t), x2(t), x3(t), x4(t), x5(t), x6(t)) are aperiodic. In order to discriminate between a multiple periodic motion that can show also a complicated behavior and a chaotic motion, and it can be observed that the time domain waveform has non-cyclical characteristics.

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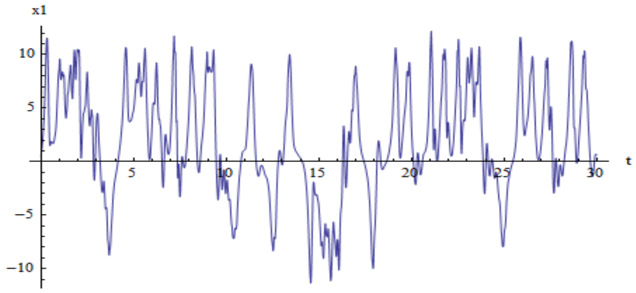

Figure 23. Time versus x1 of the novel chaotic system.

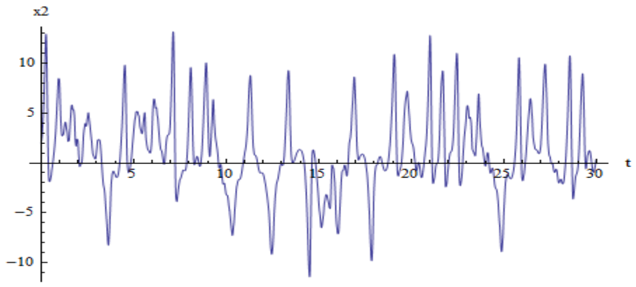

Figure 24. Time versus x2 of the novel chaotic system

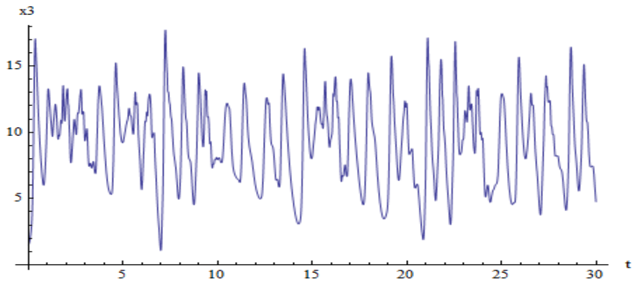

Figure 25. Time versus x3 of the novel chaotic system.

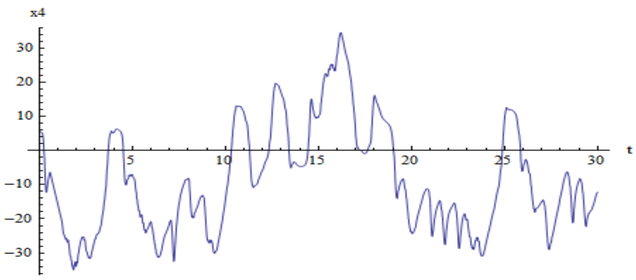

Figure 26. Time versus x4 of the novel chaotic system.

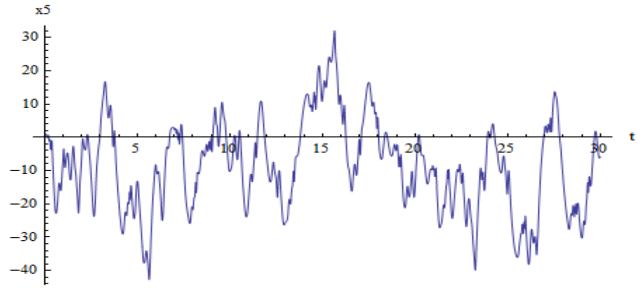

Figure 27. Time versus x5 of the novel chaotic system.

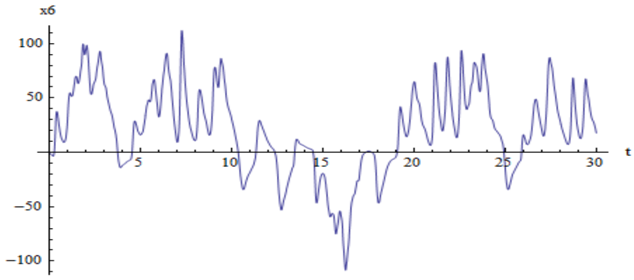

Figure 28. Time versus x6 of the novel chaotic system.

CONCLUSIONS

According to the nonlinear dynamical theory, a quantitative measure approach of the sensitive dependence on the initial conditions is calculating the Lyapunov exponent. It is the average rate of divergence (or convergence) of two neighboring trajectories. Moreover, the ten Lyapunov exponents of the nonlinear dynamical system (1) with parameters a=10.2, b=12, c=5.1, d=30, e=2.5, f=2, g=5, h=0.5, i=10, j=17 and k=4. Are obtained as follows:

L1= 4.72625, L2= 1.06765, L3= -1.26405, L4= -4.89365, L5= -14.9575and L6=-21.0403

It can be seen that the largest Lyapunov exponent is positive, indicating that the system has chaotic characteristics. Since the $L1$ and L2 are a positive Lyapunov exponent, and the rest three Lyapunov exponents are negative. Thus, the system is hyper-chaotic. This nonlinear system exhibits the complex and abundant chaotic dynamics behaviors. The topology looks like a shape of a flying butterfly flipping its wings; thus, coining the term “Butterfly Effect”.

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