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On Jordan Ideal in Prime and Semiprime Inverse Semirings with Centralizer

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Abstract
In this paper we recall the definition of centralizer on inverse semiring. Also introduce the definition of Jordan ideal and Lie ideal, some results of Vukman on centralizers on semiprime rings are generalized to inverse semirings.

Keywords: Additively inverse semiring, Jordan ideal of an inverse semiring, Semiprime inverse semirings.

الخلاصة
في هذا البحث استذكرنا تعريف التمركز في اشباه الحلقات المعكوسة وقدمنا تعريف المثاليات لي و الجوردان فيها.
و عممنا بعض نتائج الباحث Joso Vukman حول التمركزات في الحلقات شبه الأولية الى اشباه الحلقات المعكوسة.

Introduction

A semiring $(S, +, \cdot)$ with commutative addition and an absorbing zero 0, is called an additively inverse semiring which introduced by Karvaellas [5], if for every element $a \in S$ there exists a unique element $a' \in S$ such that $a + a' + a = a$ and $a' + a + a' = a'$. Let S be an inverse semiring and let T be an additive mapping from S to itself, T is defined as a left centralizer of S if $T(xy)=T(x)\, y$ for all $x,y \in S$ and right centralizer of S as (T) if $T(xy) = x\,T(y)$, T is called centralizer if it is both right and left centralizer[8]. In this paper we will represent S as an inverse semiring with $a+a'$ belong to the center of S. In [9] a commutator $[.\,,.]$ in inverse semirng is defined as $[x,y] = xy+yx'$ $= xy+y'x$, and make inclusive use of basic commuttator identities $[xz\,,\,y] = x[z,y] + [x,y]z$, $[x,zy] =[x,z]y+z[x,y]$, for all $x, y, z \in S$. A derivation d is an additive mapping from S to into itself satisfy $d(ab) = d(a)b + a\, d(b)$, for all $a,\ b \in S$ see [9]. S is prime whenever $a\,S\,b = 0$ then $a = 0$ or $b = 0$ and semiprime whenever $aSa=0$ implies that $a = 0$. S is n-tortion free if $na = 0, a \in S$ implies that $a = 0$. A non empty subset I of S is said to be an ideal in S if for $s,m \in I$, $r \in S$ imply $s + m \in I$ and $rs, sr \in I$, See [4]. An additive mapping d from inverse semiring S to itself, when $a \in S$ be a fixed element of S. Define $d: S \to S$ by $d(x) = [a,x]$ for all $x \in S$ is called derivation, for all $x, y \in S, d(xy) = [a,xy] = x[a,y] + [a,x]y = xd(y) + d(x)y$ (see[1]). When S be 2-tortion free semiprime inverse semiring and U be an additive subgroup of S then we can defined U as Jordan ideal of S if $ur+ru \in U$, for all $u \in U$,$r \in S$. In this paper we recall the definition of centralizer and illustrate this concept by example, also generalize some results of Joso Vukman[2,3,4] and Ram Awtar [6] in semiprime rings to inverse semirings. T will denote an additive mapping from S into itself satisfies the condition:

$$T(ur + ru) + u'T(r) + T(r)u' = 0. \qquad (1)$$

In particular, if $r = u$ in equation (1), then by (1), we note that:
$2T(u^2) = T(u)u + uT(u)$ for all $u \in U$.

Put $u^r = T(ur) + uT(r)'$,
$r^u = T(ru) + T(r)u'$.
This hypothesis is a corresponding case of Heba. A. Shaker [10] on prime and semiprime rings.

Preliminaries

Lemma (2.1) [5]:
Let S be an inverse semiring, for all $a, b \in S$, if $a + b = 0$, then $a = b'$.

Definition (2.2) [8]:
Let S be an inverse semiring and T be an additive mapping from S into itself. The left centralizer of S is defined as T such that: $T(xy) = T(x)y$ for all $x, y \in S$. Also right centralizer of S as T, such that $T(xy) = xT(y)$ for all $x, y \in S$. And T is called centralizer if it is both right and left centralizer.

Example (2.3):
Let S be an inverse semiring, and $M_2(S)$ be the set of all matrices of order 2 with usual addition and usual multiplication on matrices, let $T_1, T_2: M_2(S) \longrightarrow M_2(S)$ be additive mappings defined as:

$T_1\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ z & w \end{pmatrix}$, for all $x, y, z, w \in S$,

$T_2\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 0 & y \\ 0 & w \end{pmatrix}$, for all $x, y, z, w \in S$.

We have to show that T_1 is left centralizer and T_2 is the right centralizer, as follows:

$$T_1\left(\begin{pmatrix} x & y \\ z & w \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = T_1\begin{pmatrix} xa + yc & xb + yd \\ za + wc & zb + wd \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ za + wc & zb + wd \end{pmatrix}.$$

But,

$$T_1\begin{pmatrix} x & y \\ z & w \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ z & w \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ za + wc & zb + wd \end{pmatrix}$$

Thus, T_1 is the left centralizer. By the same way we can find that T_2 right centralizer.

Results

Theorem (3.1): [8]
Let S be a 2-torsion free semiprime inverse semiring, and let T be an additive mapping T from S to itself which satisfies $T(xyx) + xT(y)x' = 0$, for all $x \in S$. Then T is centralizer.

Remark (3.2):
Note that by (1), we have:
$(u)^r + (r)^u = 0$, for all $u \in U$ and $r \in S$,
since**:**

$$T(ur) + u'T(r) + T(ru) + T(r)u' = T\,(ur + ru) + u'T(r) + T(r)u' = 0.$$

Remark (3.3):
If S is a 2- torsion free semiprime inverse semiring, and since $(u)^r + (r)^u = 0$. Hence by Lemma (2.1)**,** we have:
$(u)^r = ((r)^u)'$.

Lemma (3.4):
Let S be an inverse semiring and U is a Jordan ideal of S then for all $a, b \in U$ and $x \in S$, $(ab + ba)x + x'(ab + ba) \in U$

Proof:
Since $a \in U$ and for any $x \in S$, we have, $xb + bx' \in S$, then we obtain:
$a(xb + bx') + (xb + bx')a \in U$.
But $a(xb + bx') + (xb + bx')a = axb + abx' + xba + bx'a$, since S is inverse semiring, we can replace x by $x + x' + x$ and $x'$by $x' + x + x'$ in above equation. So, it will be:

$$\begin{aligned} &axb + ab(x' + x + x') + (x + x' + x)ba + bx'a \\ &= axb + abx' + ab(x + x') + (x + x')ba + xba + bx'a \\ &= axb + abx' + xab + x'ab + bax + bax' + xba + bx'a \\ &= \{(ax + x'a)b + b(ax + x'a)\} + \{x(ab + ba) + (ab + ba)x'\}. \end{aligned}$$

The left side is in *U*. Hence, the right side $x(ab + ba) + (ab + ba)'x$ is in *U* too.

Theorem (3.5)
Let S be a 2-torison free semiprime inverse semiring then any non-zero Jordan ideal of *S* contains a non-zero ideal of *S*.

Proof:
Let $U \neq 0$ be a Jordan ideal of *S*, suppose that $a,b \in U$. By Lemma (3.4), for any $x \in S$, $xc + cx' \in U$, where $c = ab + ba$, since $c \in U$, $xc + cx \in U$. By adding $xc + cx'$ to $xc + cx$, we get $2xc \in U$, for all $x \in S$. Hence for all $y \in S$, $(2xc)y + y(2xc) \in U$.

Since$2yxc \in U$. We obtain that $2xcy \in U$ for all$x, y \in S$. That is, $2ScS \subset U$. Now, $2ScS$ is an ideal of S, so we are done. Unless $2ScS = 0$, if $2ScS = 0$.
By our assumption $ScS = 0$ and so, $cScScSc = 0$. Since S is semiprime inverse semiring then, $2ScS = 0$, this leads to $c = 0$.
Also by using the semiprimness of S that is given for all $a, b \in U$ then, $ab + ba = 0$. Let $0 \neq a \in U$, then for $x \in S$, $b = ax + xa \in U$, hence, $a(ax + xa) + (ax + xa)a = 0$. That is, $a^2x+xa^2+2axa = 0$. Now, for $a \in U$, $0 = aa + aa = 2a^2$. Hence, $a^2= 0$. The top relation then reduces to $2axa = 0$ for all $x \in S$, and so, $aSa = 0$, then $a = 0$, contrary to assumption. Thus, we have U contains a non-zero ideal of S.

Theorem (3.6):
Let S be a 2-torsion free of semiprime inverse semiring, U be a Jordan ideal of S. Suppose that t $\in S$ *commutes* with u^2 for all $u \in U$, then t commutes with every element of U .

Proof:
For all $r \in S$, let $d(r) = tr + rt'$, d is derivation by definition (2.4) [7]. Since t commutes with u^2 then, $[t, u^2] = 0$. That is, $tu^2 + u^2t' = d(u^2) = 0$, for all $u \in U$.
Linearizing on u in above equation, we get:

$$d((u+v)^2) = d(u^2 + uv + vu + v^2) = d(u^2) + d(uv + vu) + d(v^2).$$

Then$d(uv + vu) = 0$, for all $v, u \in U$. Since d is additive, hence:

$$d(uv + vu) = d(uv) + d(vu) = d(u)v + ud(v) + d(v)u + vd(u) = 0.$$

Then, for all $u, v \in U$, we have:

$$ud(v) + d(u)v + vd(u) + d(v)u = 0 \qquad (2)$$

Replace$v$ by $ur + ru$, where $r \in S$, we obtain:
$d(u)(ur + ru) + ud(ur + ru) + d(ur + ru)u + (ur + ru)d(u) = 0$.
That is:

$$d(u)ur + d(u)ru + ud(ur) + ud(ru) + d(ur)u + d(ru)u + urd(u) + rud(u) = 0 \qquad (3)$$

Thus,

$$d(u)ur + d(u)ru + ud(u)r + u^2d(r) + ud(r)u + urd(u) + d(u)ru + ud(r)u + d(r)u^2 + rd(u)u + urd(u) + rud(u) = 0$$

Replace r by ru in (3), we get:

$$d(u)uru + d(u)ru^2 + ud(u)ru + u^2d(ru) + ud(ru)u + urud(u) + d(u)ru^2 + ud(ru)u + d(ru)u^2 + rud(u)u + urud(u) + ru^2d(u) = 0$$

That is,
$d(u)uru + d(u)ru^2 + ud(u)ru + u^2d(r)u + u^2rd(u) + ud(r)u^2 + urd(u)u + urud(u) + d(u)ru^2 + ud(r)u^2 + urd(u)u + d(r)u^3 + rd(u)u^2 + rud(u)u + urud(u) + ru^2d(u) = 0$.
Use (3) to obtain:
$(d(u)ur + d(u)ru + ud(u)r + u^2d(r) + ud(r)u + 2urd(u) + d(u)ru + ud(r)u + d(r)u^2 + rd(u)u + rud(u)u + (u^2r + 2uru + ru^2)d(u) = 0$
Then, for all $u \in U, r \in S$,

$$(u^2r + 2uru + ru^2)d(u) = 0 \qquad (4)$$

U is Jordan ideal, it follows that:
$ur + ru \in U$, and $uu + uu = 2u^2 \in U$.
But,

$$\begin{aligned}4uru &= 2uru + 2uru = 2u(r + r' + r)u + 2uru \\ &= 2u(r + r')u + 2uru + 2uru \\ &= 2u(r + r' + r + r')u + 4uru \\ &= 2u(r + r')u + 2u(r + r')u + 4uru \\ &= 2u^2(r + r') + (r + r')(2u^2) + 4uru \\ &= 2u^2r + 2u^2r' + 2ru^2 + 2r'u^2 + 4uru \\ &= 2u^2r + 2uru + 2ru^2 + \{2u^2r' + r'2u^2\} \\ &= 2\{u(ur + ru) + (ur + ru)u\} + \{2u^2r' + r'2u^2\}.\end{aligned}$$

The first and the second terms on the right hand side are in U. Hence, $4uru \in U$.
Therefore, if we replace v by $4uru$ in (2), where $r \in S$ then:

$$ud(4uru) + d(u)4uru + 4urud(u) + d(4uru) = 0$$

$$0 = 4\{ud(u)ru + u^2d(ru) + d(u)uru + urud(u) + d(u)ru^2 + 4d(ru)u$$

$$= 4\{ud(u)ru + u^2d(r)u + u^2rd(u) + d(u)uru + urud(u) + d(u)ru^2 + ud(r)u^2 + urd(u)u\}.$$

Since S is a free 2-torsion then:

$$ud(u)ru + u^2d(r)u + u^2rd(u) + d(u)uru + urud(u) + d(u)ru^2 + ud(r)u^2 + urd(u)u = 0. \quad (5)$$

Replace r by ru in (5), we have:

$$ud(u)ru^2 + u^2d(ru)u + u^2rud(u) + d(u)uru^2 + uru^2d(u) + d(u)ru^3 + ud(ru)u^2 + urud(u)u = 0$$

That is,

$$ud(u)ru^2 + u^2d(r)u^2 = u^2rd(u)u + u^2rud(u) + d(u)uru^2 + uru^2d(u) + d(u)ru^3 + ud(r)u^3 + urd(u)u^2 + urud(u)u = 0$$

So,

$$\big(ud(u)ru + u^2d(r)u + u^2rd(u) + d(u)uru + d(u)ru^2 + ud(r)u^2 + urd(u)u + urd(u)\big)u + (u^2ru + uru^2)d(u) = 0.$$

By (5), and for all $u \in U, r \in S$, we get:

$$u^2ru + uru^2)d(u) = 0. \quad (6)$$

Since,$0 = d(u^2) = d(uu) = ud(u) + d(u)u$, then by Lemma (2.1) we have, $ud(u) = d(u)u'$. By multiplying on the right of (4) by u', we get $(u^2ru + 2uru^2 + ru^3)'d(u) = 0$.
Adding equation (6) to the last equation, we have:

$$(u^2ru + uru^2)d(u) + (u^2r'u + 2ur'u^2 + r'u^3)d(u) = (u^2(r + r')u + u(r + r')u^2 + ur'u^2 + r'u^3)d(u) = (u(r + r')u^2 + u(r + r')u^2 + ur'u^2 + r'u^3)d(u) = u(r + r' + r + r' + r)u^2 + r'u^3$$

$$= (ur'u^2 + r'u^3)d(u) = 0.$$

Thus, $(uru^2 + ru^3)d(u) = 0$. So:

$$(ur + ru)u^2d(u) = \text{ for } r \in S,\ u \in U \quad (7)$$

where $s \in S$ in (7), we will replace r by rs, and we mean $u \circ s$ by $us + su$ for all $u \in U$.
Since,

$$u(rs) + (rs)u = [u, r]s + r(u \circ s) = urs + r'us + rus + rsu$$

$$= urs + (r' + r)us + rsu = urs + u(r' + r)s + rsu = urs + ur's + urs + rsu = urs + rsu.$$

Then we get, $\{[u, r]s + r(u \circ s)\}\, u^2d(u) = 0$.
Since $r(us + su)u^2d(u) = 0$, by (7), so:

$$[u, r]Su^2d(u) = 0$$

Replace r by $d(u)$, we obtain:

$$[u, d(u)]Su^2d(u) = 0$$

$$ud(u)S\, u^2d(u) + d(u)u'Su^2d(u) = ud(u)Su^2d(u) + ud(u)Su^2d(u) = 0$$

Then,$2ud(u)Su^2d(u) = 0$, since S is 2-torsion free implies that $ud(u)Su^2d(u) = 0$, so $u^2d(u)Su^2d(u) = 0$, by semiprieness we get:

$$u^2d(u) = 0, \text{ for all } u \in U \quad (8)$$

In (8) replace$u$ by $u + v$, we have:

$$(u + v)^2d(u + v) = 0$$

$$u^2d(v) + v^2d(u) + (uv + vu)d(u) + (uv + vu)d(v) = 0 \quad (9)$$

Replace v by v', we get

$$u^2d(v)' + v^2d(u) + (uv' + v'u)d(u) + (uv' + v'u)d(v)' = 0.$$

Then, $u^2d(v)' + v^2d(u) + (uv' + v'u)d(u) + (uv + vu)d(u) = 0 \quad (10)$

Adding (9) to (10), we have:

$$u^2(d(v) + d(v)') + 2v^2d(u) + (uv + uv' + vu + v'u)d(u) + 2(uv + vu)d(v) = 0 \quad (11)$$

By adding (9) to (11), we have:

$$u^2(d(v) + d(v)' + 3v^2d(u) + (uv + uv' + uv + vu + vu + vu')d(u) + 3(uv + vu)d(v) = 0.$$

So,

$$u^2d(v)+3v^2d(u)+(uv+vu)d(u)+3(uv+vu)d=0 \quad (12)$$

From (2), we get:

$$u^2d(v)+(uv+vu)d(u)+(uv+vu)d(v)=v^2d(u)' \quad (13)$$

Now substitute (13) in (12), we get:
$3v^2d(u)+v^2d(u)'+2(uv+vu)d(v)=0$.
$v^2d(u)+v^2d(u)+v^2d(u)+v^2d(u)'+2(uv+vu)d(v)=0$,
that is $2v^2d(u)+2(uv+vu)d(v)=0$.
Replacing v by $2v^2$ to obtain,
$4v^4d(u)+2(2uv^2+2v^2u)d(v^2)=0$.
Since $d(v^2)=0$ for all $v\in U$, then $4v^4d(u)=0$, since S is 2-torsion free then,

$$v^4d(u)=0 \quad \text{for } u\in U, \text{and } r\in S \quad (14)$$

$$\begin{aligned} u\circ(ru+ur')+(ru+ur')u &= uru+u^2r'+ru^2+uru' \\ &= u(r+r')u+ru^2+u^2r' \\ &= u^2r+u^2r'+u^2r'+ru^2 \\ &= ru^2+u^2r' \quad \in U \end{aligned}$$

and so, $2(ru^2+u^2r')\in U$. It follows that $4u^2r$ *and* $4ru^2$ are in *U*. Therefore, replacing u by $4u^2r$ where $r\in S$ in (4), we get $v^4d(4u^2r)=4v^4d(u^2r)=4v^4d(u^2)r+4v^4u^2d(r)=0$ for all $u\in U,\ r\in S$. In view of (14), $2v^4d(u^2)=0$.
Hence, $4v^4u^2d(r)=0$. In particular,

$$u^6d(r)=0, \text{for } u\in U, r\in S \quad (15)$$

Let $M=\{x\in S \mid xd(r)=0, \text{for all } r\in S\}$.
By Lemma (3.2) [7], M is right and left ideal. Let $\overline{S}=S/M$, then by Lemmas (3.3) [7], (3.4) [7], (3.5) [7], $\bar{S}$ is 2-torsion free semiprime inverse semiring.
Then by (14), $\bar{u}^6=0$, for all $\bar{u}\in\overline{U}$, where $\overline{U}$ A Jordan Ideal of $\bar{S}$. We will show that if $u^6=0$ for all $u\in U$, where U is Jordan ideal of 2-torsion free semiprime inverse semiring S, then $u=0$ for all $u\in U$. For $u\in U, r\in S$ we have $2u^2\in U$, and so:

$$(2u^2)\,(2u^2)+(2u^2)\,(2u^2)=8u^2\in U.$$

Therefore, $8(u^4r+ru^4)\in U$, and hence,
$0=8^6(u^4r+ru^4)^6$.
Multiply on the right by u^4r, to obtain $8^6(u^4r)^7=0$. Hence, $(u^4r)^7=0$. If for some u, $u^4\neq 0$, then u^4S is non zero right ideal of *S*, in which the seventh power of every element is zero by lemma of levizkis theorem on inverse semirings**,** S would have a non zero nilpotent ideal. This is impossible for semiprime inverse semiring.
Hence, $u^4=0$ for all $u\in U$. By repeating the above argument twice can show that $u=0$ for all $u\in U$. Hence, $U=0$. And by this we infer that $\overline{U}=0$, that is $U\subset M$and by the definition of M, $ud(r)=0$, for $u\in U, r\in S$. Now replace r byrx, for $x\in S$, then $uSd(r)=0$. As $tS\subset S$, $utSd(r)=0$. But$tu\,S\,d(r)=0$, thus we infer that:

$$d(u)\,S\,d(r)=0, \text{for } r\in S, u\in U.$$

In particular, $d(u)S\,d(r)=0$, for $u\in U$. This says that$(d(u)S)^2=0$, which implies that $d(u)=0$ for all $u\in U$. Because S is semiprime, hence *t* commutes with every element of *U*.

Theorem (3.7):

Let *S* be a 2-torsion free semiprime inverse semiring, *U* be a Jordan ideal of *S*, suppose that $t\in S$ commutes with every element of [*U*,*U*], then *t* commutes with every element of *U*.

Proof:

For all $r\in S$, $u,v\in U$, we have:

$$\begin{aligned} (uv+vu)r+r'(uv+vu) &= uvr+vur+ruv'+rvu' \\ &= uvr+vur+(r+r'+r)uv'+(r+r'+r)vu' \\ &= uvr+vur+ruv'+u(r'+r)v'+rvu'+v(r'+r)u' \end{aligned}$$

$=uvr+vur+ruv'+urv+urv'+rvu'+vru+vru'=\{u(vr+rv)+(vr+rv)u'\}+\{v(ur+ru)+(ur+ru)v'\}$.
Thus,

$$[U\circ U,S]\subset[U,U] \quad (16)$$

Again for $u, v \in U$ and $r \in S$, we have:

$$(uv + vu')r + r'(uv + vu')$$
$$= uvr + vur' + ruv' + rvu$$
$$= uvr + vur' + (r + r' + r)uv' + (r + r' + r)vu$$
$$= uvr + vur' + ruv' + u(r' + r)v' + rvu + v(r' + r)u$$
$$= uvr + vur' + ruv' + urv + urv' + rvu + vr'u + vru$$
$$= \{u(vr + rv) + (vr + rv)u\} + \{v'(ur + ru) + (ur + ru)v'\}$$

Thus,

$$\big[[U,U],S\big] \subset U \circ U \qquad (17)$$

Suppose $F = U \circ U + [U,U]$, clearly that F is an additive subgroup of *S*. In view of (16) and (17), we obtain:

$[F,S] = [U \circ U + [U,U],S] = [U \circ U,S] + \big[[U,U],S\big] \subset [U,U] + U \circ U = F$.

Hence, F is a lie ideal of S , from equation (16) and the hypothesis yield:

$$\big[[U \circ U,t],t\big] \subset \big[[U,U],t\big] = 0$$

Therefore,

$\big[[F,t],t\big] = \big[[U \circ U,t],t\big] + \big[[U,U],t\big],t] = 0$.

By Theorem (3.11) [7], we have $[t,F] = 0$.

But:

$[t, U \circ U] \subset [t,[U,U] + U \circ U] = 0$.

Hence for $u \in U, [t, 2u^2] = 0$.

So, $[t,u^2] = 0$ for all $u \in U$.

Therefore, by Theorem 3.6, we infer that $[t,u] = 0$ for all $u \in U$.

Thus, *t* commutes with every element of *U*.

Lemma (3.8):

Let S be a 2-torsion free semiprime inverse semiring, U be a Jordan ideal, for all $u \in U$, $r \in S$ if $u^2 \in Z(S)$, then $(u^2)^r = 0$.

Proof:

Since $(u^2)^r = T(u^2r) + u^2T(r)'$, and by the following:

1. $T(ur + ru) + u'T(r) + T(r)u' = 0$,
2. $T(u^2r + ru^2) + u^2\, T(r)' + T(r)'u^2 = 0$,

for all $r \in S, u \in U$, and since $u^2 \in Z(S)$, then:

$$T(2u^2r) + 2u^2T(r)' = 0.$$

Since *S* is 2-torsion free, then:

$$(u^2)^r = T(u^2r) + u^2T(r)' = 0.$$

Lemma (3.9):

Let S be a 2-torsion free prime inverse semiring, and U be a Jordan ideal. Then for all $r \in S\ and\ u \in U, T(uru) + uT(r)u' = 0$.

Proof:

Replace ®by $(u.2r + 2r.u)$ in equation (1), then we get:

$T(u(u.2r + 2r.u) + (u.2r + 2r.u)u + u'T(u.2r + 2r.u) + T(u.2r + 2ru)u' = 0$

$\Rightarrow T(2u^2r + 2uru + 2uru + 2ru^2 + 2u'$
$(uT(r) + T(r)u) + 2(uT(r) + T(r)u)u' = 0$

$\Rightarrow\ T(2u^2r + 2ru^2) + 4T(uru + 2u'uT(r)$
$+2u'T(r)u + 2uT(r) + 2T(r)uu' = 0$

That is,

$T(2u^2r + 2ru^2) + 4T(uru) + 2u'uT(r) + 4u'T(r)u + 2T(r)uu' = 0.$

Since $uu + uu \in U$, and U is Jordan ideal, so, $2u^2 \in U$, and then:

$T(2u^2r + T(2ru^2) + 4T(uru) + 2u^2T(r)' + 4u'T(r)u + 2T(r)'u^2 = 0.$

Since,

1. $2T(u^2r) + 2T(ru^2) + 2u^2T(r)' + 2T(r)'u^2 = 0,$
2. $T(u^2r + ru^2) + u^2T(r)' + T(r)'u^2 = 0,$

so $2T(u^2r) + 2u^2T(r)' = 0.$

Since *S* is 2-torsion free, this implies:

$$T(u^2r) + u^2T(r)' = 0,$$

and then:

$$4T(uru) + 4u'T(r)u = 0,$$
$$T(uru) + u'T(r)u = 0.$$

Corollary (3.10):

Let S be a 2-torsion free prime inverse semiring, and U be a Jordan ideal. Then:

$$T(urv + vru) + uT(r)v' + vT(r)u' = 0,$$

for all $r \in S,\ u, v \in U.$

Proof:

If we replace *u* by $u + v$ in the equation: $T(uru) + uT(r)u' = 0$, of lemma (3.9), we obtain that:

$$T(u + v)r(u + v) + (u + v)\, T(r)\, (u + v)' = 0$$
$$T(uru) + T(urv) + T(vru) + T(vrv) + uT(r)u' + uT(r)v' + vT(r)u' + vT(r)v' = 0,$$
$$T(uru) + uT(r)u' + T(urv) + T(vru) + T(vrv) + vT(r)v' + vT(r)u' + uT(r)v' = 0.$$

Then, $T(urv + vru) + uT(r)v' + vT(r)u' = 0$.

Lemma (3.11):

If S is 2-torsion free prime inverse semiring such that $tv^2+v^2t = 0$, U be a Jordan ideal of S, then, for all $t \in S, v, u \in U$, $t = 0$.

Proof:

By linearizing $t v^2+v^2t = 0$ on v, we get:

$t(u+v)^2 + (u+v)^2t = 0,$

$t(u^2+uv+vu+v^2)+(u^2+uv+vu+v^2)t$

$= tu^2 + t(uv+vuv) + tv^2 + u^2t + (uv+vu)t + v^2t = 0$, by submission $t(uv+vu) + (uv+vu)t = 0$. Replace v by $u+2v^2 \in U$, in the later equation, we get:

$$t(u(u+2v^2) + (u+2v^2)u) + (u(u+2v^2) + (u+2v^2)u)t$$
$$= t(u^2+2uv^2+u^2+2v^2u) + (u^2+2uv^2+u^2+2v^2u)t$$
$$= 2tu^2 + 2u^2t + 2t(uv^2 + v^2u) + 2(uv^2+v^2u)t = 0.$$

Since S is 2-torsion free, then:

$$t(uv^2+v^2u) + (uv^2+v^2u)t = 0 \qquad (18)$$

Since $t v^2+v^2t = 0$ then by Lemma (2.1), we get:

$tv^2 = v^2t'$,

so by (18) we obtain:

$tuv^2 + tv^2u + uv^2t + v^2ut = 0,$

$tuv^2 + v^2tu' + v^2u't + v^2ut = 0.$

So, $tuv^2 + v^2t'u + v^2u't + ut'v^2 = 0$

That is,

$[t,u]v^2+v^2[t,u]' = 0, [\,[t,u], v^2\,] = 0$ for all $v, u \in U$.

So, by Theorem 3.6, we have:

$$\big[[t,u], v\big] = 0, \text{ for all } v, u \in U. \qquad (19)$$

Now, by Jacobi identities we obtain: $[\,[u,v],t\,] + [\,[v,t],u] + [\,[t,u],v] = 0$

In the view of (18) the second and the third terms are zeros.

So, $[\,[u,v],t\,] = 0$ for all $u, v \in U$.

For all $v \in U$, $t \in S$ so, $t v^2 = 0$, linearizing on v we get:

$t(u+v)^2 = t(u^2+uv+vu+v^2) = tu^2 + tuv + tvu + tv^2 = t(uv+vu) = 0,$

(i.e) $t(uv+vu)u = tuvu + tvu^2 = 0.$

So, $tuvu = tuvtu = 0$ for all $v, u \in U$.

(i,e) $(tu)\,U\,(tu) = 0$, by Theorem (3.5),

(i.e) $(tu)\,I\,(tu) = 0,$

(i.e) $(tu)IS\,(tu) = 0,$

$(tu)IS\,(tu)I = 0$. Since S is prime, therefore, $(tu)I = 0$, and so, $(tu)SI = 0$.

But, $I \neq 0$, so we obtain:

$tu = 0$, for all $u \in U$, then $tU = 0$.

So, by Theorem (3.5), we get:

$tI = 0,$

(i.e) $tSI = 0$. But S is prime inverse semiring, and $I \neq 0$, so, $t = 0$.

Lemma (3.12):

Let S be a 2-torsion free prime inverse semiring, U be a Jordan ideal of S then, for all $r \in S$, $v \in U$, $[v^2,r](v^2)^r=0$ and $(v^2)[v^2,r] = 0$.

Proof:

By equation (1), and for all $r \in S, u \in U$, we have:

$T(uu+uu) + u'T(u) + T(u)u' = 0$

$2T(u^2) + u'T(u) + T(u)u' = 0$ for all $u \in U$.

Then, by using Theorem (3.1), on U, T is centralizer. Thus:

$T(uv) = T(u)\,v$

$T(uv) + T(u)v' = 0$, for all $u, v \in U$. By multiply the both sides from left by $(uv+vu')$, we have:

$(uv+vu')(T(uv) + T(u)v') = 0.$

Replace u by $2vr+2rv$ for all, $r \in S$, we have:

$((2vr+2rv)v + v'(2vr+2rv)T((2vr+2rv)v)+T(2vr+2rv)v'$

$=2vrv + 2rv^2+2v^2r'+2vr'v)(T(2vrv+2rv^2))+ 2vT(r)v' + 2T(r)vv'$

$= 2vrv + 2v'rv + 2rv^2+2v^2r')\,T\,(2vrv+2rv^2) + 2vT(r)v' + 2T(r)vv')$

$= 2(v+v')rv + 2rv^2+2v^2r')(T(2rv^2) + 2T(r)'v^2)$

$$= 2rv(v+v') + 2rv^2 + 2v^2r')(2T(v^2r) + 2v^2T(r) = 0.$$

That is,

$2rv^2 + 2r'v^2 + 2rv^2 + 2v^2r')(v^2)^r = 0,$

$2(r+r'+r)v^2 + 2v^2r' = 0$, (By the properties of inverse semirings).

Thus:

$(rv^2+v^2r)(v^2)^r = 0$, (Since S is 2-torsion free), so:

$[v^2, r](v^2)^r = 0.$

Similarly, we can prove that $(v^2)^r[v^2, r] = 0.$

Corollary (3.13):

Let S be a 2-torsion free prime inverse semiring, U be a Jordan ideal of S then, for all $r \in S,\ v \in U$, then:

(1) $[u^2, r](u^2)^s + [u^2, s](u^2)^r = 0$.

(2) $(u^2)^s[u^2, r] + (u^2)^r[u^2, s] = 0.$

Proof:

Replacing r by $r+s$ for all, $s \in S$, in the equation of Lemma 3.12**,** we have:

$$[u^2, r + s](u^2)^{r+s} = 0, \qquad (20)$$

and,

$$(u^2)^r[u^2, r + s] = 0 \qquad (21)$$

Take (20), by our hypothesis of T mentioned at the end of the Introduction, we have:

$$\begin{aligned}(u^2)^{r+s} &= T(u^2(r + s)) + u^2T(r + s)' \\ &= T(u^2r + u^2s) + (u^2)'T(r)(u^2)'T(s) \\ &= T(u^2r) + u^2T(r) + T(u^2s) + u^2T(s) \\ &= (u^2)^r + (u^2)^s.\end{aligned}$$

Then, the equation (20) becomes:

$[u^2, r](u^2)^s + [u^2, s](u^2)^r = 0.$

To prove (21): we replace r by $r + s$ in the second part of Lemma (3.12), last term we get:

$$\begin{aligned}(u^2)^{r+s} &= T(u^2(r + s) + (u^2)'T(r + s) \\ &= T(u^2r) + T(u^2s) + (u^2)'T(r) + (u^2)'T(s) \\ &= T(u^2r) + u^2T(r)' + T(u^2s) + u^2T(s)' \\ &= (u^2)^r + (u^2)^s\end{aligned}$$

Then, $(u^2)^{r+s}[u^2, (r + s)] = 0,$

$(u^2)^r + (u^2)^s\, [u^2, (r + s)] = 0,$

Since $(u^2)^r[u^2, r] = 0,$ and $(u^2)^s[u^2, s] = 0$.

Thus, $(u^2)^r[u^2, s] + (u^2)^s[u^2, r] = 0.$

Lemma (3.14):

Let S be a 2-torsion free prime inverse semiring, U be a Jordan ideal of S, for all $u \in U, r \in S,\ (u^2)r = 0$ **.**

Proof:

By (21) of Corollary(3.13), we have:

$$(u^2)^s\,[u^2, r] + (u^2)^r\,[u^2, s] = 0\ , \qquad (22)$$

for all $u \in U, r \in S$**.** So:

$[u^2, z](u^2)^r\,[u^2, s] + [u^2, z](u^2)^s\,[u^2, r] = 0,$

By equation (22), we obtain

$[u^2, z](u^2)^r\,[u^2, s] = [u^2, r](u^2)^z\,[u^2, s]' = [u^2, r](u^2)^s\,[u^2, z]$

Then,

$$[u^2, r](u^2)^s\,[u^2, z] + [u^2, z](u^2)^s[u^2, r] = 0 \qquad (23)$$

Replace z by zt in equation (23) and by using of Jacobi identities, we obtain:

$[u^2, r](u^2)^s\,[u^2, zt] + [u^2, zt](u^2)^s\,[u^2, r] = 0$

$[u^2, r](u^2)^s\, z[u^2, t] + [u^2, r](u^2)^s[u^2, z]t + z[u^2, t](u^2)^s[u^2, r] + [u^2, z]t(u^2)^s[u^2, r] = 0.$

That is,

$$\begin{aligned}&[u^2, r](u^2)^s\, z[u^2, t] \\ &\quad + z'[u^2, r](u^2)^s[u^2, t] \\ &\quad + [u^2, z]((u^2)^s[u^2, r]t' \\ &\quad + [u^2, z]t(u^2)^s[u^2, r]) \\ &\quad = 0,\end{aligned}$$

$$\begin{aligned}&[u^2, r](u^2)^s z[u^2, t] \\ &\quad + z'[u^2, r](u^2)^s[u^2, t] \\ &\quad + [u^2, z](u^2)^s[u^2, r]t' \\ &\quad + [u^2, z]t(u^2)^s[u^2, r] \\ &\quad = 0,\end{aligned} \qquad (24)$$

$$\begin{aligned}&([u^2, r](u^2)^s z \\ &\quad + z'[u^2, r](u^2)^s)[u^2, t] \\ &\quad + [u^2, z]((u^2)^s[u^2, r]t' \\ &\quad + t(u^2)^s[u^2, r]) = 0,\end{aligned}$$

$$\begin{aligned}&[[u^2, r](u^2)^s, z][u^2, t] \\ &+ [u^2, z][(u^2)^r[u^2, s], t] = 0\end{aligned}$$

Since $[u^2,z] \in S$, replace z by $z[u^2,z]$ in equation (24), we get:

$[[u^2, r](u^2)^s,\ z[u^2, z]][u^2, t] + [u^2, z[u^2, z]]\,[(u^2)^r[u^2, s], t\,] = 0.$

By equation (24), we have:

$[u^2, z]'\,[[u^2, r](u^2)^s, z][u^2, t] + [[u^2, r](u^2)^s, z][u^2, z][u^2, t] = 0.$

This means:

$$\big[[u^2, r](u^2)^s, z], [u^2, z]\big][u^2, t] = 0, \qquad (25)$$

for all $s, t, r, z \in S, u \in U$.

Put $t = ct$ in equation (24) and using Jacobi identities, we obtain:

$$\big[[u^2r](u^2)^s, z], [u^2, z]\big][u^2, ct] = 0$$

This means:

$$\begin{aligned}&\big[[u^2, r](u^2)^s, z], [u^2, z]\big]c[u^2, t] \\ &+ \Big[\big[[u^2, r](u^2)^s, z\big], [u^2, z]\Big]\,[u^2c]t = 0\end{aligned}$$

$\Rightarrow \left[\left[[u^2,r](u^2)^s,z\right],[u^2,z]\right] c[u^2,t]=0$

$\Rightarrow \left[\left[[u^2,r](u^2)^s,z\right],[u^2,z]\, s[u^2,t]\right]=0$

Since S is prime, so, either:

$$[u^2,t]=0, \text{for all } t\in S,$$

or,

$$\left[\left[[u^2,r](u^2)^s,z\right],[u^2,z]\right]=0.$$

So, if $[u^2,t]=0$, then $u^2\in Z(S)$.

So, by Lemma (3.11), we have:

$(u^2)^r=0$, for all $u\in U$ and $r\in S$.

If,

$\left[\left[[u^2,r](u^2)^s,z\right],[u^2,z]\right]=0,$ for all $r,s,z\in S, u\in U$.

(i.e.),

$$\left[[u^2,r]\,(u^2)^s,z\right][u^2,z] + [u^2,z]'\left[[u^2,r](u^2)^s,z\right]=0\,.$$

(i.e.),

$$\left[[u^2,r](u^2)^s,z\right][u^2,z] = [u^2,z]\left[[u^2,z](u^2)^s,z\right] \qquad (26)$$

Replace t by z in equation (24), we obtain:

$$\left[[u^2,r](u^2)^s,z\right][u^2,z] + [u^2,z][(u^2)^r[u^2,s],z]=0.$$

By (26), we obtain:

$$[u^2,z][(u^2)^r[u^2,s],z] + [u^2,z]\left[[u^2,r](u^2)^s,z\right]=0.$$

$$\Rightarrow [u^2,z]\left[(u^2)^r[u^2,s]\right]+\left[[u^2,r](u^2)^s,z\right] =0,$$

and by (22), we get:

$$[u^2,z][(u^2)^r[u^2,s]+[u^2,s]'\ (u^2)^r,z]=0.$$

$[u^2,z]\left[\left[(u^2)^r,[u^2,s]\right],z\right]=0,$ for all $r,s,z\in S,$ and for $u\in U$. By linearizing on z, we obtain:

$$[u^2,z+t]\left[\left[(u^2)^r,[u^2,s]\right],z+t\right][u^2,z]\left[\left[(u^2)^r,[u^2,s]\right],t\right] + [u^2,t]\left[\left[(u^2)^r,[u^2,s]\right],z\right] = 0, \text{for all } r,s,t,z\in S, u\in U \qquad (27)$$

Put $t=ut^2$ in equation (27), we obtain:

$$[u^2,z]\left[\left[(u^2)^r,[u^2,s]\right],u^2t\right]+ [[u^2,u^2t][(u^2)^r,[u^2,s],z]=0$$

$$\Rightarrow [u^2,z](u^2\left[\left[(u^2)^r,[u^2,s]\right],t\right]+ \left[[(u^2)^r,[u^2,s],u^2]t\right)+ ([u^2,u^2]t+u^2[u^2,t])\left[\left[(u^2)^r,[u^2,s]\right],z\right]=0.$$

Now since $[u^2,u^2]=0$ then the third term is zero, and in view of equation (27) the second term is (0), so, the last equation will be:

$$u^2[u^2,t]\left[\left[(u^2)^r,[u^2,s]\right],z\right]+ [u^2,z]u^2\left[\left[(u^2)^r,[u^2,s]\right],t\right]=0$$

In view of (27), we have: $u^2[u^2,z]'\left[\left[(u^2)^r,[u^2,s]\right],t\right]+ [u^2,z]u^2\left[\left[(u^2)^r,[u^2,s]\right],t\right]=0$

This means:

$\left[[u^2,z],u^2\right]\left[[(u^2)^r,[u^2,s],t]\right]=0,$ for all $r,s,t,z\in S, u\in U$.

Then:

$\left[[u^2,z],u^2\right]S\left[\left[(u^2)^r,[u^2,s]\right],t\right]=0.$

Since S is prime inverse semiring, then either:

$\left[[u^2,z],u^2\right]=0$ for all $u\in U, r\in S$,

or,

$\left[\left[(u^2)^r,[u^2,s]\right],t\right]=0,$ for all $r,s,t\in S, u\in U$

If $\left[[u^2,z],u^2\right]=0,$ or $\left[u^2,[u^2,z]\right]=0,$ then by Theorem (3.1) in [7], we obtain:

$$u^2\in Z(S),$$

thus,

$(u^2)^r=0$ for all $,u\in U, r\in S$.

If, $\left[\left[(u^2)^r,[u^2,s]\right],t\right]=0,$ for all $r,s,t\in S, u\in U$. Then:

$$\left[(u^2)^r,[u^2,s]\right]\in Z(S),$$

this means:

$(u^2)^r[u^2,s]+[u^2,s]'(u^2)^r \in Z(S)$.

Put

$\rho=(u^2)^r[u^2,s]$ and $\lambda=[u^2,s](u^2)^r$.

Now, trivially we get:

$\rho=0,$ and $\lambda=0$.

So,

$(\rho+\lambda')^3=\lambda\rho'\lambda+\rho\lambda'\rho.$

Now, since $\left[(u^2)^r,[u^2,s]\right]\in Z(S)$, then,

$[u^2,s]\left[(u^2)^r,[u^2,s]\right]=\left[(u^2)^r,[u^2,s]\right][u^2,s].$

By expanding, and using Corollary (3.13), Lemma (3.12), itself, we obtain:

$$[u^2,s]'\,[u^2,s](u^2)^r = (u^2)^r[u^2,s][u^2,s] \quad (28)$$

and so, since $\left[(u^2)^r,[u^2,s]\right] \in Z(S)$, we obtain:

$(u^2)^r\left[(u^2)^r,[u^2,s]\right] = \left[(u^2)^r,[u^2,s]\right](u^2)^r$.

Again by expanding and using Lemma (3.12), Corollary (3.13), we get:

$$(u^2)^r(u^2)^r[u^2,s] = [u^2,s]'^{(u^2)^r(u^2)^r} \quad (29)$$

Now,

$$\rho\lambda = ((u^2)^r[u^2,s])([u^2,s](u^2)^r).$$

By (27), we have

$\rho\lambda = [u^2,s]'[u^2,s](u^2)^r(u^2)^r$,

and from (28), we have:

$$\rho\lambda = [u^2,s]'^{(u^2)^r(u^2)^r}[u^2,s]' = [u^2,s](u^2)^r(u^2)^r[u^2,s] = \lambda\rho.$$

So,

$(\rho+\lambda')^3 = \lambda\rho'\lambda + \rho\lambda'\rho = \lambda'\rho\rho + \rho'\lambda\lambda = 0$.

Now, since S is prime and $\rho+\lambda' \in Z(S)$, then by Lemma (3.7) in [7], the center of S has no non-zero nilpotent element. Then,

$$\rho+\lambda' = 0.$$

This means,

$$\left[(u^2)^r,[u^2,s]\right] = 0, \text{ for all } r,s \in S, u \in U. \quad (30)$$

Put $s = st$ in equation (30), we get:

$[(u^2)^r, s[u^2,t] + [u^2,s]t] = 0$, by Jacobie identities we have:

$s\left[(u^2)^r,[u^2,t]\right] + [(u^2)^r,s][u^2,t] + \left[(u^2)^r,[u^2,s]\right]t + [u^2,s]\,[(u^2)^r,t] = 0$

In view of equation (30), the first and third terms of above are (0) so, we obtain:

$[(u^2)^r,s][u^2,t] + [u^2,s][(u^2)^r,t] = 0$, for all $r,s,t \in S, u \in U$.

Replace $s = [u^2,s]$ in last equation, we have: $\left[(u^2)^r,[u^2,s]\right][u^2,t] + \left[u^2,[u^2,s]\right][(u^2)^r,t] = 0$.

Again by (30), we obtain: $\left[u^2,[u^2,s]\right][(u^2)^r,t] = 0$, for all $r,s,t \in S, u \in U$.

So,

$\left[u^2,[u^2,s]\right]S\,[(u^2)^r,t] = 0$.

Since S is prime inverse semiring, then either:

$$\left[u^2,[u^2,s]\right] = 0,$$

or,

$$[(u^2)^r,t] = 0.$$

If,

$\left[u^2,[u^2,s]\right] = 0$ for all $s \in S,\ u \in U$,

then by Theorem (3.1) in [7]:

$u^2 \in Z(S)$, $(u^2)^r = 0$, for all $r \in S, u \in U$.

If, $[(u^2)^r,t] = 0$, for all $t \in S$. This means, $(u^2)^r \in Z(S)$.

Since by Lemma (3.12), $(u^2)^r[u^2,r] = 0$, for all $r \in S, u \in U$. Thus, if for some r and u, $(u^2)^r \neq 0$, since S is prime inverse semiring, so:

$[u^2,r] = 0$, then $u^2 \in Z(S)$.

$\Longrightarrow (u^2)^r = 0$.

Thus, for all $r \in S, u \in U, (u^2)^r = 0$.

Finally we can prove the following main theorem.

Theorem (3.15):

Let S be a 2-torsion free prime inverse semiring, U be a Jordan ideal of S and $T: S \to S$ be an additive mapping such that:

$$T(ur+ru) + u'T(r) + T(r)u' = 0, \quad (31)$$

for all $u \in U$ *and* $r \in S$. Then:

$T(ur) + u'T(r) = 0$, for all $u \in U$ and $r \in S$.

Proof:

Put $r = ur$ in equation (30), then:

$$T(uur+uru) + u'T(ur) + T(ur)u' = 0, \Longrightarrow T(u^2r+uru) + u'T(ur) + T(ur)u' = 0 \quad (32)$$

So,

$$T(u^2r+uru) = T(u^2r) + T(uru) = T(u^2r) + uT(r)u \quad (33)$$

But, by Lemma (3.14), we have:

$$(u^2)^r = 0, \text{for all } r \in S, u \in U.$$

Then, $(u^2)^r = 0 = T(u^2r) + u^2T(r)'$.

By Lemma (2.1), we have:

$T(u^2r) = u^2T(r)$.

So, equation (33) becomes:

$$T(u^2r + uru) = u^2T(r)' + uT(r)u' \qquad (34)$$

In view of (32) and (34), we get:
$u^2T(r) + uT(r)u' + u'T(ur) + T(ur)u' = 0,$
$\Rightarrow$
$u(uT(r) + T(ur)' + \big(uT(r) + T(ur)\big)u' = 0,$
$\Rightarrow u(u)^{r'} + (u)^r u' = 0,$ for all $r \in S, u \in U.$
Linearizing the above equation on u, we get:
$(u+v)'(u+v)^r + (u+v)^r\,(u+v)' = 0,$
$\Rightarrow u'u^r + u'v^r + v'u^r + v'v^r + u^r u' + u^r v' + v^r u' + v^r v' = 0,$
$\Rightarrow u'v^r + v'u^r + u^r v' + v^r u' = 0.$
Replace v by $2v^2$ and using Lemma (3.14), we get:
$u'(2v^2)^r + (2v^2)'u^r + u^r(2v^2)' + (2v^2)^r u' = 0$
$\Rightarrow \; 2(v^2)'u^r + u^r(2v^2)' = 0,$ for all $v, u \in U, r \in S.$
Also by Lemma 3.11, we get:
$(u)^r = 0,$ for all $u \in U$ and $r \in S.$
This means:
$T(ur) = uT(r),$ for all $u \in U$ and $r \in S$.

References

[1] Golan, J. S., *The theory of Semirings With Applications in Mathematics and Theoretical Computer Science*, Longman Scientific and Technical, UK, 1992.
[2] Joso Vukman, *Centralizers on semiprime rings*, Comment. Math Univ. Carolinae 42,V.2, 237-245. 2001.
[3] Joso Vukman, *An identity related to centralizers in semiprime rings*, Comment. Math Univ. Carolinae 40 , V.3, 447-456. 1999.
[4] Joso Vukman, *Centralizers on prime and semiprime rings*, Comment. Math. Univ. Carolinae 38,V.2, 231-240. 1997.
[5] Karvellas, *P.H. Inversive semirings*, J.Austral Math Soc. 18: 277-288. 1974.
[6] Ram Awtar, *Jordan structure in semiprime rings*, Can.j.math, 3, VxxvIII, No.5,1067-1072. 1976.
[7] Rawnaq, KH. And Abdulrahman H. Majeed, *On Lie Structure in Semiprime Inverse Semirings*, IRAQ JOURNAL OF SCIENCE, to appear in issue (12), Volume (60) 2019.
[8] Shafiq S. and Aslam, M. and JAVED, M.A. *Centralizer of Inverse Semiring*, General Algebra and Applications 3671-84(2016).
[9] Shafiq, S. and Aslam, M., *On Jordan mapping of inverse semirings*, De Gruyter, Open Math. 15: 1123-1131, 2017.
[10] Shaker. H.A, *Centralizers on prime and semiprimerings*, Baghdad University (Iraq). Msc. Thesis, 2005.