Research Article Open Access

Supra $\widehat{\omega}$-separation axioms

Nadia Kadum Humadi*, Haider Jebur Ali
Department of Mathematics, College of Science, Mustansiriyah University, IRAQ.
*Correspondent author email: m.a.nadia313@gmail.com,

Article Info

Received
30/06/2019

Accepted
02/08/2019

Published
15/01/2020

Abstract

The purpose of this paper is to introduce new types of supra separation axioms by using supra $\widehat{\omega}$-open sets and supra $\hat{\eta}$-open sets in the supra spaces and illustrate the relation between them, and to introduce new forms of supra* continuous functions, perfectly supra* continuous functions and supra* homeomorphism functions.

Keywords: supra $\widehat{\omega}T_0$-space, supra $\widehat{\omega}T_1$-space, supra $\widehat{\omega}T_2$-space, supra* $\widehat{\omega}^*$-homeomorphism function, supra* $\widehat{\omega}^{**}$-homeomorphism function.

الخلاصة

الغرض من هذا البحث هو لتقديم انواع جديدة من بديهيات الفصل الفوقية بأستخدام المجموعات المفتوحة- $\widehat{\omega}$ الفوقية والمجموعات المفتوحة -$\hat{\eta}$ الفوقية في الفضاءات الفوقية وتوضيح العلاقة بينهم, ولتقديم صيغ جديدة من الدوال المستمرة الفوقية*, الدوال المستمرة الفوقية* التامة والدوال التشاكلية الفوقية*.

Introduction

In 1983, Mashhour introduced the concept of supra topology [6]. The supra closure for a subset $\mathcal{W}$ of a supra space X was defined as the intersection of all supra closed subsets of X containing $\mathcal{W}$, while the supra interior of $\mathcal{W}$ defined as the union of all supra open subsets of X contained in $\mathcal{W}$. The researcher in [1], defined the supra compact spaces. Also many researchers wrote about the supra separation axioms, and we introduced in this research definitions of two sets $\hat{\eta}, \widehat{\omega}$ in supra spaces and new forms of supra separation axioms such as supra $\widehat{\omega}T_0$, supra $\widehat{\omega}T_1$, supra $\widehat{\omega}T_2$, supra $\hat{\eta}T_0$, supra $\hat{\eta}T_1$, and supra $\hat{\eta}T_2$, also new forms of supra* continuous functions, perfectly supra* continuous functions and new forms of supra* homeomorphism functions by using supra $\widehat{\omega}$-open and supra $\hat{\eta}$-open sets. We presented some theorems, propositions and remarks and we supported them by examples.

1- Supra $\widehat{\omega}$-Continuous and supra* $\hat{\eta}$-continuous functions.*

We introduced some new types of supra* continuous and perfectly supra*continuous functions by using su. $\hat{\eta}$-open, su. $\widehat{\omega}$-open sets and illustrated the relation between them. We used the abbreviation "su." to refer to "supra".

Definition (1.1) [3]: Let X be a non- empty set and μ be a sub collection of the power set of X, then μ is a supra topology on X if:

1- $\emptyset, X \in \mu$.

2- μ is closed under the arbitrary union, any set $\mathcal{W} \in \mu$ is called supra open set and its complement is supra closed set. The pair (X, μ) is called a supra space.

Definition (1.2) [6]: Let $(X, \text{Ʈ})$ be a topological space, μ is called a supra topology associated with Ʈ if Ʈ$\subset \mu$.

Remark (1.3): Any topology is su. topology, since every topology includes $\emptyset, X$ and it is closed under the infinite union. This remark is irreversible

Example (1.4): In the su. space(X, μ), where X= {1, 2, 3}, μ= {$\emptyset$, X, {1}, {2}, {1, 2}, {2, 3}, {1, 3}}, μ is su. topology on X but not topology since $\{1, 3\} \cap \{2, 3\} = \{3\} \notin \mu$.

Definition (1.5):

1- A subset $\mathcal{W}$ of a su. space (X, μ) is called a su. $\widehat{\omega}$-open set if for any $s \in \mathcal{W}$, there is $V \in \mu$

such that $s \in V$ and V-$\mathcal{W}$ is countable. $\mathcal{W}^c$ is called a su. $\widehat{\omega}$-closed set.

2- A subset $\mathcal{W}$ of a su. space (X, μ) is called a su. ῃ-open set if for any $s \in \mathcal{W}$, there is $V \in \mu$ such that $s \in V$ and V-$\mathcal{W}$ is finite. $\mathcal{W}^c$ is called a su. ῃ-closed set.

3- The su. $\widehat{\omega}$-closure of a subset $\mathcal{W}$ of a su. space (X, μ) is the intersection of all su. $\widehat{\omega}$-closed subsets of X which contain $\mathcal{W}$, and we denote it by $cl^{\mu}_{\widehat{\omega}}(\mathcal{W})$. While the su. $\widehat{\omega}$-interior of $\mathcal{W}$ is the union of all su. $\widehat{\omega}$-open subsets of X which contained in $\mathcal{W}$, and we denote it by $Int^{\mu}_{\widehat{\omega}}(\mathcal{W})$. By the same way we can define su. ῃ-closure for $\mathcal{W}$ (denoted by $cl^{\mu}_{ῃ}(\mathcal{W})$) and su. ῃ-interior for $\mathcal{W}$ (denoted by $Int^{\mu}_{ῃ}(\mathcal{W})$).

Remark (1.6):

1- Any su. open set is su $\widehat{\omega}$-open (resp. su. ῃ open) set.

2- Any su. closed set is su $\widehat{\omega}$-closed (resp. su. ῃ -closed) set.

Definition (1.7): Let $(X, \mu_X), (Y, \mu_Y)$ be a topological spaces and $\mathrm{T}_X \subset \mu_X, \mathrm{T}_Y \subset \mu_Y$. The function $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is called:-

1- Su*. continuous function. If the inverse image of any su. open (resp. su. closed) set in Y is a su. open (resp. su. closed) set in X [6].

2- Su*. $\widehat{\omega}$-continuous function. If the inverse image of any su. open (resp. su. closed) set in Y is a su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in X.

3- Su*. ῃ-continuous function. If the inverse image of any su. open (resp. su. closed) set in Y is a su. ῃ-open (resp. su ῃ-closed) set in X.

4- Strongly su*. $\widehat{\omega}$-continuous function. If the inverse image of any su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in Y is a su. open (resp. su. closed) set in X.

5- Strongly su*. ῃ-continuous function. If the inverse image of any su. ῃ-open (resp. su. ῃ-closed) set in Y is a su. open (resp. su. closed) set in X.

6- Su*. $\widehat{\omega}$-irresolute function. If the inverse image of any su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in Y is a su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in X.

7- Su*. ῃ-irresolute function. If the inverse image of any su. ῃ-open (resp. su. ῃ-closed) set in Y is a su. ῃ-open (resp. su. ῃ-closed) set in X.

8- Perfectly su*. continuous. If the inverse image of any su. open (resp. su. closed) set in Y is a su. clopen set in X [8].

9- Totally su*. $\widehat{\omega}$-continuous. If the inverse image of any su. open (resp. su. closed) set in Y is a su. $\widehat{\omega}$-clopen set in X.

10- Totally su*. ῃ-continuous. If the inverse image of any su. open (resp. su. closed) set in Y is a su. ῃ-clopen set in X.

11- Perfectly su*. $\widehat{\omega}$-continuous. If the inverse image of any su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in Y is a su. clopen set in X.

12- Perfectly su*. ῃ-continuous. If the inverse image of any su. ῃ-open (resp. su. ῃ-closed) set in Y is a su. clopen set in X.

13- Perfectly su*. $\widehat{\omega}$-irresolute. If the inverse image of any su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in Y is a su. $\widehat{\omega}$-clopen set in X.

14- Perfectly su*. ῃ-irresolute. If the inverse image of any su. ῃ-open (resp. su. ῃ-closed) set in Y is a su. ῃ-clopen set X.

Example (1.8): Let X=Y={1, 2, 3}, $\mu_X = \{\emptyset, X, \{1\}, \{3\}, \{1, 3\}, \{2, 3\}, \{1, 2\}\}$and $\mu_Y = \{\emptyset, Y, \{3\}, \{1, 2\}\}$, so $f: X \longrightarrow Y$ defined as $f(1)=2$, $f(2)=1$, $f(3)=3$ is su*. continuous, su*. $\widehat{\omega}$-continuous, su*. ῃ-continuous, su*. $\widehat{\omega}$-irresolute, su*. ῃ-irresolute function, but not strongly su*. $\widehat{\omega}$-continuous and not strongly su*. ῃ-continuous function, since $\{1\}$ is su. $\widehat{\omega}$-open and su. ῃ-open set in Y but $f^{-1}(\{1\}) = \{2\}$ is not su. open set in X. Also, it is perfectly su*. continuous, totally su*. $\widehat{\omega}$-continuous, totally su*. ῃ-continuous, perfectly su*. ῃ-irresolute, perfectly su*.$\widehat{\omega}$-irresolute, but not perfectly su*. ῃ-continuous, and not perfectly su*. $\widehat{\omega}$-
continuous function.

Remark (1.9):

1- Every perfectly su*. continuous function is su*. continuous function.

2- Every totally su*. $\widehat{\omega}$-continuous (resp. totally su*. ῃ-continuous) function is su*. $\widehat{\omega}$-continuous (resp. su*. ῃ-continuous) function.

3- Every perfectly su*. $\widehat{\omega}$-continuous (resp. perfectly su*. ῃ-continuous) function is strongly su*. $\widehat{\omega}$-continuous (resp. strongly su*. ῃ-continuous) function.

4- Every perfectly su*. $\widehat{\omega}$-irresolute (resp. perfectly su*. ῆ-irresolute) function is su*. $\widehat{\omega}$-irresolute (resp. su*. ῆ-irresolute).

Example (1.10): Let $(\mathcal{R}, \daleth_{cof})$ be the co-finite topological space and $\daleth_{cof} \subset \mu_{cof}$, so $I_{\mathcal{R}}: (\mathcal{R}, \mu_{coc}) \longrightarrow (\mathcal{R}, \daleth_{cof})$ is su*. continuous, su*. $\widehat{\omega}$-continuous, su*. ῆ-continuous, strongly su*. $\widehat{\omega}$-continuous, strongly su*. ῆ-continuous, su*. $\widehat{\omega}$-irresolute and su*. ῆ-irresolute function but not totally su*. $\widehat{\omega}$-continuous, not totally su*. ῆ-continuous, not perfectly su*. $\widehat{\omega}$-continuous, not perfectly su*. $\widehat{\omega}$-irresolute, not perfectly su*. ῆ-continuous, not perfectly su*. ῆ-irresolute and not perfectly su*. continuous.

2- Su. separation axioms by using su. $\widehat{\omega}$-open and su. ῆ*-open sets.*

At the beginning we presented definitions of some separation axioms by using su. $\widehat{\omega}$-open and su. ῆ-open sets, and we provided the relation between them, also we connected them with several types of su*.continuous, su*. open and su*. closed functions.

Definition (2.1): The su. space (X, μ) is called:-

1- A su. T_0-space [6], if for each different elements x, y in X, there is $\mathcal{W} \in \mu$ such that $x \in \mathcal{W}, y \notin \mathcal{W}$.

2- A su. $\widehat{\omega}T_0$-space, if for each different elements x, y in X, there is a su. $\widehat{\omega}$-open set $\mathcal{W}$ in X such that $x \in \mathcal{W}, y \notin \mathcal{W}$.

3- A su. ῆT_0-space, if for each different elements x, y in X, there is a su. ῆ-open set $\mathcal{W}$ in X such that $x \in \mathcal{W}, y \notin \mathcal{W}$.

4- A su. T_1-space [6], if for each different elements x, y in X, there are $\mathcal{W}_1, \mathcal{W}_2 \in \mu$ with $x \in \mathcal{W}_1, y \notin \mathcal{W}_1$ and $y \in \mathcal{W}_2, x \notin \mathcal{W}_2$.

5- A su. $\widehat{\omega}T_1$-space, if for each different elements x, y in X, there are su. $\widehat{\omega}$-open sets $\mathcal{W}_1, \mathcal{W}_2$ in X with $x \in \mathcal{W}_1, y \notin \mathcal{W}_1$ and $y \in \mathcal{W}_2, x \notin \mathcal{W}_2$.

6- A su. ῆT_1-space, if for each different elements x, y in X, there are su. ῆ-open sets $\mathcal{W}_1, \mathcal{W}_2$ with $x \in \mathcal{W}_1, y \notin \mathcal{W}_1$ and $y \in \mathcal{W}_2, x \notin \mathcal{W}_2$.

7- A su. T_2-space [6], if for each different elements x, y in X, there are disjoint $\mathcal{W}_1, \mathcal{W}_2 \in \mu$ with $x \in \mathcal{W}_1$ and $y \in \mathcal{W}_2$.

8- A su. $\widehat{\omega}T_2$-space, if for each different elements x, y in X, there are disjoint su. $\widehat{\omega}$-open sets $\mathcal{W}_1, \mathcal{W}_2$ in X with $x \in \mathcal{W}_1$ and $y \in \mathcal{W}_2$.

9- A su. ῆT_2-space, if for each different elements x, y in X, there are disjoint su. ῆ-open sets $\mathcal{W}_1, \mathcal{W}_2$ with $x \in \mathcal{W}_1$ and $y \in \mathcal{W}_2$.

Example (2.2): 1- Let X= {1, 2, 3} and μ_X= {$\emptyset$, X, {1, 2}, {2, 3}, {1, 3}}, so (X, μ_X) is su. T_0-space, su. $\widehat{\omega}T_0$-space and su. ῆT_0-space, su. T_1-space, su. $\widehat{\omega}T_1$-space, su. ῆT_1-space, su. $\widehat{\omega}T_2$-space, su. ῆT_2-space, but not T_2-space.

2- Let X= {1, 2, 3} and μ_X= {$\emptyset$, X, {1}, {2}, {1, 2}, {1, 3}, {2, 3}}, so (X, μ_X) is su. T_2-space, su. $\widehat{\omega}T_2$-space, su. ῆT_2-space.

Remark (2.3): Suppose X is a su. space, then, if X is:-

1- Su. T_i-space, then it is su. $\widehat{\omega}T_i$-space and su. ῆT_i-space, i=0, 1, 2.

2- Su. ῆT_i-space, then it is su. $\widehat{\omega}T_i$-space, i=0, 1, 2.

3- Su. $\widehat{\omega}T_i$-space, then it is su.$\widehat{\omega}T_{i-1}$-space, i=1, 2.

4- Su. ῆT_i-space, then it is su. ῆT_{i-1}-space, i=1, 2.

5- Su. $\widehat{\omega}T_2$-space (resp. ῆT_2-space), then it is su. $\widehat{\omega}T_0$-space (resp. ῆT_0-space).

Example (2.4):

1. $(\mathcal{Z}, \mu_{ind})$ is su. $\widehat{\omega}T_0$-space, su. ῆT_0-space also su. $\widehat{\omega}T_2$-space but not su. T_0-space, not su. T_2-space and not su. ῆT_2-space.

2- Let X= {1, 2, 3} and μ_X= {$\emptyset$, X, {1, 2}, {2, 3}}, so (X, μ_X) is su. $\widehat{\omega}T_1$-space, su. ῆT_1-space, but not su. T_1-space.

3- $(\mathcal{R}, \mu_{cof})$ is su. $\widehat{\omega}T_1$-space and su. $\widehat{\omega}T_0$-space, but it is not su. $\widehat{\omega}T_2$-space.

Proposition (2.5): If $\mathcal{W}_i,\ i \in I$ is u. $\widehat{\omega}$- open (resp. su. ῆ-open) subsets of a su. space (X, μ_X) then $\bigcup_{i \in I} \mathcal{W}_i$ is a su. $\widehat{\omega}$-open (resp. su. ῆ-open) subset of (X, μ_X).

Proof: Suppose $e \in \bigcup_{i \in I} \mathcal{W}_i \Longrightarrow e \in \mathcal{W}_{\alpha_i}$, for some $\alpha_i \in I$, thus there is $G \in \mu_X$ containing e

and G-$\mathcal{W}_{\alpha_i}$ is countable (resp. finite, but G-$\bigcup_{i\in I}\mathcal{W}_i \subseteq G$-$\mathcal{W}_{\alpha_i}$ (since $\mathcal{W}_{\alpha_i} \subseteq \bigcup_{i\in I}\mathcal{W}_i \Longrightarrow X$- $\bigcup_{i\in I}\mathcal{W}_i \subseteq X$-$\mathcal{W}_{\alpha_i} \Longrightarrow$ $G \cap (X$-$\bigcup_{i\in I}\mathcal{W}_i) \subseteq G \cap (X$-$\mathcal{W}_{\alpha_i}) \Longrightarrow G$-$\bigcup_{i\in I}\mathcal{W}_i \subseteq G$-$\mathcal{W}_{\alpha_i}$), hence G- $\bigcup_{i\in I}\mathcal{W}_i$ is a countable (resp. a finite) set (because G-$\mathcal{W}_{\alpha_i}$ is a countable (resp. finite) set and any subset of countable (resp. finite) set is countable (resp. finite)). Therefore $\bigcup_{i\in I}\mathcal{W}_i$ is a su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set.

Definition (2.6): Suppose H is a subset of a su. space X, whenever for any element $x \in H$ there is a su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) subset U of X containing x and $U \subseteq H$, then x is a su. $\widehat{\omega}$-interior (resp. su. $\hat{\eta}$-interior) point to H.

Proposition (2.7):

1- Consider X as a su. space and H as a subset of X, then H is a su. $\widehat{\omega}$-open set if $H = Int^{\mu}_{\widehat{\omega}}(H)$.

2- Consider X a su. space and H as a subset of X, then H is a su. $\hat{\eta}$-open set iff $H = Int^{\mu}_{\hat{\eta}}(H)$.

Proof: Let H be a su. $\widehat{\omega}$-open set, since $Int^{\mu}_{\widehat{\omega}}(H)$ is the largest su. $\widehat{\omega}$-open set in X contained in H), so $Int^{\mu}_{\widehat{\omega}}(H) \subseteq H$, now to prove $H \subseteq Int^{\mu}_{\widehat{\omega}}(H)$. Let $x \in H \subseteq H$ and since H is su. $\widehat{\omega}$-open set, so $x \in Int^{\mu}_{\widehat{\omega}}(H)$, and since x is arbitrary point in H, so each point in H is su $\widehat{\omega}$-interior point, but $\bigcup_{x\in H}\{x\} = H$, hence $H \subseteq Int^{\mu}_{\widehat{\omega}}(H)$, therefore $Int^{\mu}_{\widehat{\omega}}(H) = H$. Conversely, if $Int^{\mu}_{\widehat{\omega}}(H) = H$, and since $Int^{\mu}_{\widehat{\omega}}(H)$ is su. $\widehat{\omega}$-open set, therefore H is a su. $\widehat{\omega}$-open set.

Definition (2.8) [2]: Whenever (X, μ_X) is a su. space and (Y, μ_Y) is a su. sub space of X, then $\mathcal{W} \in \mu_Y$ iff $\mathcal{W} = U \cap Y$ in which $U \in \mu_X$.

Proposition (2.9): In case $\mathcal{W}$ is a su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set in a su. space (X, μ), so $\mathcal{W} \cap Y$ is a su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set in (Y, μ_Y) whenever Y is a su. sub space of X.

Proof: Consider $x \in \mathcal{W} \cap Y \Longrightarrow x \in \mathcal{W}$ and $x \in Y$, so there is $G \in \mu_X$, with $x \in G$ and G-$\mathcal{W}$ is countable (resp. finite), but $(G$-$\mathcal{W}) \cap Y \subseteq (G$-$\mathcal{W}) \Longrightarrow (G$-$\mathcal{W}) \cap Y$ is countable (resp. finite), and $(G$-$\mathcal{W}) \cap Y = (G \cap Y)$-$(\mathcal{W} \cap Y)$ is countable (resp. finite), where $G \cap Y$ is a su. open set in Y (from definition (2.8)), which implies $\mathcal{W} \cap Y$ is a su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set in Y.

Definition (2.10): The function $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is called:-

1- Su*. closed function, if $f(V)$ is su. closed set in Y, for any su. closed set V in X [9].

2- Su*. open function, if $f(V)$ is su. open set in Y, for any su. open set V in X [5].

3- Su*. $\widehat{\omega}$-closed (resp. su*. $\widehat{\omega}$-open) function, if $f(V)$ is su. $\widehat{\omega}$-closed (resp. su. $\widehat{\omega}$-open) set in Y, for any su. closed (resp. su. open) set V in X.

4- Totally su*. $\widehat{\omega}$-closed (resp. totally su*.$\widehat{\omega}$ -open) function, if $f(V)$ is su. closed (resp. su. open) set in Y, for any su. $\widehat{\omega}$-closed (resp. su. $\widehat{\omega}$-open) set V in X.

5- Strongly su*. $\widehat{\omega}$-closed (resp. strongly su*. $\widehat{\omega}$-open) function, if $f(V)$ is su. $\widehat{\omega}$-closed (resp. su. $\widehat{\omega}$-open) set in Y, for any su. $\widehat{\omega}$-closed (resp. su. $\widehat{\omega}$-open) set V in X.

6- Su*. $\hat{\eta}$-closed (resp. su*. $\hat{\eta}$-open) function, if $f(V)$ is su. $\hat{\eta}$-closed (resp. su. $\hat{\eta}$-open) set in Y, for any su. closed (resp. su. open) set V in X.

7- Totally su*. $\hat{\eta}$-closed (resp. totally su*. $\hat{\eta}$-open) function, if $f(V)$ is su. closed (resp. su. open) set in Y, for any su. $\hat{\eta}$-closed (resp. su. $\hat{\eta}$-open) set V in X.

8- Strongly su*. $\hat{\eta}$-closed (resp. strongly su*. $\hat{\eta}$-open) function, if $f(V)$ is su. $\hat{\eta}$-closed (resp. su. $\hat{\eta}$-open) set in Y, for any su. $\hat{\eta}$-closed (resp. su. $\hat{\eta}$-open) set V in X.

Example (2.11):

1- X= {1, 2} and μ_X= {$\emptyset$, X, {1}}, and Y={1, 2, 3}, μ_Y={$\emptyset$, Y, {1}, {3}, {1,2}, {1,3}, {2,3}} so $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$ such that $f(a) = a$ for any $a \in X$, is su*. closed and su*. open, su*. $\widehat{\omega}$-closed, su*. $\widehat{\omega}$-open, strongly su*. $\widehat{\omega}$-open and strongly su*. $\widehat{\omega}$-closed, su*. $\hat{\eta}$-closed, su*. $\hat{\eta}$-open, strongly su*. $\hat{\eta}$-open, strongly su*. $\hat{\eta}$-closed, totally su*. $\widehat{\omega}$-closed, and totally su*. $\hat{\eta}$-closed function, but neither totally su*. $\hat{\eta}$-open nor totally su*. $\widehat{\omega}$-open function.

2- A function $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$, where X= {1, 2}, μ_X= {$\emptyset$, X, {1}}, Y= {1, 2, 3, 4} and μ_Y={$\emptyset$, Y, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 4}, {2, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {3, 4}}such that $f(1)$=1 and $f(2)$=2, then f satisfies all the definitions in (2.10).

Theorem (2.12): Su. $\widehat{\omega}T_i$-space (resp. su. ŋ̂T_i), i=0, 1, 2 is a hereditary property and a topological property.

Proof: Take Y as a su. sub space of a su. space X and x, y as distinct points in Y, hence x, y are distinct points in X which is a su. $\widehat{\omega}T_0$-space (resp. ŋ̂T_0-space), so there exists a su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) subset $\mathcal{W}$ of X, with $x \in \mathcal{W}, y \notin \mathcal{W}$. We have $\mathcal{W} \cap Y$ is a su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) subset of Y (proposition (2.9)) with $x \in \mathcal{W} \cap Y, y \notin \mathcal{W} \cap Y$ (because $x \in \mathcal{W}$ and $x \in Y$ but $y \notin \mathcal{W}$). Therefore Y is a su. $\widehat{\omega}T_0$-space (resp. su. ŋ̂T_0-space). Which means su. $\widehat{\omega}T_0$-space (resp. su. ŋ̂T_0-space) is a hereditary property. Now to prove su. $\widehat{\omega}T_0$-space (resp. ŋ̂T_0-space) is a topological property. Suppose $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is a surjective, strongly su*. $\widehat{\omega}$-open (resp. strongly su*. ŋ̂-open) function, in which X is a su. $\widehat{\omega}T_0$-space (resp. su. ŋ̂T_0-space) and y_1, y_2 are different points in Y, then there are different points x_1, x_2 in X with $f(x_1) = y_1$, $f(x_2) = y_2$ (since f is a surjective function), so there exists a su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) subset $\mathcal{W}$ of X with $x_1 \in \mathcal{W}$ and $x_2 \notin \mathcal{W}$ (because X is a su. $\widehat{\omega}T_0$-space (resp. su. ŋ̂T_0-space) where $f(x_1) = y_1 \in f(\mathcal{W})$ and $f(x_2) = y_2 \notin f(\mathcal{W})$, in which $f(\mathcal{W})$ is a su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) subset of Y (because f is strongly su*. $\widehat{\omega}$-open (resp. strongly su*. ŋ̂-open) function, therefore Y is a su. $\widehat{\omega}T_0$-space (resp. su. ŋ̂T_0-space). Which means the su. $\widehat{\omega}T_0$-space (resp. su. ŋ̂T_0-space) is a topological property. By the same way we can prove the rest properties.

Theorem (2.13): If $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is injective function, then the su. space (X, μ_X) is:-

1- A su. T_0-space, whenever Y is su. T_0-space and f is perfectly su*. continuous function.

2. A su. $\widehat{\omega}T_0$-space, whenever Y is su. T_0-space and f is perfectly su*. continuous function.

3- A su. ŋ̂T_0-space, whenever Y is su. T_0-space and f is perfectly su*. continuous function.

4- A su. $\widehat{\omega}T_0$-space, whenever Y is su. T_0-space and f is totally su*. $\widehat{\omega}$-continuous function.

5- A su. ŋ̂T_0-space, whenever Y is su. T_0-space and f is totally su*. ŋ̂-continuous function.

6- A su. $\widehat{\omega}T_0$-space, whenever Y is su. T_0-space and f is perfectly su*. $\widehat{\omega}$-continuous function.

7- A su. ŋ̂T_0-space, whenever Y is su. T_0-space and f is perfectly su*. ŋ̂-continuous function.

8- A su. $\widehat{\omega}T_0$-space, whenever Y is su. $\widehat{\omega}T_0$-space and f is perfectly su*. $\widehat{\omega}$-continuous function.

9- A su. ŋ̂T_0-space, whenever Y is su. ŋ̂T_0-space and f is perfectly su*. ŋ̂-continuous function.

10- A su. T_0-space, whenever Y is su. $\widehat{\omega}T_0$-space and f is perfectly su*. $\widehat{\omega}$-continuous function.

11- A su. T_0-space, whenever Y is su. ŋ̂T_0-space and f is perfectly su*. ŋ̂-continuous function.

12- A su. $\widehat{\omega}T_0$-space, whenever Y is su. T_0-space and f is perfectly su*. $\widehat{\omega}$-irresolute function.

13- A su. ŋ̂T_0-space, whenever Y is su. T_0-space and f is perfectly su*. ŋ̂-irresolute function.

14- A su. $\widehat{\omega}T_0$-space, whenever Y is su. $\widehat{\omega}T_0$-space and f is perfectly su*. $\widehat{\omega}$-irresolute function.

15- A su. ŋ̂T_0-space, whenever Y is su. ŋ̂T_0-space and f is perfectly su*. ŋ̂-irresolute function.

Proof: 1- Consider $x_1 \neq x_2$ are any points in X, since f is injective, so $f(x_1) \neq f(x_2)$ in Y which is su. T_0-space. Then there is $U \in \mu_Y$ in which $f(x_1) \in U$ and $f(x_2) \notin U$, hence U^c is su. closed subset of Y, therefore $f^{-1}(U^C) = \left(f^{-1}(U)\right)^C$ is su. clopen subset of X (because f is perfectly su*. continuous), hence $f^{-1}(U)$ is su. open subset of X where $f^{-1}\big(f(x_1)\big) = x_1 \in f^{-1}(U)$ and $f^{-1}\big(f(x_2)\big) = x_2 \notin f^{-1}(U)$, therefore X is su. T_0-space. We can prove the other properties by the same way.

Hint: The previous theorem is true when we replace each T_0 by T_1or T_2.

3- Su. $\widehat{\omega}$-compact and su. ŋ̂-compact spaces.

In this part we submitted definitions of new types of su. compact spaces which are su. $\widehat{\omega}$ -

compact and su. ŋ̂-compact spaces, and new types of su*. Homeomorphism functions.

Definition (3.1): 1- A class ç= $\{\mathcal{W}_\alpha |\ \alpha \in\wedge\}$ of su. $\widehat{\omega}$-open subsets $\mathcal{W}_\alpha$ of (X,μ) is a su. $\widehat{\omega}$-open cover to a subset S of X whenever $\subseteq \bigcup_{\alpha\in\wedge} \mathcal{W}_\alpha$, when $S = X$, then $\{\mathcal{W}_\alpha \mid \alpha \in\wedge\}$ is a su. $\widehat{\omega}$-open cover to X. If $\mathcal{W}_\alpha$ is su. ŋ̂-open set, then ç is called a su. ŋ̂-open cover to S.

2- A subset S of (X,μ) is a su. $\widehat{\omega}$-compact if any su. $\widehat{\omega}$-open cover for S possesses a finite sub cover, when $X = S$, then X is a su. $\widehat{\omega}$-compact space.

3- A subset S of (X,μ) is a su. ŋ̂-compact if any su. ŋ̂-open cover for S possesses a finite sub cover, when $X = S$, then X is a su. ŋ̂-compact space.

Example (3.2): 1- Let X= {1, 2, 3}, μ_X= {$\emptyset$,X, {1, 2}, {1, 3}} is su. compact, su. $\widehat{\omega}$-compact and su. ŋ̂-compact space.

Remark (3.3): 1- If X is a su. $\widehat{\omega}$-compact space, then it is a su. compact.

2- If X is a su. ŋ̂-compact space, then it is a su. compact.

Theorem (3.4) [4]: A su. closed subset of a su. compact space is a su. compact.

Proposition (3.5): 1- A su. $\widehat{\omega}$-closed subset $\mathcal{M}$ of a su. $\widehat{\omega}$-compact space (X,μ_X), is a su. $\widehat{\omega}$-compact set.

2- A su. ŋ̂-closed subset $\mathcal{M}$ of a su. ŋ̂-compact space (X,μ_X), is a su. ŋ̂-compact set.

Proof: 1- Consider (X,μ_X) is a su. $\widehat{\omega}$-compact space and $\mathcal{M}$ is a su. $\widehat{\omega}$-closed set in X, let $\{\mathcal{W}_\alpha\}_{\alpha\in\wedge}$ be a su. $\widehat{\omega}$-open cover for $\mathcal{M} \Longrightarrow \mathcal{M} \subseteq \bigcup_{\alpha\in\wedge} \mathcal{W}_\alpha$, but $X = \mathcal{M} \cup \mathcal{M}^C \Longrightarrow X \subseteq \{\bigcup_{\alpha\in\wedge} \mathcal{W}_\alpha\} \cup \mathcal{M}^C$ and since $\mathcal{M}$ is su. $\widehat{\omega}$-closed set in X, then $\mathcal{M}^C$ is su. $\widehat{\omega}$-open, this means $\{\mathcal{W}_\alpha | \alpha \in\wedge, \mathcal{M}^C\}$ is a su. $\widehat{\omega}$-open cover for X, but X is a su. $\widehat{\omega}$-compact, so any su. $\widehat{\omega}$-open cover for X possesses a finite sub cover, hence $X \subseteq \left(\bigcup_{i=1}^{n} \mathcal{W}_{\alpha_i}\right) \cup \mathcal{M}^C$, but $\mathcal{M} \subseteq X \Longrightarrow \mathcal{M} \subseteq \left(\bigcup_{i=1}^{n} \mathcal{W}_{\alpha_i}\right) \cup \mathcal{M}^C$, since $\mathcal{M} \cap \mathcal{M}^C = \emptyset \Longrightarrow \mathcal{M} \subseteq \bigcup_{i=1}^{n} \mathcal{W}_{\alpha_i}$, then $\{\mathcal{W}_{\alpha_i}\}_{i=1}^{n}$ is a finite sub cover of the su. $\widehat{\omega}$-open cover $\{\mathcal{W}_\alpha\}_{\alpha\in\wedge}$ for $\mathcal{M}$, therefore $\mathcal{M}$ is a su. $\widehat{\omega}$-compact.

Theorem (3.6) [7]: The continuous image of su. compact space is su. compact.

Proposition (3.7): If the function $f: (X,\mu_X) \longrightarrow (Y,\mu_Y)$ is:-

1- Su*. $\widehat{\omega}$-continuous, so the image of each su. $\widehat{\omega}$-compact set in X is a su. compact set in Y.

2- Strongly su*. $\widehat{\omega}$-continuous, so the image of each su. compact set in X is a su. $\widehat{\omega}$-compact set in Y.

3- Su*. $\widehat{\omega}$-irresolute, so the image of each su. $\widehat{\omega}$-compact set in X is a su. $\widehat{\omega}$-compact set in Y.

4- Su*. ŋ̂-continuous, so the image of each su. ŋ̂-compact set in X is a su. compact set in Y.

5- Strongly su*. ŋ̂-continuous, so the image of each su. compact set in X is a su. ŋ̂-compact set in Y.

6- Su*. ŋ̂-irresolute, so the image of each su. ŋ̂-compact set in X is a su. ŋ̂-compact set in Y.

Proof: Let f be a su*. $\widehat{\omega}$-continuous function and $\mathcal{W}$ be a su. $\widehat{\omega}$-compact set in X. Take $\{V_\alpha\}_{\alpha\in\wedge}$ to be a su. open cover to $f(\mathcal{W})$, where each $V_\alpha \in \mu_Y, \alpha \in\wedge$., then $f(\mathcal{W}) \subseteq \bigcup_{\alpha\in\wedge} V_\alpha$, but f is su*. $\widehat{\omega}$-continuous, hence $\mathcal{W} \subseteq f^{-1}\left(f(\mathcal{W})\right) \subseteq f^{-1}\left(\bigcup_{\alpha\in\wedge} V_\alpha\right) = \bigcup_{\alpha\in\wedge}(f^{-1}(V_\alpha))$, then $\{f^{-1}(V_\alpha)\}_{\alpha\in\wedge}$ is a su. $\widehat{\omega}$-open cover for $\mathcal{W}$, since $\mathcal{W}$ is su. $\widehat{\omega}$-compact, so each su. $\widehat{\omega}$-open cover to it possesses a finite sub cover, hence $\mathcal{W} \subseteq \bigcup_{i=1}^{n} f^{-1}\left(V_{\alpha_i}\right)$ and by take the image of both sides we get $f(\mathcal{W}) \subseteq f\left(\bigcup_{i=1}^{n} f^{-1}\left(V_{\alpha_i}\right)\right) = \bigcup_{i=1}^{n} f(f^{-1}(V_{\alpha_i})) \subseteq \bigcup_{i=1}^{n} V_{\alpha_i} \Longrightarrow f(\mathcal{W}) \subseteq \bigcup_{i=1}^{n} V_{\alpha_i}$, which means that $\{V_{\alpha_i}\}_{i=1}^{n}$ is a finite sub cover for the su. open cover $\{V_\alpha\}_{\alpha\in\wedge}$, so $f(\mathcal{W})$ is a su. compact subset of Y. The rest of the possibilities can be proved by the same way.

Theorem (3.8): Let $f: (X,\mu_X) \longrightarrow (Y,\mu_Y)$ be a surjective function, if:-

1- X is su. T_1-space and f is su*. open function, then Y is su. T_1-space.

2- X is su. T_1-space and f is su*. open function, then Y is su. $\widehat{\omega}T_1$-space.

3- X is su. T_1-space and f is su*. open function, then Y is su. ŋ̂T_1-space.

4- X is su. $\widehat{\omega}T_1$-space and f is totally su*. $\widehat{\omega}$-open function, then Y is su. T_1-space.

5- X is su. ŋ̂T_1-space and f is totally su*. ŋ̂-open function, then Y is su. T_1-space.

6- X is su. $\widehat{\omega}T_1$-space and f is totally su*. $\widehat{\omega}$-open function, then Y is su. $\widehat{\omega}T_1$-space.

7- X is su. ῆT_1- space and f is totally su*. ῆ-open function, then Y is su. ῆT_1-space.
8- X is su. $\widehat{\omega}T_1$-space and f is strongly su*. $\widehat{\omega}$-open function, then Y is $\widehat{\omega}T_1$- space.
9- X is su. ῆT_1-space and f is strongly su*. ῆ-open function, then Y is ῆ T_1-space.
10- X is su. T_1-space and f is strongly su*. $\widehat{\omega}$-open function, then Y is $\widehat{\omega}T_1$- space.
11- X is su. T_1-space and f is strongly su*. ῆ-open function, then Y is ῆT_1-space.

Proof: Suppose a, b be distinct two points in Y. So there are two distinct points x, y in X in which $f(x) = a, f(y) = b$ (because f is onto and by definition of functions), since X is su. T_1-space, so there are two su. open sets $\mathcal{W}, \mathcal{B}$ in X with $x \in \mathcal{W},\ y \notin \mathcal{W}$ and $y \in \mathcal{B},\ x \notin \mathcal{B}$, hence, $f(x) = a \in f(\mathcal{W}),\ f(y) = b \notin f(\mathcal{W})$ and $f(y) = b \in f(\mathcal{B}), f(x) = a \notin f(\mathcal{B})$ where $f(\mathcal{W}), f(\mathcal{B})$ are su. open sets in Y (since f is su*. open function), then Y is su. T_1-space.

Hint: Theorem (3.8) remains true when we replace T_1 by T_0 or T_2, also it is true if we use the types of su*. closed function instead of the types of su*. open function.

Definition (3.9): A bijective function f from a su. space X into a su. space Y is.
1- Su*. homeomorphism function, if f and f^{-1} are su*. continuous [5].
2- Su*. $\widehat{\omega}$-homeomorphism function, if f and f^{-1} are su*. $\widehat{\omega}$-continuous.
3- Su*. ῆ-homeomorphism function, if f and f^{-1} are su*. ῆ-continuous.
4- Su*. $\widehat{\omega}^*$-homeomorphism function, if f and f^{-1} are su*. $\widehat{\omega}$-irresolute.
5- Su*. ῆ*-homeomorphism function, if f and f^{-1} are su*. ῆ-irresolute.
6- Su*. $\widehat{\omega}^{**}$-homeomorphism function, if f and f^{-1} are strongly su*. $\widehat{\omega}$-continuous.
7- Su. ῆ**-homeomorphism function, if f and f^{-1} are strongly su*. ῆ-continuous.

Definition (3.10): A bijective function f from a su. space X into a su. space Y is:-
1- Su*. homeomorphism function, if it is su*. continuous and su*. open (or su*. closed) function **.**
2- Su*. $\widehat{\omega}$-homeomorphism function, if it is su*. $\widehat{\omega}$-continuous and su*. $\widehat{\omega}$-open (or su*. $\widehat{\omega}$-closed) function.
3- Su*. ῆ-homeomorphism function, if it is su*. ῆ-continuous and su*. ῆ-open (or su*. ῆ-closed) function.
4- Su*. $\widehat{\omega}^*$-homeomorphism function, if it is su*. $\widehat{\omega}$-irresolute and strongly su*. $\widehat{\omega}$-open (or strongly su*. $\widehat{\omega}$-closed) function.
5- Su*. ῆ*-homeomorphism function, if it is su*. ῆ-irresolute and strongly su*. ῆ-open (or strongly su*. ῆ-closed) function.
6- Su*. $\widehat{\omega}^{**}$-homeomorphism function, if it is strongly su*. $\widehat{\omega}$-continuous and totally su*. $\widehat{\omega}$-open (or totally su*. $\widehat{\omega}$-closed) function.
7- Su*. ῆ**-homeomorphism function, if it is strongly su*. ῆ-continuous and totally su*. ῆ-open (or totally su*. ῆ-closed) function.

Proposition (3.11):
1- Each su. topology finer than su. T_0 is also su.T_0.
2- Each su. topology finer than su. $\widehat{\omega}T_0$ is also su. $\widehat{\omega}T_0$.
3- Each su. topology finer than su. T_1 is also su. T_1.
4- Each su. topology finer than su. $\widehat{\omega}T_1$ is also su. $\widehat{\omega}T_1$.

Proof: Let $a \neq b$ be two elements in a su. space X and μ, μ^* are two su. topologies defined on X, where μ^* is finer than μ, and μ is a su. T_0-topology on X, so there is $U \in \mu$ containing a but not b, since $\mu \subseteq \mu^*$, hence $U \in \mu^*$ too, then μ^* is a su.T_0-topology on X. By the same way we can prove the rest properties.

Theorem (3.12) [6]: A space X is a su. T_1-space, iff for any $x \in X, \{x\}$ is su. closed set.

Corollary (3.13): A space X is su. $\widehat{\omega}T_1$-space iff any singleton subset $\{x\}$ of X is su. $\widehat{\omega}$-closed.

Proof: Suppose any singleton $\{x\}$ is a su. $\widehat{\omega}$-closed subset of X, and let $d \neq e \in X$, so $\{d\}^c, \{e\}^c$ are su. $\widehat{\omega}$-open sets containing e, d respectively, which lead us to X is su. $\widehat{\omega}T_1$-space. Conversely, let X be a su. $\widehat{\omega}T_1$-space, let $e \in X$, and $e \in \{d\}^c$, so $d \neq e$ and there exists a su. $\widehat{\omega}$-open set $\mathcal{W}$ in X with $e \in \mathcal{W}, d \notin \mathcal{W}$, so $e \in \mathcal{W} \subseteq \{d\}^c$, thus $\{d\}^c$ is su. $\widehat{\omega}$-open set (by proposition (2.7)), therefore $\{d\}$ is su. $\widehat{\omega}$-closed, but d is arbitrary element in X, that means every singleton subset in X is su. $\widehat{\omega}$-closed.

Definition (3.14): The su. space X is called su. $\widehat{\omega}$-space, if each su. $\widehat{\omega}$-open subset from X, is su. open.

Theorem (3.15): A su. space X is su. $\widehat{\omega}T_0$-space iff $cl^{\mu}_{\widehat{\omega}}(x) \neq cl^{\mu}_{\widehat{\omega}}(y)$ for each non-equal points x, y in X.

Proof: Suppose $cl^{\mu}_{\widehat{\omega}}(x) \neq cl^{\mu}_{\widehat{\omega}}(y)$ for each distinct points x, y in X, so there is at least one element in one of them and not in the other, say $a \in cl^{\mu}_{\widehat{\omega}}(x), a \notin cl^{\mu}_{\widehat{\omega}}(y)$, and suppose $x \notin cl^{\mu}_{\widehat{\omega}}(y)$, because if $x \in cl^{\mu}_{\widehat{\omega}}(y)$ then $cl^{\mu}_{\widehat{\omega}}(x) \subseteq cl^{\mu}_{\widehat{\omega}}\left(cl^{\mu}_{\widehat{\omega}}(y)\right) = cl^{\mu}_{\widehat{\omega}}(y) \Longrightarrow a \in cl^{\mu}_{\widehat{\omega}}(x) \subseteq cl^{\mu}_{\widehat{\omega}}(y)$ and that is a contradiction, therefore $x \in X - cl^{\mu}_{\widehat{\omega}}(y)$, Now $X - cl^{\mu}_{\widehat{\omega}}(y)$ is su. $\widehat{\omega}$-open set containing x but not y, that implies X is su. $\widehat{\omega}T_0$-space. Conversely, If X is su. $\widehat{\omega}T_0$-space and $x \neq y$ are arbitrary elements in X, so there is a su. $\widehat{\omega}$-open set U of X with $x \in U$ and $y \notin U$, then $X - U$ is su. $\widehat{\omega}$-closed set contains y but not x, from definition of $cl^{\mu}_{\widehat{\omega}}(y)$ we get $cl^{\mu}_{\widehat{\omega}}(y) \subseteq X - U$, which means $x \notin cl^{\mu}_{\widehat{\omega}}(y)$ but $x \in cl^{\mu}_{\widehat{\omega}}(x)$, so that $cl^{\mu}_{\widehat{\omega}}(x) \neq cl^{\mu}_{\widehat{\omega}}(y)$.

Corollary (3.16): A su. space X is su. ωT_0-space iff $x \notin cl^{\mu}_{\omega}(y)$ or $y \notin cl^{\mu}_{\omega}(x)$ for each distinct points x, y in X.

Theorem (3.17): The composition between:-

1- Perfectly su*. continuous function and Perfectly su*. $\widehat{\omega}$-continuous function is Perfectly su*. $\widehat{\omega}$-continuous function.

2- Perfectly su*. continuous function and perfectly su*. $\hat{\eta}$-continuous function is perfectly su*. $\hat{\eta}$-continuous function.

3- Totally su*. $\widehat{\omega}$-continuous function and perfectly su*. $\widehat{\omega}$-continuous function is perfectly su*. $\widehat{\omega}$-irresolute function.

Proof:

1- Take $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$ as perfectly su*. continuous, $g: (Y, \mu_Y) \longrightarrow (\mathcal{Z}, \mu_{\mathcal{Z}})$ as perfectly su*. $\widehat{\omega}$-continuous and $\mathcal{M}$ is su. $\widehat{\omega}$-closed set in $\mathcal{Z}$, so $g^{-1}(\mathcal{M})$ is su. clopen set in Y, then $f^{-1}\left(g^{-1}(\mathcal{M})\right) = (g \circ f)^{-1}(\mathcal{M})$ is su. clopen set in X, thus $g \circ f$ is perfectly su*. $\widehat{\omega}$-continuous function.

3- Take $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$ as totally su*. $\widehat{\omega}$-continuous, $g: (Y, \mu_Y) \longrightarrow (\mathcal{Z}, \mu_{\mathcal{Z}})$ as perfectly su* $\widehat{\omega}$-continuous and $\mathcal{M}$ is su. $\widehat{\omega}$-closed set in $\mathcal{Z}$, so $g^{-1}(\mathcal{M})$ is su. clopen set in Y, then $f^{-1}\left(g^{-1}(\mathcal{M})\right) = (g \circ f)^{-1}(\mathcal{M})$ is su. $\widehat{\omega}$-clopen set in X, thus $g \circ f$ is perfectly su*. $\widehat{\omega}$-irresolute function.

The rest of properties can be proved in the same way.

References

[1] T. M. Al-Shami, ‘‘Supra Semi- Compactness Via Supra Topological Spaces, Vol. 12, Journal of Taibah University for Science, 2018, p. 338-343.

[2] T. M. Al-Shami, ‘‘Some results related to supra topological spaces, 7(4), Journal of Advanced Studies in Topology, 2016, p.283-294.

[3] N. Chandramathi, K. Bhuvaneswari, and S. Bharathi, ‘‘On ω^{μ}-Closed Sets and Continuous Functions In Supra Topologicsal Space, Vol. 3, International Journal of Mathematical Archive, 2012, p.712-721

[4] T. H. Jassim, ‘‘On supra compactness in supratopologicol spaces, Vol. 14(3), Tikrit Journal of Pure Science, 2009, p. 57-69.

[5] T. H. Jassim, R. B. Yasseen and Nabeel E. Arif, ‘‘On Some Separation Axioms of Supra Topological Spaces, Vol. 13, Tikrit Journal of Pure Science, 2008, p. 59-62.

[6] A. S. Mashhour, A. A. Allam, F. S. Mahmoud and F. H. Khedr, **‘‘**On Supra Topological Spaces, Indian Journal of Pure and Applied Mathematics, 1983 April 14, p. 502-510.

[7] J. Thomas and S. Jacob John. ‘‘μ-Compactness in Generalized Topological spaces, Vol. 3, Journal of advanced studies in Topology, 2012, p. 18-22.

[8] P. M. Trinita, and I. Arockiarani, ‘‘Some Stronger Forms of g^{μ}b–continuous Functions, Vol. 1, IOSR Journal of Engineering (IOSRJEN), 2011, p. 111-117.

[9] L. Vidyarani, and M. Vigneshwaran, ‘‘Some forms of N-closed maps in supra Topological spaces, Vol. 6, IOSR Journal of Mathematics, 2013, p. 13-17.