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Statistical Distribution of Rainfall in Kurdistan-Iraq Region

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Abstract

Study the statistical distribution for rainfall is important to know the behaviour of the rainfall series and to know the most frequently rainfall amount in each month. Five statistical distributions were applied on Sulaimani, Erbil and Duhok rainfall series for the period (1941-2017) except Duhok (1944-2017). These distributions were Gamma (3P), Weibul(3P), Earlang (3P), Normal and General extreme value. Kolmogrove-Semirnov, Anderson-Darling and Chi-Square goodness of fit test were used to know the best fit distribution from these five distributions.The results shows that General extreme value distribution is the best fit distribution for Jan, Dec and Oct in the three cities. Weibull (3p) distribution is the best fit distribution for the three cities in march and April also for almost months in Erbil. There is no best fit common distribution for all the three cities..

Keywords: Gamma, Weibul and Earlang distribution, Kolmogrove-Semirnov, Anderson-Darling and Chi-Square test.

الخلاصة

دراسة التوزيعات الاحصائية للامطار تفيد في معرفة سلوك السلسلة الزمنية للامطارومعرفة كمية الامطار الاكثرترددا في كل شهر. تم تطبيق خمس انواع من التوزيعات الحصائية على سلاسل امطار السليمانية,اربيل ودهوك ضمن الفترة (١٩٤٤-٢٠١٧) وكانت التوزيعات: كاما، وبيل، ايرلنك، التوزيع الطبيعي وتوزيع القيمة العظمى العامة. تم استخدام قياس كولمكروف-سمينوف، اندرسون دارلنك ومربع كاي للجودة لمعرفة التوزيع الاكثر انطباقا من بين التوزيعات الخمس. اوضحت النتائج ان توزيع القيمة العظمى العام هوالافضل للاشهركانون اللاول,كانون الثاني وتشرين الاول,اما توزيع ويبل فكان الانسب لشهر اذارونيسان في المحطات الثلاثة ولغالبية الاشهر في اربيل.ولايوجد توزيع ينطبق على كافة الاشهر للمحطات الثلاثة.

Introduction

Rainfall is one of the most climate elements where it's the source of the water supply to dames, ground water and agriculture. Hydrologic frequency analysis is a method used for evaluation of the probability of hydrologic events, which are averaged out in statistical point of view, common hydrologic engineering designs, such as a dam height, design discharges, etc. are determined by the results of frequency analysis[1]. Many studies on rainfall statistical distribution have been done. Vivekanandan N. 2014 used number of probability distributions such as Exponential, Extreme Value Type-1, Extreme Value Type-2, Generalized Extreme Value and Normal in rainfall analysis. Generally, Method of Moments was used for determination of parameters of the distributions [2]. Amin M. T., Rizwan M. and Alazba A. A, 2016 applied Normal, log-normal, log-Pearson type-III and Gumbel max distribution on Pakistan rainfall, Based on the scores of goodness of fit tests, the normal distribution was found to be the best-fit probability distribution [3]. Alghazali and Alawadi, 2014 fitted Normal, Gamma and Weibull distributions on thirteen Iraqi stations of monthly rainfall observations. Chi-Square and Kolmogorov-Smirnov tests used to show a suitable distribution. Gamma distribution was suitable for five stations, Normal and Weibull distributions were not suitable for any station [4]. Mohamed T.M. and Ibrahim A. A., 2016 used five distributions on Sudan rainfall, namely Normal, Log normal, Gamma, Weibull and exponential distribution. Three statistical goodness of fit test were used on the basis of the minimum value of test statistic. The normal and gamma distribution were selected as the best fit probability distribution for the annual rainfall in Sudan during the period of the study, respectively [5]. Salama A. M., Gado T.A. and Zeidan B. A., 2018 examined six popular probability distributions, Normal , Log-Normal , Gumbel , Pearson Type III, Log-Pearson Type III, and Generalized extreme value , and compared for their abilities in the estimation of annual maximum rainfalls in Egypt. The results indicated that the Log-Normal and Log-Pearson Type III distributions are the best models for describing the distribution of daily annual maximum rainfalls in most stations in Egypt [6].

The selected sites are Sulaimani, Erbil and Duhok weather stations in Kurdistan-Iraq region as in Figure1 are 884.8 m, 420 m, 575m above the sea level respectively and they receive different amount of rainfall.

Normal Distribution

The normal distribution or, as it is often called, the Gauss distribution is the most important distribution in statistics. The distribution is given by [7]:

$$F(X) = p(X \leq x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}} \left(\frac{x-\mu}{\sigma}\right)^2 \quad (1)$$

where μ is the mean, σ the standard deviation. For μ = 0 and σ = 1 we refer to this distribution as the standard normal distribution.

Weibull distribution

Weibull distribution is one of the best distributions and has wide applications in diverse disciplines especially in meteorology. The probability density function f (x) and the cumulative distribution function F(x) of the 2-parameter Weibull distribution are given by [8]:

$$f(x) = \frac{\alpha}{\beta} x^{\alpha-1} exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right) \quad (2)$$

$$F(x) = 1 - exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right) \quad (3)$$

for x > 0, α >0, β >0. α is the dimensionless shape parameter and β is the scale parameter of precipitation series. The maximum likelihood method (MLM) estimates are obtained by solvingiiteratively.

$$\frac{1}{\alpha} = \frac{\sum_{i=1}^{n}(x_i)^{\alpha} \ln(x_i)}{\sum_{i=1}^{n}(x_i)^{\alpha}} + \frac{1}{n}\sum_{i=0}^{n}(lnx) = 0 \quad (4)$$

$$\beta = \left(\frac{1}{n}\sum_{i=1}^{n}(x_i)^{\alpha}\right)^{\frac{1}{\alpha}} \quad (5)$$

The probability density function f (x) and the cumulative distribution function F(x) of the 3-parameter Weibull distribution are given by:

$$f(x) = \frac{\alpha}{\beta^{\alpha}}(x\Theta)^{\alpha-1} \exp\left(-\left(\frac{x-\theta}{\beta}\right)^{\alpha}\right) \quad (6)$$

$$F(x) = 1 - exp\left(-\left(\frac{x-\theta}{\beta}\right)^{\alpha}\right) \quad (7)$$

for x > θ, α>0, β> 0, α is the dimensionless shape parameter, β is the scale parameter, θ is the location parameter. The MLM estimates are obtained by solving iteratively [8,9]:

$$\frac{1}{\alpha} = \frac{\sum_{i=1}^{n}(x_I - \theta)^{\alpha} \ln(x_i - \theta)}{\sum_{i=1}^{n}(x_I - \theta)^{\alpha}} + \frac{1}{n}\sum_{i=0}^{n}(lnx - \theta) \quad (8)$$

$$\beta = \left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \theta)^{\alpha}\right)^{\frac{1}{\alpha}} \quad (9)$$

$$\frac{\alpha}{1-\alpha} = \frac{1}{n}\frac{\sum_{i=1}^{n}(x_i - \theta)^{\alpha}}{\sum_{i=1}^{n}(x_i - \theta)^{\alpha-1}}\sum_{i=1}^{n}\frac{1}{x_i - \theta} \quad (10)$$

Gamma distribution

The gamma distribution involves the notion of gamma function. First, The gamma function, Γ(α), is a generalization of the notion of factorial. The gamma function is defined as:

$$\Gamma(\alpha) = \int_{0}^{\infty} X^{\alpha} e^{-X} dx \quad (11)$$

The Gamma distribution can also be used to model the amounts of a rainfall in a region. A gamma distribution was postulated because precipitation occurs only when water particles can form around dust of sufficient mass, and

waiting the aspect implicit in the gamma distribution.
Gamma distribution is widely used in hydrologic analysis. The probability distribution function of a random variable x having a gamma distribution is:

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} X^{\alpha-1} e^{-\frac{X}{\beta}} \quad (12)$$

α:shape parameter
β:scale parameter
which represents 2-parameters gamma function.
The Cumulative Distribution Function is:

$$F(x) = \frac{\Gamma_{x/\beta(\alpha)}}{\Gamma_{(\alpha)}} \quad (13)$$

3-Parameter Gamma Distribution is given by:

$$f(x) = \frac{(X-\gamma)^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}} e^{-(\frac{X-\gamma}{\beta})} \quad (14)$$

γ:location parameter

The Cumulative Distribution Function is [10]:

$$F(x) = \frac{\Gamma_{(x-\gamma)/\beta(\alpha)}}{\Gamma_{(\alpha)}} \quad (15)$$

General extreme value distribution

Extreme value theory deals with the stochastic behavior of the extreme values in a process. The generalized extreme value distribution is defined by the following distribution function:

$$F(x) = \exp-(1 + k\frac{x-\mu}{\sigma})^{-\frac{1}{k}} \quad (16)$$

For $1 + k\frac{x-\mu}{\sigma} >$, k the shape parameter, μ the location parameter and σ>0 the scale parameter, the density function is given by [11]:

$$f(x) = \frac{1}{\sigma}\left(1 + k\frac{x-\mu}{\sigma}\right)^{-\frac{1}{k}-1} e^{-\left(1+k\frac{x-\mu}{\sigma}\right)^{-\frac{1}{k}}} \quad (17)$$

Erlang Distribution

The Erlang variate is the sum of a number of exponential variates. It was developed as the distribution of waiting time and message length in telephone traffic. The Erlang variate is a gamma variate with shape parameters c, an integer, where:

Range 0 ≤ x < ∞.
Scale parameterb β > 0.
Shape parameter m > 0
For the 2- parameter Erlang distribution:

$$Distrbution\ function\ F(x) =$$

$$= 1 - [exp\left(-\frac{X}{\beta}\right)\left(\sum_{i=1}^{C=1}\frac{\left(\frac{X}{\beta}\right)^{i}}{i!}\right) \quad (18)$$

$$Probabilty\ density\ function\ f(x)$$

$$= \frac{\left(\frac{X}{\beta}\right)^{m-1}\exp\left(-\frac{X}{\beta}\right)}{\beta(m-1)!} \quad (19)$$

While for the 3- parameter Erlang distribution:

$$Distrbution\ function\ F(x) = 1$$

$$= -[exp\left(-\frac{X-\gamma}{\beta}\right)\left(\sum_{i=1}^{C=1}\frac{\left(\frac{X-\gamma}{\beta}\right)^{i}}{i!}\right) \quad (20)$$

$$Probabilty\ density\ function\ f(x)$$

$$= \frac{\left(\frac{X-\gamma}{\beta}\right)^{m-1}\exp\left(-\frac{X-\gamma}{\beta}\right)}{\beta(m-1)!} \quad (21)$$

γ: location parameter [12]

Goodness of Fit Tests

The goodness of fit (GOF) tests measures the compatibility of a random sample with a theoretical probability distribution function. In other words, these tests show how well the distribution selected fits to the data. The general procedure consists of defining a test statistic which is some function of the data measuring the distance between the hypothesis and the data, and then calculating the probability of obtaining data which have a still larger value of this test statistic than the value observed, assuming the hypothesis is true. This probability is called the confidence level.

Chi square test

The Chi-Squared test is used to determine if a sample comes from a population with a specific distribution. The first step in chi-square test is to arrange the number of observation into a set of class intervals .We compare observed frequencies with corresponding expected frequencies calculated on the basis of a null hypothesis with stated trial assumptions. Then calculate a quantity which summarizes the disagreement between observed and expected frequencies, and test whether it is so large that it would not likely occur by chance.

Let the observed frequency for class *i* be o_i, and let the expected frequency for that same class be e_i, where:

$$\sum_{i=1}^{n} e_i = \sum_{i=1}^{n} o_i \qquad (22)$$

$$x^2_{calculated} = \sum_{i=1}^{n} \frac{(o_i - e_i)^2}{e_i} \qquad (23)$$

However, like other tests of significance, the chi-squared test for frequency distributions becomes more sensitive as the number of degrees of freedom increases, and that increases as the number of classes increases. Thus, we should make the number of classes as large as we can. If the calculated value of χ^2 is greater than the corresponding tabulated or computer value of χ^2, the null hypothesis must be rejected at the level of significance equal to the stated upper-tail χ^2 probability. The chi-squared test for frequency distributions appears in various forms depending on just what trial assumptions are used to give null hypotheses. In each case the expected frequency for any class or cell is the product of two quantities: the total frequency for all classes and the probability that a randomly chosen item will fall in that particular class [11].

Kolmogrove-Smirnov. Test

Underlying The Kolmogorov–Smirnov (K-S) test is a goodness-of-fit test used to determine whether an underlying probability distribution differs from a hypothesized distribution when given a finite data set.

The step-by-step procedure for executing K-S test for given a set of sample values $x_1, x_2, \ldots, x_i$ observed from a population *X*, is as follows:

- The sample values are arranged in increasing order of magnitude, denoted by (x_i).
- The observed distribution functions $S(x_i)$ are determined from the relation:

$S(x_i)=i/N$ N
is the total number of observations.

- Distribution function $F(x_i)$ at each xi by using the hypothesized distribution is obtained and the deviations D_2 are determined from Equation:

$D_2=S(x_i)-F(x_i)$

- The maximum absolute value of D_2, obtained from the last Equation, is compared with critical value shown in statistical tables. If D_2 is less than the critical value the tested distribution is suitable for describing the observed data, otherwise the tested distribution is not suitable for describing the observed data [13].

The Anderson-Darling test

The Anderson-Darling test makes use of the specific distribution in calculating critical values. This has the advantage of allowing a more sensitive test and the disadvantage that critical values must be calculated for each distribution.

The Anderson - Darling test statistic is defined by:

$A_2= -N-S$

Where:

$$S = \sum_{i=1}^{N} \frac{2i-1}{N}[lnF(Y_i)+ln(1-F(Y_{N+1-i})] \qquad (24)$$

F is the cumulative distribution function of the specified distribution. Note that the Yi are the ordered data. The critical values for the Anderson-Darling test are dependent on the specific distribution that is being tested.

The test is a one-sided test and the hypothesis that the distribution is of a specific form is rejected if the test statistic, *A*, is greater than the critical value. Note that for a given distribution, the Anderson-Darling statistic may be multiplied by a constant (which usually depends on the sample size, n). This is what should be compared against the critical values [14].

Results and Discussion

Sulaimani city

Figures (2a-i) Shows the Sulaimani rainfall histograms with the five applied distributions (Gamma(3P), Weibul(3P), Earlang (3P), Normal and General extreme value) of Jan, Feb, Mar, Apr, May, Oct, Nov, Dec and total rainfall values respectively, for the period (1941-2017). According to the goodness fit tests gamma (3P) is best fit distribution for Jan, Feb, Dec and total rainfall with 80-120 mm and 85-110 mm, 75-110 mm and 700-800 mm most frequency rainfall values respectively. Weibull (3P) is adequate for mar with 82-122 mm rainfall value. Apr and May rainfall are best fit with Earlang (3p) with 55-85 mm and 0-20 mm mostly repeated rainfall values respectively. General extreme value is fit to May and Oct rainfall with 0-20 mm and 0-27 mm rainfall range respectively. The most fitted distributions to the data in sulamani city are found in figures (2a), (2c), (2d), (2h) and (2i).

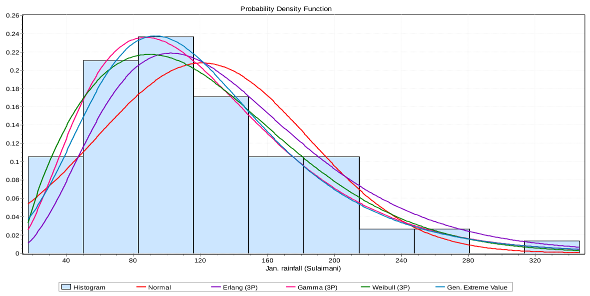
Erbil city

The Five distributions were fitted on Erbil histograms of monthly and total rainfall as in figures (3a-i) for the period 1941-2017. Goodness of fit tests shows that general extreme value distribution is best fit distribution to Jan, Feb, May, Oct, Nov, Dec and total rainfall , 40-80 mm, 60-80 mm, 0-10 mm, 0-9 mm, 0-20 mm, 40-80 mm and 290-360 mm are the high frequency values of rainfall amounts. Weibull (3P) is the most suitable distribution to Mar and Apr rainfall with a high frequency value of 30-57mm and 0-10 mm respectively. The most fitted distributions to the data in Erbil city are found in figures (3a), (3b) and (3i).

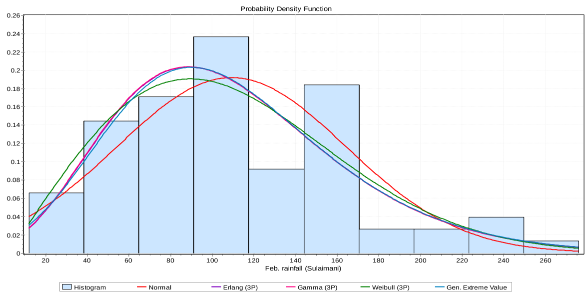
Duhok city

The three goodness of fit tests on the five types of distributions show that each distribution is appropriate for a given month in Duhok city as shown in figures (4a-i). According to the three goodness of fit tests General extreme value is fit on Jan, May, Oct, Dec and total rainfall with a high frequency value of 40-80,0-10 mm, 0-9 mm , 40-80 mm and 480-560 mm respectively. In Feb, Mar, Apr Weibull (3P) is the most suitable distribution with 60-80mm, 30-50mm; 37-55mm frequently rainfall amount ranges respectively. Gamma (3p) is adequate to Nov rainfall with most repeated rainfall value of 0-20mm. The most fitted distributions to the data in Dhouk city are found in figures (4a) and (4i). In general According to the figures (2i), (3i), (4i) we can say that the total rainfall in sul.,arbil and duhok cites respectively most suitable test for these distribution (gamma 3p, weibll 3p, earlang 3p ,normal and general extreme value) are shown fitted this data.

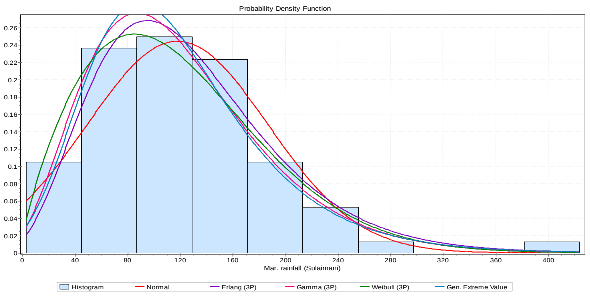
And we can conclude that the best suitable fitting for these data at Sulimani city, The reason for this is due to the fact that the Sulaimani city more rainy compared to the cities of Erbil and Dhouk in the winter and suitable for rainfall at annual Rain.



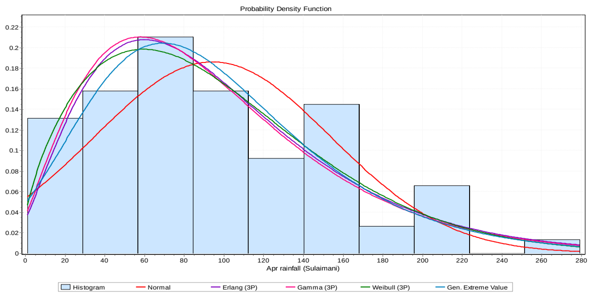
(a) Jan. rainfall distribution – Sulaimani



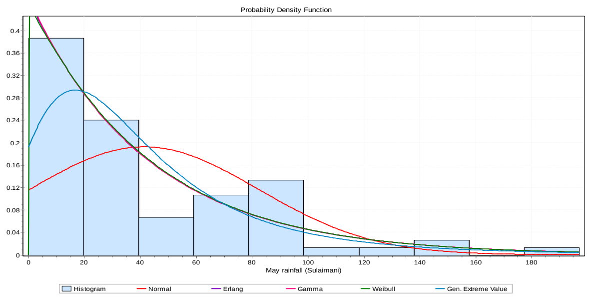
(b) Feb. rainfall distribution – Sulaimani



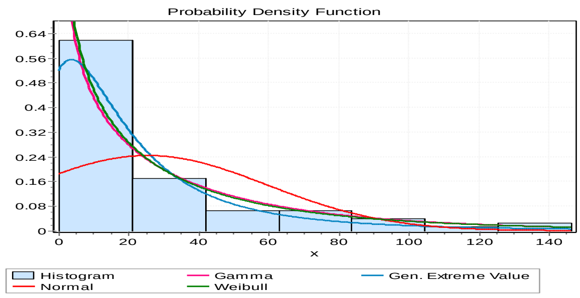
(c): Mar. rainfall distribution – Sulaimani



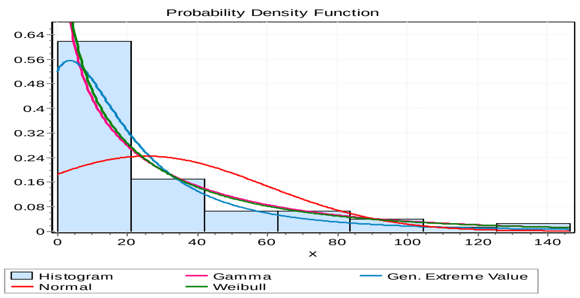
(d) Apr. rainfall distribution – Sulaimani



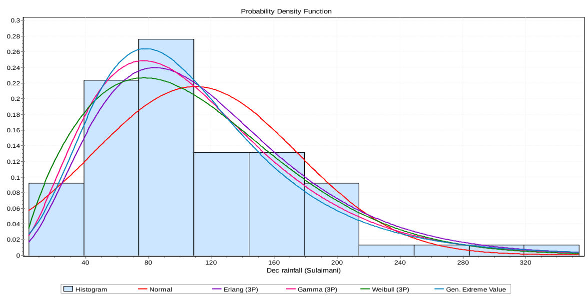
(e): May rainfall distribution – Sulaimani



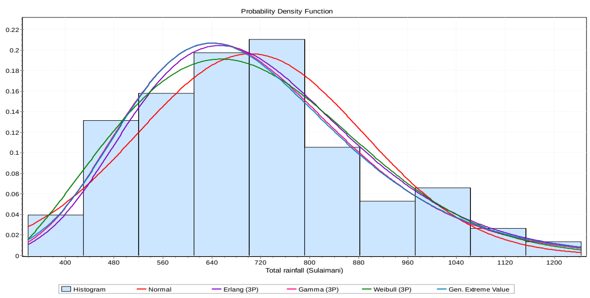
(f) Oct. rainfall distribution – Sulaimani



(g) Nov. rainfall distribution – Sulaimani

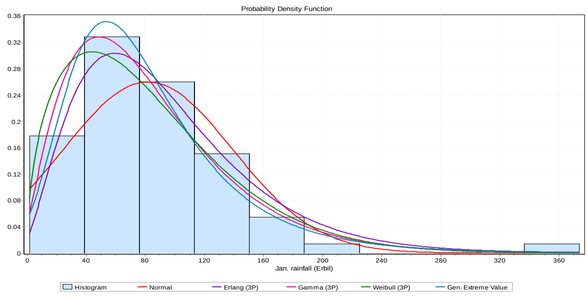


(h) Dec. rainfall distribution – Sulaimani

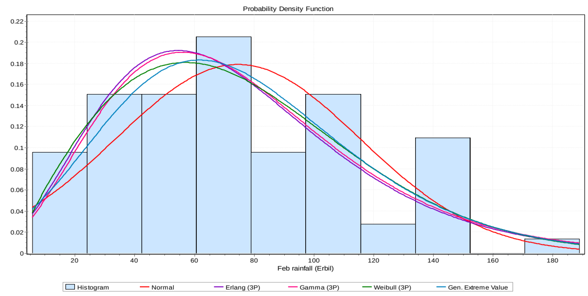


(i) Total. rainfall distribution – Sulaimani Sulaimani

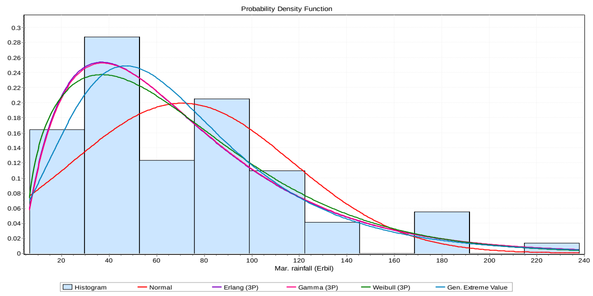
Figure 2: Monthly rainfall distribution – Sulaimani.



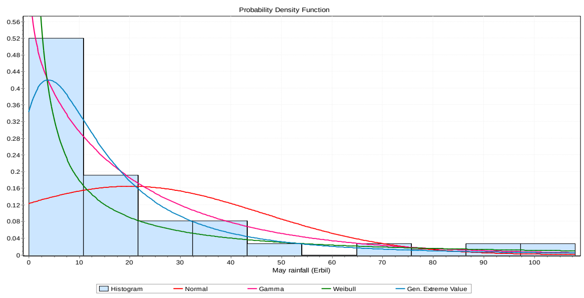
(a) Jan. rainfall distribution – Erbil



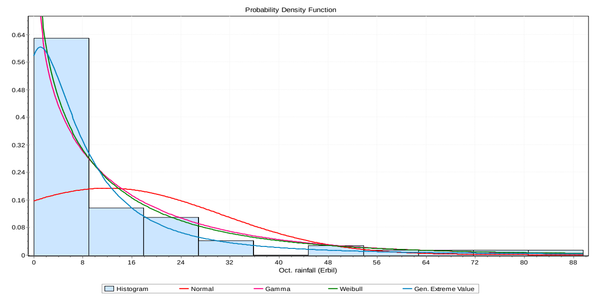
(b) Feb. rainfall distribution – Erbil



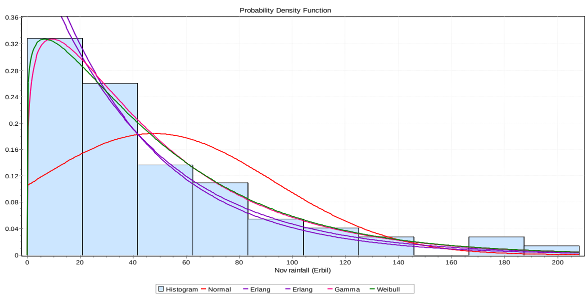
(c) Mar. rainfall distribution – Erbil



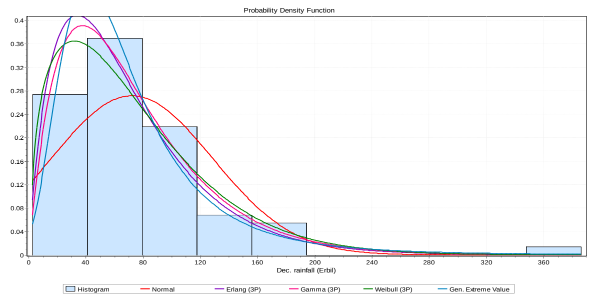
(e) May rainfall distribution – Erbil



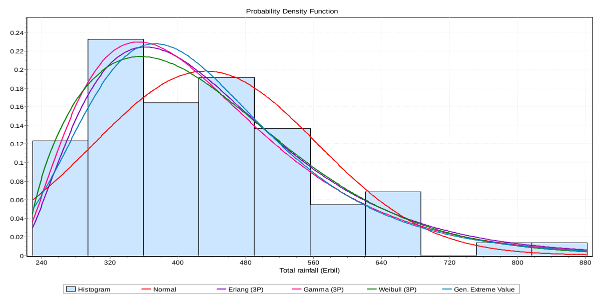
(f) Oct. rainfall distribution – Erbil



(g) Nov. rainfall distribution – Erbil

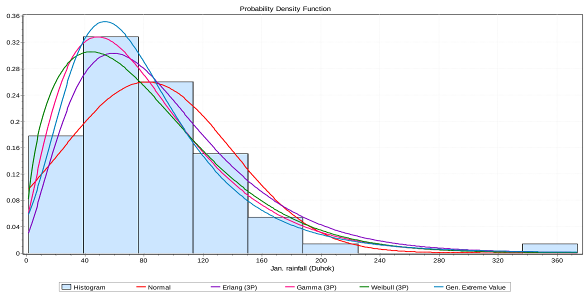


(h) Dec. rainfall distribution – Erbil

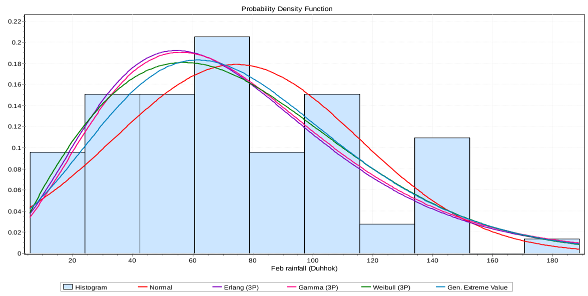


(i) Total. rainfall distribution – Erbil

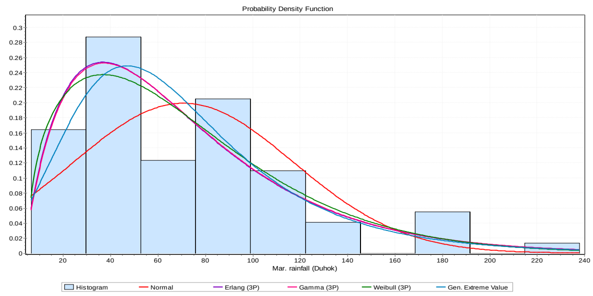
Figure 3: Monthly rainfall distribution – Erbil.



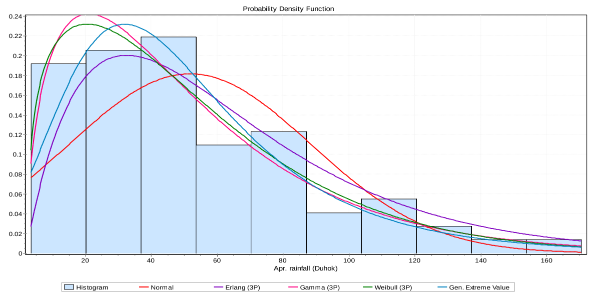
(a) Jan. rainfall distribution – Duhok



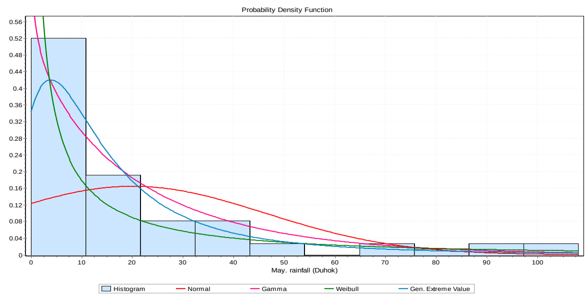
(b) Feb. rainfall distribution – Duhok



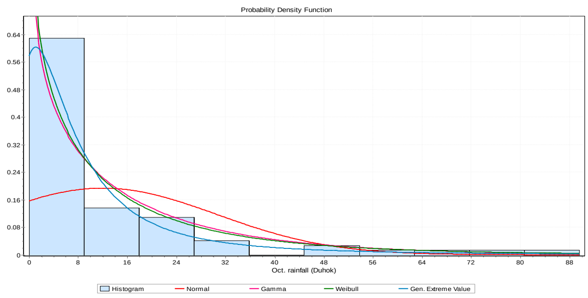
(c) Mar. rainfall distribution – Duhok



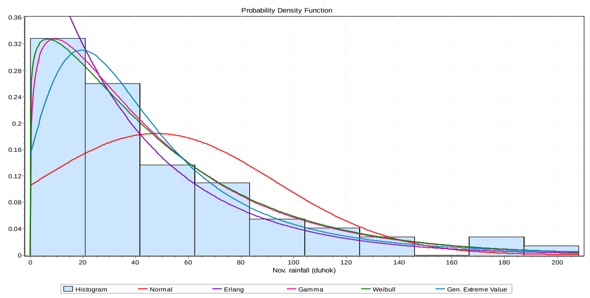
(d) Apr. rainfall distribution – Duhok



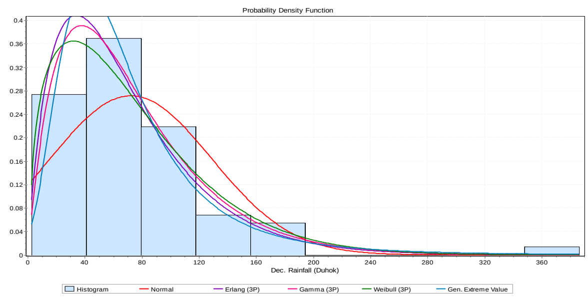
(e) May rainfall distribution – Duhok



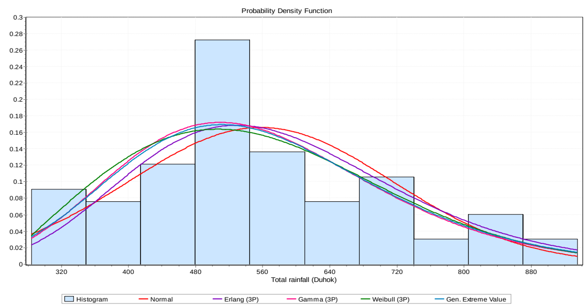
(f) Oct. rainfall distribution – Duhok



(g) Nov. rainfall distribution – Duhok



(h) Dec rainfall distribution – Duhok



(i) Total. rainfall distribution – Duhok

Figure 4: Monthly rainfall distribution – Duhok.

Table 1: Sulaimani distribution parameters.

Month	Distribution	Parameters
Jan.	Normal	σ=63.202 μ=121.29
	Gamma(3P)	α=3.5533 β=33.75 γ=1.365
	Weibull (3P)	α=1.7829 β=121.38 γ=13.314
	Erlang (3P)	m=4 β=33.75 γ=1.365
	General extreme value	k=-0.01397 σ=51.017 μ=92.54
Feb.	Normal	σ=54.927 μ=109.41
	Gamma(3P)	α=6.9873 β=20.851 γ=-36.287
	Weibull (3P)	α=2.056 β=121.15 γ=1.9917
	Erlang (3P)	m=7 β=20.851 γ=-36.287
	General extreme value	k=-0.08945 σ=48.044 μ=85.606
Mar.	Normal	σ=68.735 μ=118.12
	Gamma(3P)	α=4.7491 β=30.666 γ=-27.51
	Weibull (3P)	α=1.5689 β=134.48
	Erlang (3P)	m=5 β=30.666 γ=-27.51
	General extreme value	k=-0.01883 σ=53.764 μ=88.073
Apr.	Normal	σ=59.676 μ=94.376
	Gamma(3P)	α=2.9443 β=36.284 γ=-12.453
	Weibull (3P)	α=1.6604 β=108.48 γ=-2.7408
	Erlang (3P)	m=3 β=36.284 γ=-12.453
	General extreme value	k=-0.04012 σ=50.156 μ=67.345
May	Normal	σ=40.867 μ=41.435
	Gamma(3P)	α=1.028 β=40.308
	Weibull (3P)	α=1.0075 β=43.292
	Erlang (3P)	m=1 β=40.308
	General extreme value	k=0.19144 σ=25.134 μ=21.118
Oct.	Normal	σ=34.165 μ=25.464
	Gamma(3P)	α=0.55552 β=45.839
	Weibull (3P)	α=0.7726 β=28.741
	Erlang (3P)	No fit
	General extreme value	k=0.38051 σ=14.763 μ=8.1565
Nov.	Normal	σ=68.088 μ=82.143
	Gamma(3P)	α=1.4555 β=56.438
	Weibull (3P)	α=1.1265 β=87.817
	Erlang (3P)	m=1 β=56.438
	General extreme value	k=0.03569 σ=53.334 μ=49.411
Dec.	Normal	σ=64.898 μ=109.85
	Gamma(3P)	α=3.7787 β=32.757 γ=-13.928
	Weibull (3P)	α=1.7723 β=123.4 γ=0.04781
	Erlang (3P)	m=4 β=32.757 γ=-13.928
	General extreme value	k=0.03441 σ=48.871 μ=79.923
Total	Normal	σ=183.9 μ=703.64
	Gamma(3P)	α=8.8151 β=61.822 γ=158.67
	Weibull (3P)	α=2.2905 β=446.93 γ=307.64
	Erlang (3P)	m=9 β=61.822 γ=158.67
	General extreme value	k=-0.09372 σ=161.82 μ=624.06

Table 2: Erbil distribution parameters.

Month	Distribution	Parameters
Jan.	Normal	σ=57.22 μ=81.956
	Gamma(3P)	α=2.6819 β=33.24 γ=-7.1905
	Weibull (3P)	α=1.5093 β=90.973 γ=-0.1337
	Erlang (3P)	m=3 β=33.24 γ=-7.1905
	General extreme value	k=0.07314 σ=39.092 μ=56.358
Feb.	Normal	σ=40.882 μ=74.629
	Gamma(3P)	α=5.0701 β=18.645 γ=-19.901
	Weibull (3P)	α=1.9181 β=84.719 γ=-0.59355
	Erlang (3P)	m=5 β=18.645 γ=-19.901
	General extreme value	k=-0.11804 σ=37.107 μ=57.127
Mar.	Normal	σ=46.39 μ=70.774
	Gamma(3P)	α=2.0117 β=33.575 γ=3.2312
	Weibull (3P)	α=1.4378 β=72.068 γ=5.3037
	Erlang (3P)	m=2 β=33.575 γ=3.2312
	General extreme value	k=0.05708 σ=34.291 μ=48.937
Apr.	Normal	σ=36.711 μ=51.8
	Gamma(3P)	α=1.6256 β=30.688 γ=1.9121
	Weibull (3P)	α=1.3131 β=52.959 γ=2.8451
	Erlang (3P)	m=2 β=30.688 γ=1.9121
	General extreme value	k=0.08191 σ=26.602 μ=34.112
May	Normal	σ=26.259 μ=20.041
	Gamma(3P)	α=0.89184 β=25.237
	Weibull (3P)	α=0.89825 β=21.261
	Erlang (3P)	No fit
	General extreme value	k=0.41113 σ=10.219 μ=7.2336
Oct.	Normal	σ=18.484 μ=12.053
	Gamma(3P)	α=0.42525 β=28.344
	Weibull (3P)	α=0.80918 β=13.894
	Erlang (3P)	No fit
	General extreme value	k=0.47653 σ=6.0324 μ=3.2572
Nov.	Normal	σ=45.152 μ=47.837
	Gamma(3P)	α=1.1225 β=42.617
	Weibull (3P)	α=1.1164 β=50.61
	Erlang (3P)	m=1 β=42.617
	General extreme value	k=0.24737 σ=25.393 μ=25.055
Dec.	Normal	σ=56.416 μ=72.371
	Gamma(3P)	α=2.1106 β=34.527 γ=-0.50071
	Weibull (3P)	α=1.3745 β=77.26 γ=1.9647
	Erlang (3P)	m=2 β=34.527 γ=-0.50071
	General extreme value	k=0.19103 σ=31.585 μ=46.859
Total	Normal	σ=131.34 μ=433.03
	Gamma(3P)	α=2.8981 β=78.813 γ=204.62
	Weibull (3P)	α=1.654 β=235.52 γ=222.34
	Erlang (3P)	m=3 β=78.813 γ=204.62
	General extreme value	k=-0.00693 σ=105.48 μ=372.87

Table 3: Duhok distribution parameters.

Month	Distribution	Parameters
Jan.	Normal	σ=57.22 μ=81.956
	Gamma(3P)	α=2.6819 β=33.24 γ=-7.1905
	Weibull (3P)	α=1.5093 β=90.973 γ=-0.1337
	Erlang (3P)	m=3 β=33.24 γ=-7.1905
	General extreme value	k=0.07314 σ=39.092 μ=56.358
Feb.	Normal	σ=40.882 μ=74.629
	Gamma(3P)	α=5.0701 β=18.645 γ=-19.901
	Weibull (3P)	α=1.9181 β=84.719 γ=-0.59355
	Erlang (3P)	m=5 β=18.645 γ=-19.901
	General extreme value	k=-0.11804 σ=37.107 μ=57.127
Mar.	Normal	σ=46.39 μ=70.774
	Gamma(3P)	α=2.0117 β=33.575 γ=3.2312
	Weibull (3P)	α=1.4378 β=72.068 γ=5.3037
	Erlang (3P)	m=2 β=33.575 γ=3.2312
	General extreme value	k=0.05708 σ=34.291 μ=48.937
Apr.	Normal	σ=36.711 μ=51.8
	Gamma(3P)	α=1.6256 β=30.688 γ=1.9121
	Weibull (3P)	α=1.3131 β=52.959 γ=2.8451
	Erlang (3P)	m=2 β=30.688 γ=1.9121
	General extreme value	k=0.08191 σ=26.602 μ=34.112
May	Normal	σ=26.259 μ=20.041
	Gamma(3P)	α=0.5825 β=34.405
	Weibull (3P)	α=0.89825 β=21.261
	Erlang (3P)	α=0.89825 β=21.261
	General extreme value	k=0.41113 σ=10.219 μ=7.2336
Oct.	Normal	σ=18.484 μ=12.053
	Gamma(3P)	α=0.42525 β=28.344
	Weibull (3P)	α=0.80918 β=13.894
	Erlang (3P)	No fit
	General extreme value	k=0.47653 σ=6.0324 μ=3.2572
Nov.	Normal	σ=45.152 μ=47.837
	Gamma(3P)	α=1.1225 β=42.617
	Weibull (3P)	α=1.1164 β=50.61
	Erlang (3P)	m=1 β=42.617
	General extreme value	k=0.24737 σ=25.393 μ=25.055
Dec.	Normal	σ=56.416 μ=72.371
	Gamma(3P)	α=2.1106 β=34.527 γ=-0.50071
	Weibull (3P)	α=1.3745 β=77.26 γ=1.9647
	Erlang (3P)	m=2 β=34.527 γ=0-0.50071
	General extreme value	k=-0.12986 s=142.73 m=492.02
Total	Normal	s=156.7 m=557.99
	Gamma(3P)	a=10.593 b=48.354 g=45.792
	Weibull (3P)	a=2.1299 b=356.07 g=242.36
	Erlang (3P)	m=11 β=48.354 γ=45.792
	General extreme value	k=-0.12986 s=142.73 m=492.02

Conclusions

This study investigated the statistical distribution of rainfall in Sulaimani, Erbil and Duhok stations. The results show that general extreme value distribution is the best fit distribution for Jan, Dec and Oct in the three cities. General extreme value is best adequate distribution for almost months in Erbil. In Mar the Weibull(3p) distribution is the best fit distribution for the three cities. In general for all the three cities Weibull(3p) distribution is mostly appropriate in Mar and Apr. In May, Oct, and Nov Earlang distribution cannot be applied to the frequency histogram for the three cities. There is no best fit common distribution for all the three cities. High observed frequency rainfall values were observed in Jan and the lowest were observed in Oct. There is no fit distribution exists for all months in the three cities. The most fitted distribution to the data occurred in Jan and annual rainfall for the three sites.

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