

# Effect of Inclined Magnetic Field on Peristaltic Flow of Carreau Fluid through Porous Medium in an Inclined Tapered Asymmetric Channel

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## Abstract

During this article, we have a tendency to show the peristaltic activity of magnetohydrodynamics flow of carreau fluid with heat transfer influence in an inclined tapered asymmetric channel through porous medium by exploitation the influence of non-slip boundary conditions. The tapered asymmetric channel is often created because of the intrauterine fluid flow induced by myometrial contraction and it had been simulated by asymmetric peristaltic fluid flow in an exceedingly two dimensional infinite non uniform channel, this fluid is known as hereby carreau fluid, conjointly we are able to say that one amongst carreau's applications is that the blood flow within the body of human. Industrial field, silicon oil is an example of carreau fluid. By exploitation, the perturbation technique for little values of weissenberg number, the nonlinear governing equations in the two-dimensional Cartesian coordinate system is resolved under the assumptions of long wavelength and low Reynolds number. The expressions of stream function, temperature distribution, the coefficient of heat transfer, frictional forces at the walls of the channel, pressure gradient are calculated. The effectiveness of interesting parameters on the inflow has been colluded and studied.

**Keywords:** Peristaltic transport, Magnetic field, Heat transfer, non-Slip effects, Porous medium, inclined tapered asymmetric channel.

## الخلاصة

خلال هذا البحث، لدينا محاولة ابراز التأثير المغناطيسي الهيدروديناميكا للتدفق التمعجي للسائل كارو مع تأثير نقل الحرارة في قناة غير المتناظرة مستدقة ميلا من خلال الوسط المسامي باستخدام الشروط الحدودية غير قابلة للانزلاق. غالباً ما يتم إنشاء القناة غير المتناظرة المستدقة بسبب تدفق السوائل داخل الرحم الناجمة عن انكماش رحمي وأنه قد تم محاكاة عن طريق تدفق السوائل في قناة غير منتظمة وذات بعدين، هذا السائل المعروف بالمانع كارو، بالإضافة نحن هنا نستطيع أن نقول أن واحداً بين التطبيقات في كارو أن تدفق الدم داخل جسم الإنسان. الميدان الصناعي، زيت السيليكون مثال للمانع كارو. باستغلال تقنية اضطراب للقيم قليلاً من عدد ويسينبيرج، يتم حل المعادلات غير الخطية التي تغطي الجريان في نظام إحداثي ديكارتي ثنائي الأبعاد تحت الافتراضات لطول الموجه الطويلة وانخفاض عدد رينولدز. التعبيرات الخاصة بدالة التدفق، توزع درجات الحرارة، ومعامل للحرارة، قوي الاحتكاك في جدران القناة، الضغط التدريجي. قد احتسبت. تأثير المعاملات ذات الاهتمام في الجريان قد درست.

## Introduction

The peristaltic transmit is a crucial mechanism for transporting blood, wherever crosswise of the artery is shrunken or enlarged sporadically by the propagation of progressive wave. It represents an irreplaceable role in moving several physiographic fluids in the body in numerous standing like a piddle transport from the urinary organ to the bladder out of the duct, transport of spermatozoa in the ducts efferent

of the male generative tract and also the movement of ovum in the fallopian tubes. Roller and finger pumps using viscous fluids also operate on this principle gastro-intestinal tract, bile ducts and other glandular ducts. The precept of peristaltic transport has been hard-done by for made up implementation like hygienic fluid transport, blood pumps in heart lungs machine and transport of corrosive fluids where the touch of the fluid with the machinery elements is prohibited since the primary

realization of [1, 2] comprehensive analytical studies have been under taken that involve such fluids. Vital studies to the subject embrace the works in [3-6, 7]

Although most prior studies of peristaltic motion have concentrate on Newtonian fluids, there are furthermore studies comprising non-Newtonian fluids, such that the shear stress might rely on the shear rate (the rapport between shear rate and shear stress isn't linear), each shear stress and shear rate is also time subordinate and also the fluid might have viscous additionally as resilient characteristics (see [8, 9]). As a result of the several rheological particular of non-Newtonian fluids, there occurs no unique international constituent linkage between stress and rate of strain by that all the non-Newtonian fluids can be deliberate. So, sundry models of non-Newtonian fluids are advised. Complexness in non-Newtonian fluids starts because of the non-linear terms showing in their constitutional relationships. Many researchers through-about various models underneath completely different approximations and geometries by forward the fluid content as a Newtonian fluid that is appropriate in some specific cases like piddle transport ([10]).The samples of non-Newtonian fluids contains semi-solid food referred to as bolus in esophagus, semi-liquid food (chyme) in stomach and intestines, blood in arteries or veins, cervical mucus, seed and gamete in reproductory tracts. Wherever as just in case of industrial fluids waste inside the sanitary ducts, toxic materials, metal alloys, oil and grease in vehicles or mechanics, nuclear slurries within the nuclear reactor and many others.

Leverage of applied magnetic field (MHD) on peristaltic efficacy is paramount in references to well-tried issues of the stir of the semi conductive physiological fluids, (e. g.), blood and blood pump machines, magnetic drug forcing and relevant method of human digestive system, also it is beneficial in treating gastro paresis, chronic constipation and morbid obesity also magnetic resonance imaging (MRI) that is employed for identification of brain, vascular diseases and every one the form within the studies [11-13, 14], the uniform MHD has been used. There are few makes an

attempt within which stimulated magnetic field is employed. (See [15-16, 17])

Consideration of porous medium is that a material which contains a number of small holes distributed over the material. Flows over porous medium subsist infiltration of fluids and seep of water in riverbeds. Movement of underground, water and oil are some vital samples of flows through porous medium. An oil reservoir largely contents of substance formation like limestone and sandstone in which oil is entrapped. There are several samples of natural porous medium like beach sand, rye bread, wood, filter, loaf of bread, human lung and gall bladder, [18].

Action of heat transfer in peristaltic relocate of fluid is kind of important in food process, oxygenation, hemodialysis, tissues condition, heat convection for blood flow from the pores of tissues and radiation between environment and its surface, [19].

Mass transfer is beneficial within the aforementioned processes; particularly mass transfer cannot be underneath calculable once nutrients diffuse out from the blood to neighboring tissues. More mass transfer involvement is kind of prevailing in distillation, chemical impurities diffusion, membrane separation and combustion method. It ought to be remarked that connection between fluxes and driving potentials dwell once each heat and mass transfer act at the same time. Yet mass flux and composition gradients are because of temperature gradient (whose is named soret action), [19].

The aim of this article is to study the peristaltic flow of a carreau fluid with constant viscosity pattern out of porous medium underneath collective actions of MHD and heat transfer analysis in sloping tapered asymmetric canal. The influence of mass transfer is considered. The effects of magnetic field (which is simulated by Hartmann number), Darcy number, penchant angle of magnetic field, amplitudes of the waves, non-uniform parameter of the channel, phase difference of the channel and other variables are taken into account in the study.

$(x, y)$  Cartesian coordinates in a wave frame

$(\bar{x}, \bar{y})$	Cartesian coordinates in a fixed frame
$\bar{p}$	The pressure in the wave of reference
$\bar{p}$	The pressure in the fixed frame of reference
$d$	Mean-half width of the channel
$t$	Time
$(u, v)$	Velocity components of wave frame of reference
$(\bar{u}, \bar{v})$	Velocity components of fixed frame of reference
$a_1, a_2$	Amplitude of the channel at the lower and upper walls respectively
$Q(\bar{x}, \bar{t})$	Instantaneous flux in a fixed frame
$\bar{Q}$	Time average flux
$\tau$	Stress tensor
$\Gamma$	The time constant
$\alpha$	Angle of inclination of channel
$\mu_\infty$	The infinity shear rate viscosity
$\sigma$	The electrical conductivity of the fluid
$g$	Acceleration to gravity
$C_\rho$	Specific heat at constant pressure
$Pr$	Prandtl number
$Br$	Brinkman number
$T_1$	Temperature of the fluid at upper wall
$c_1$	Concentration of the fluid at upper wall
$D_T$	The thermal diffusion ratio
$D_\beta$	The diffusion coefficient of the diffusing species
$a, b$	Non-dimensional amplitudes of the lower and upper walls respectively
$n$	Power-law index
$Da$	Darcy number
$Re$	The Reynolds number
$M$	Hartmann Number
$We$	The Weissenberg number
$\lambda$	Wave length of the channel
$c$	Wave speed
$\theta'$	The time-average of the flow flux in a wave frame
$B_0$	Constant transverse magnetic field

$k_0$	Permeability
$\mu_0$	The zero shear rate viscosity
$\delta$	The wave number
$\beta$	Angle of inclination of magnetic field
$\eta$	Gravity parameter
$\rho$	The density of the fluid
$k$	The porosity parameter
$k_1$	Thermal conductivity of the fluid
$\theta$	Non-dimensional temperature
$Ec$	Eckert number
$Fr$	Fraud number
$T_0$	Temperature of the fluid at lower wall
$c_0$	Concentration of the fluid at lower wall
$\phi$	Non-dimensional concentration
$T_m$	Mean fluid temperature
$\psi$	Stream function

## Materials and Methods

Deem the peristaltic outflow of an incompressible electrically connector carreau fluid into a two dimensional inclined asymmetric tapered canal filled with porous material. Let  $\bar{Y} = H_1$  and  $\bar{Y} = H_2$  be severally the lower and upper wall boundaries of the tapered asymmetric channel. We have a tendency to assume that infinite wave train traveling with velocity  $c$  along the non-uniform walls. We elect a rectangular coordinates system for the canal with  $\bar{x}$  along the direction of wave propagation and parallel to the center line and  $\bar{Y}$  transverse to it. An sloping magnetic field  $(B_0 \sin \beta, B_0 \cos \beta, 0)$  will be applied in the  $\bar{XY}$ -direction. Heat and mass transfer of the fluid will be analyzed. The wall of tapered asymmetric channel are given by the equations:

$$H_2(\bar{x}, \bar{t}) = d + m'\bar{x} + a_2 \sin\left[\frac{2\pi}{\lambda}(\bar{x} - c\bar{t})\right] \quad (1)$$

$$H_1(\bar{x}, \bar{t}) = -d - m'\bar{x} - a_1 \sin\left[\frac{2\pi}{\lambda}(\bar{x} - c\bar{t}) + \phi\right] \quad (2)$$

Where  $2d$  is the width of the channel,  $m'$  ( $m' \ll 1$ ) is the non-uniform parameters, the

phase difference  $\phi$  varies in the range  $0 \leq \phi \leq \pi$ ,  $\phi = 0$  represents to symmetric channel with waves out the phase (i. e.), both walls move towards the outward or inward simultaneously and further  $\phi$  and  $a_1, a_2, d$  satisfies the condition for the divergence channel at the inlet of flow:

$$a_1^2 + a_2^2 + 2a_1a_2 \cos \phi \leq (2d)^2, [20] \tag{3}$$

### Constitutive Equations and Governing Equations of the Problem

The constitutive equations for a Carreau fluid (following [21]) is:

$$\bar{\tau} = -(\mu_\infty + (\mu_0 - \mu_\infty)(1 + (\Gamma \dot{y})^2)^{\frac{n-1}{2}}) \dot{y} \tag{4}$$

Where  $\bar{\tau}$  is the extra stress tensor and is the shear rate which is defined by:

$$\dot{y} = \sqrt{\frac{1}{2} \sum_i \sum_j \dot{y}_{ij} \dot{y}_{ji}} = \sqrt{\frac{1}{2} \Pi} \tag{5}$$

Where  $\Pi$  is the second invariant of strain-rate tensor? We consider in the constitutive equation (4). The case for which  $\mu_\infty = 0$  and so we can write

$$\bar{\tau} = -\mu_0 [(1 + (\Gamma \dot{y})^2)^{\frac{n-1}{2}}] \dot{y} \tag{6}$$

The governing equations of carreau fluid in a fixed frame are given by:

$$\frac{\partial \bar{U}}{\partial X} + \frac{\partial \bar{V}}{\partial Y} = 0, \tag{7}$$

$$\rho \left( \frac{\partial \bar{U}}{\partial t} + \bar{U} \frac{\partial \bar{U}}{\partial X} + \bar{V} \frac{\partial \bar{U}}{\partial Y} \right) = -\frac{\partial \bar{P}}{\partial X} - \frac{\partial \bar{\tau}_{XX}}{\partial X} - \frac{\partial \bar{\tau}_{XY}}{\partial Y} - \sigma B_0^2 \cos \beta (\bar{U} \cos \beta - \bar{V} \sin \beta) - \frac{\mu_0}{k_0} \bar{U} - \rho g \sin \alpha \tag{8}$$

$$\rho \left( \frac{\partial \bar{V}}{\partial t} + \bar{U} \frac{\partial \bar{V}}{\partial X} + \bar{V} \frac{\partial \bar{V}}{\partial Y} \right) = -\frac{\partial \bar{P}}{\partial Y} - \frac{\partial \bar{\tau}_{XY}}{\partial X} - \frac{\partial \bar{\tau}_{YY}}{\partial Y} + \sigma B_0^2 \sin \beta (\bar{U} \cos \beta - \bar{V} \sin \beta) - \frac{\mu_0}{k_0} \bar{V} + \rho g \cos \alpha. \tag{9}$$

$$\rho C \left( \frac{\partial T}{\partial t} + \bar{U} \frac{\partial T}{\partial X} + \bar{V} \frac{\partial T}{\partial Y} \right) = k_1 \left[ \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right] + 2\mu_0 \left[ \left( \frac{\partial \bar{U}}{\partial X} \right)^2 + \left( \frac{\partial \bar{V}}{\partial Y} \right)^2 \right] + \mu_0 \left( \frac{\partial \bar{V}}{\partial X} + \frac{\partial \bar{U}}{\partial Y} \right)^2 + \frac{\mu_0}{k_0} (\bar{U})^2 + \sigma B_0^2 (\bar{U} \cos \beta - \bar{V} \sin \beta)^2 \tag{10}$$

$$\left( \frac{\partial c}{\partial t} + \bar{U} \frac{\partial c}{\partial X} + \bar{V} \frac{\partial c}{\partial Y} \right) = D_\beta \left( \frac{\partial^2 c}{\partial X^2} + \frac{\partial^2 c}{\partial Y^2} \right) + \frac{D_T}{T_m} \tag{11}$$

Where the components of shear stress  $\bar{\tau}$  in the fixed frame can be written as:

$$\begin{aligned} \bar{\tau}_{xx} &= -2\mu_0 [(1 + (\Gamma \dot{y})^2)^{\frac{n-1}{2}}] \frac{\partial \bar{U}}{\partial X}, \\ \bar{\tau}_{xy} &= -\mu_0 [(1 + (\Gamma \dot{y})^2)^{\frac{n-1}{2}}] \left( \frac{\partial \bar{U}}{\partial Y} + \frac{\partial \bar{V}}{\partial X} \right), \\ \bar{\tau}_{yy} &= -\mu_0 [(1 + (\Gamma \dot{y})^2)^{\frac{n-1}{2}}] \frac{\partial \bar{V}}{\partial Y}, \\ \dot{y} &= \sqrt{2 \left( \left( \frac{\partial \bar{U}}{\partial X} \right)^2 + \left( \frac{\partial \bar{V}}{\partial Y} \right)^2 \right) + \left( \frac{\partial \bar{U}}{\partial Y} + \frac{\partial \bar{V}}{\partial X} \right)^2}, \end{aligned}$$

The non-dimensional parameters are as follows:

$$\begin{aligned} x &= \frac{\bar{X}}{\lambda}, \quad y = \frac{\bar{Y}}{d}, \quad t = \frac{c \bar{t}}{\lambda}, \quad u = \frac{\bar{U}}{c}, \quad v = \frac{\bar{V}}{\delta c}, \quad \delta = \frac{d}{\lambda}, \\ h_1 &= \frac{\bar{H}_1}{d}, \quad h_2 = \frac{\bar{H}_2}{d}, \quad m = \frac{m' \lambda}{d}, \quad a = \frac{a_1}{d}, \quad b = \frac{a_2}{d}, \\ \tau &= \frac{d}{c \mu_0} \bar{\tau}, \quad p = \frac{d^2 \bar{P}}{c \lambda \mu_0}, \quad \text{We} = \frac{\Gamma c}{d}, \quad M = \sqrt{\frac{\sigma}{\mu_0}} B_0 d, \end{aligned}$$

$$k = \frac{1}{\sqrt{Da}} \frac{d}{\sqrt{k_0}}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad \text{Re} = \frac{\rho c d}{\mu_0},$$

$$\text{Pr} = \frac{\mu_0 C \rho}{k_1}, \quad \text{Ec} = \frac{c^2}{C (T_1 - T_0)}, \quad \text{Fr} = \frac{c^2}{g d}, \quad \phi = \frac{c - c_0}{c_1 - c_0},$$

$$\eta = \frac{-d^2 \rho g}{c \mu_0} = \frac{-\text{Re}}{\text{Fr}}, \quad t_{xx} = \frac{\lambda}{\mu_0 c} \bar{t}_{XX}, \quad t_{xy} = \frac{d}{\mu_0 c} \bar{t}_{XY},$$

$$t_{yy} = \frac{d}{\mu_0 c} \bar{t}_{YY}, \quad y = \frac{d}{c} \bar{y}, \quad \text{Br} = \text{Pr Ec}, \quad \text{Sc} = \frac{\mu_0}{\rho D \beta},$$

$$\text{Sr} = \frac{\rho D_T (T_1 - T_0)}{\mu_0 T_m (c_1 - c_0)}, \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{12}$$

Using the above non-dimensional quantities in Eq. (1-11), the resulting equations are:

$$\text{Re} \delta \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \delta^2 \frac{\partial}{\partial x} t_{xx} + \frac{\partial}{\partial y} t_{xy} - (M^2 \cos^2 \beta + K^2)u + M^2 \cos \beta \sin \beta \delta v + \eta \sin \alpha. \quad (13)$$

$$\text{Re} \delta^3 \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} - \delta^2 \frac{\partial}{\partial x} t_{xy} - \delta \frac{\partial}{\partial y} t_{yy} + M^2 \delta \sin \beta \cos \beta u - (M^2 \sin^2 \beta + k^2) \delta^2 v - \eta \delta \cos \alpha. \quad (14)$$

$$\text{Re Pr} \delta \left( \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \left[ \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right] + 2\delta^2 \text{Br} \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right) + \text{Br} \left( \delta^2 \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + \text{Br} (k^2 + M^2 \cos^2 \beta) u^2 - 2uvm^2 \text{Br} \delta \cos \beta \sin \beta + M^2 \text{Br} \delta^2 v^2 \sin^2 \beta \quad (15)$$

$$\text{Re Sc} \delta \left( \frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} \right) = \left[ \delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right] + \text{Sr Sc} \left( \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (16)$$

Where,

$$t_{xx} = -2\delta \left[ 1 + \left( \frac{n-1}{2} \right) (we)^2 (\dot{y})^2 \right] \frac{\partial u}{\partial x},$$

$$t_{xy} = -\left( 1 + \left( \frac{n-1}{2} \right) (we)^2 (\dot{y})^2 \right) \left( \frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right),$$

$$t_{yy} = -2\delta \left( 1 + \left( \frac{n-1}{2} \right) (we)^2 (\dot{y})^2 \right) \frac{\partial v}{\partial y},$$

$$\dot{y} = \sqrt{2\delta^2 \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right) + \left( \frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right)^2},$$

Continuity equation is automatically satisfied under the assumption of long wavelength ( $\delta \ll 1$ ) and low Reynolds number, then the Eqs. (13-16) can be written as by using stream function  $\psi$  :

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} t_{xy} - (M^2 \cos^2 \beta + k^2) \frac{\partial \psi}{\partial y} + \eta \sin \alpha \quad (17)$$

$$\frac{\partial p}{\partial y} = 0, \text{ it is noted that } p \text{ does not dependent of } y \quad (18)$$

$$0 = \frac{\partial^2 \theta}{\partial y^2} + \text{Br} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \text{Br} (M^2 \cos^2 \beta + k^2) \left( \frac{\partial \psi}{\partial y} \right)^2 \quad (19)$$

$$0 = \frac{\partial^2 \phi}{\partial y^2} + \text{Sr Sc} \frac{\partial^2 \theta}{\partial y^2} \quad (20)$$

Where,

$$t_{xy} = -\left( 1 + \left( \frac{n-1}{2} \right) (we)^2 (\dot{y})^2 \right) \frac{\partial^2 \psi}{\partial y^2}$$

$$(\dot{y})^2 = \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2$$

The appropriate boundary conditions of this problem are given by:

$$\left. \begin{aligned} \psi = \frac{F}{2}, \frac{\partial \psi}{\partial y} = 0, \theta = 1, \phi = 1 \text{ at } (y = h_2) \\ \psi = \frac{-F}{2}, \frac{\partial \psi}{\partial y} = 0, \frac{\partial \theta}{\partial y} = 0, \frac{\partial \phi}{\partial y} = 0 \text{ at } (y = h_1) \end{aligned} \right\} \quad (21)$$

In which

$$h_2 = 1 + mx + b \sin(2\pi(x - t))$$

$$h_1 = -1 - mx - a \sin(2\pi(x - t) + \phi)$$

That satisfies,  $a^2 + b^2 + 2ab \cos \phi \leq 4$

### Perturbation Resolution

It's obvious that the produced equations of motion (Eq. (16)) and equation of heat (Eq. (18)) are not linear because it contains unknown  $\psi$  of some powers which must be solved to yield the design stream function of fluid and the temperature function as well as concentration function of the using fluid. Due to that non linearity, it is difficult to solve it. Thus we have a tendency to use the perturbation technique to find the resolution. We expand  $\psi, F, p, \theta$  and  $\phi$  for series of small weissenberg number as follows:

$$\psi = \psi_0 + (we)^2 \psi_1 + O((we)^4)$$

$$\begin{aligned}
 F &= F_0 + (We)^2 F_1 + O((we)^4) \dots \\
 p &= p_0 + (We)^2 p_1 + O((we)^4) \dots \\
 \theta &= \theta_0 + (We)^2 \theta_1 + O((we)^4) \dots \\
 \varphi &= \varphi_0 + (We)^2 \varphi_1 + O((we)^4) \dots
 \end{aligned}
 \tag{22}$$

Now substituting Eq. (22) into Eqs. (17-20) and boundary conditions (21), we get the following order of systems.

**For the system of order Zero** ( $We^{(0)}$ )

$$0 = \left(\frac{\partial^4 \psi_0}{\partial y^4}\right) + (M^2 \cos^2 \beta + k^2) \frac{\partial^2 \psi_0}{\partial y^2} \tag{23}$$

$$\frac{\partial p_0}{\partial x} = -\frac{\partial^3 \psi_0}{\partial y^3} - (M^2 \cos^2 \beta + k^2) \frac{\partial \psi_0}{\partial y} + \eta \sin \alpha \tag{24}$$

$$0 = \frac{\partial^2 \theta_0}{\partial y^2} + Br \left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2 + Br(M^2 \cos^2 \beta + K^2) \left(\frac{\partial \psi_0}{\partial y}\right)^2 \tag{25}$$

$$0 = \frac{\partial^2 \varphi_0}{\partial y^2} + SrSc \frac{\partial^2 \theta_0}{\partial y^2} \tag{26}$$

Along with the corresponding boundary conditions:

$$\begin{aligned}
 \psi_0 &= \frac{F_0}{2}, \frac{\partial \psi_0}{\partial y} = 0, \theta_0 = 1, \varphi_0 = 1 \text{ at } (y = h_2) \\
 \psi_0 &= \frac{-F_0}{2}, \frac{\partial \psi_0}{\partial y} = 0, \frac{\partial \theta_0}{\partial y} = 0, \frac{\partial \varphi_0}{\partial y} = 0 \\
 &\text{at } (y = h_1)
 \end{aligned}
 \tag{27}$$

**For the System of order two** ( $We^{(2)}$ )

$$\begin{aligned}
 0 &= \frac{\partial^4 \psi_1}{\partial y^4} + 3\left(\frac{n-1}{2}\right) \left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2 \frac{\partial^4 \psi_0}{\partial y^4} + 6\left(\frac{n-1}{2}\right) \left(\frac{\partial^2 \psi_0}{\partial y^2}\right) \\
 &\left(\frac{\partial^3 \psi_0}{\partial y^3}\right)^2 + (M^2 \cos^2 \beta + K^2) \left(\frac{\partial^2 \psi_1}{\partial y^2}\right)
 \end{aligned}
 \tag{28}$$

$$\begin{aligned}
 \frac{\partial p_1}{\partial x} &= -\frac{\partial^3 \psi_1}{\partial y^3} - 3\left(\frac{n-1}{2}\right) \left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2 \frac{\partial^3 \psi_0}{\partial y^3} - (M^2 \cos^2 \beta \\
 &+ K^2) \left(\frac{\partial \psi_1}{\partial y}\right)
 \end{aligned}
 \tag{29}$$

$$\begin{aligned}
 0 &= \frac{\partial^2 \theta_1}{\partial y^2} + 2Br \left(\frac{\partial^2 \psi_0}{\partial y^2} \frac{\partial^2 \psi_1}{\partial y^2}\right) + 2Br(M^2 \cos^2 \beta + K^2) \\
 &\left(\frac{\partial \psi_0}{\partial y} \frac{\partial \psi_1}{\partial y}\right)
 \end{aligned}
 \tag{30}$$

$$0 = \frac{\partial^2 \varphi_1}{\partial y^2} + SrSc \frac{\partial^2 \theta_1}{\partial y^2} \tag{31}$$

With the appropriate boundary conditions:

$$\begin{aligned}
 \psi_1 &= \frac{F_1}{2}, \frac{\partial \psi_1}{\partial y} = 0, \theta_1 = 0, \varphi_1 = 0 \text{ at } (y = h_2) \\
 \psi_1 &= \frac{-F_1}{2}, \frac{\partial \psi_1}{\partial y} = 0, \frac{\partial \theta_1}{\partial y} = 0, \frac{\partial \varphi_1}{\partial y} = 0 \text{ at } (y = h_1)
 \end{aligned}
 \tag{32}$$

### Results and Discussion

The above results are discussed quantitatively, let the instantaneous volume rate of the flow  $F(x, t)$ , periodic in  $(x - t)$  as:

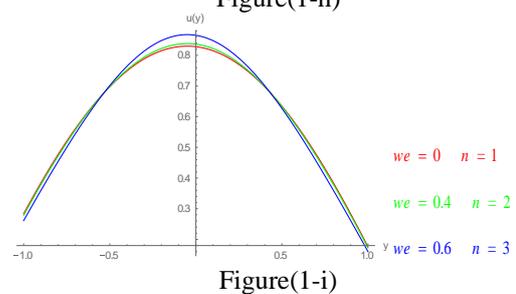
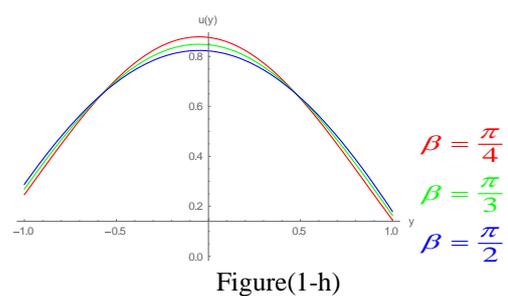
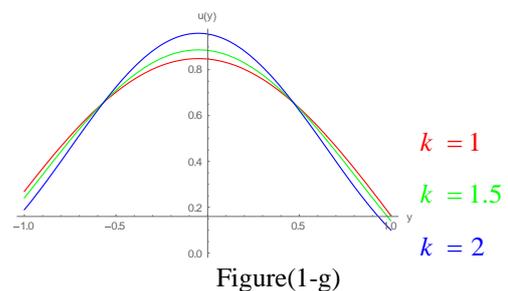
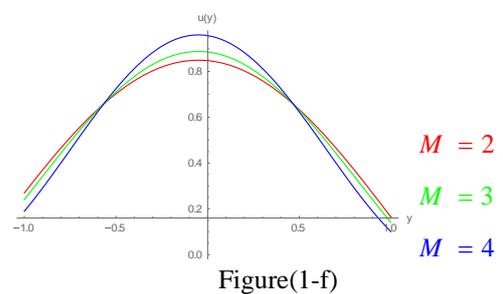
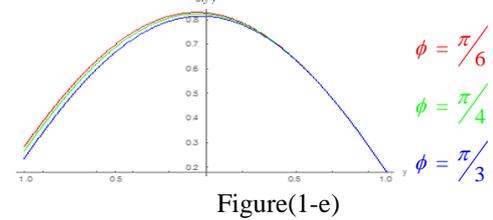
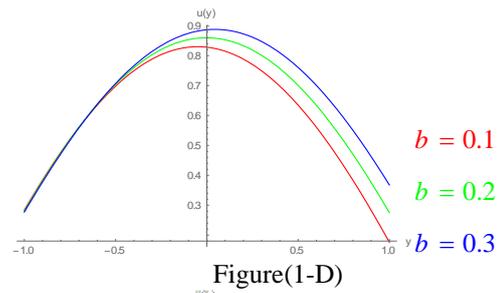
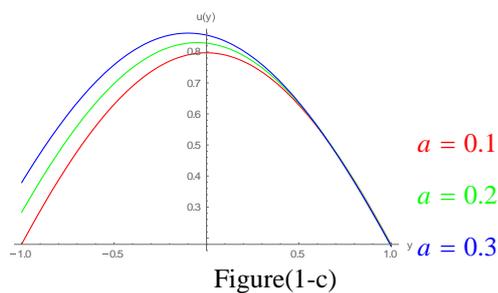
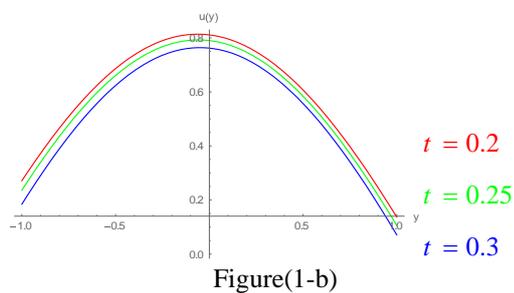
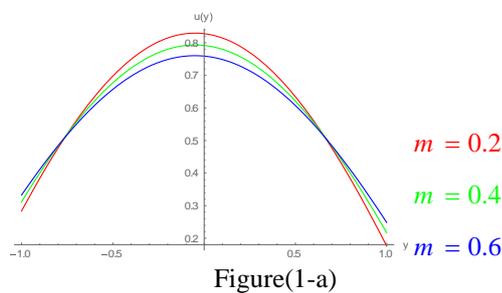
$$F(x, t) = \theta' + a \sin(2\pi(x - t) + \phi) + b \sin(2\pi(x - t)), [22] \tag{33}$$

In this section, we present the behavior of solutions of the carreau fluid flow with peristalsis through graphs.

#### Velocity allocation

Effectiveness of various parameters on the velocity allocation is clarified in Figure ((1), (a-j)). These figures are scraped at the fixed values of  $(x=0.3)$ . Figure ((1), a) displayed the effect of non-uniform parameter  $m$  on velocity distribution, it is noticed that the value of velocity decrease with an increase of  $m$  in the central part of channel but the flow of fluid will be bend at two point of the edges of the walls and then the velocity will be increased. The effect of the time on velocity distribution is illustrated in Figure (1, b), it observed that an increase in this parameter lead to reduce in magnitude of velocity. Figure (1, c) showed the impact of parameter  $a$ , it observed that velocity will be increased by clear way in the lower part of the channel and slight increase in the core and upper part of the channel. Figure (1,d) displayed the influence of parameter  $b$  on velocity, which is behaved by similar way to effect of  $a$  but in the upper wall of the channel, in the same time the effect of  $\phi$  is opposite manor of impact of  $a$  and its influence will be showed in the Figure (1, e). Figure (1,f,g) illustrated the impact of  $M$  and  $k$ , which are causes increase in the value of velocity in the central part of channel and reduce in velocity magnitude in the walls of channel, in fact it is due that magnetic field applied in transverse direction yield resistance to fluid particles which decrease the velocity. The effect of  $\beta$  on velocity is shown in Figure (1, h) which is

observed that the increasing in  $\beta$  causes an increase in velocity in the walls of channel and decreasing in core part of channel. Figure (1, i) give the graph of influence of carreau's parameters ( $we, n$ ) which are noticed that there is an increase in velocity in the central of channel and decrease in the ends of walls of channel, however in this case we can say that for Newtonian fluid ( $we = 0$  or  $n = 1$ ) the velocity is less than Newtonian fluid (carreau fluid) the impact of  $\theta'$  on velocity is shown in Figure(i, j), it is clear that an increase in this parameter lead to increase in velocity of fluid at all part of channel distribution is parabolic.



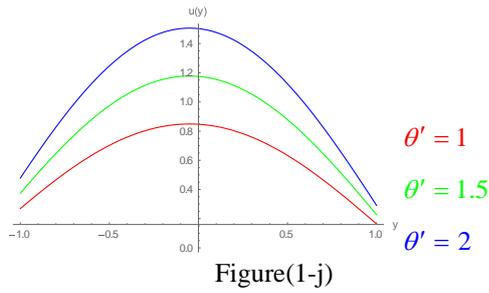
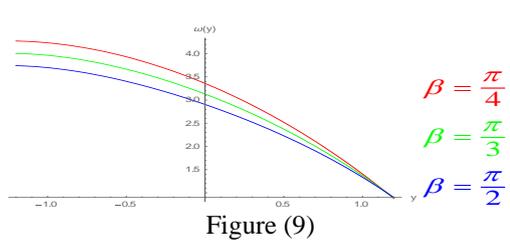
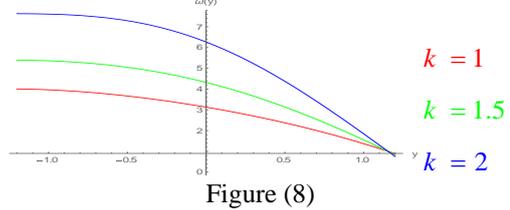
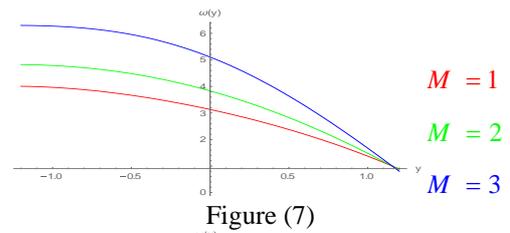
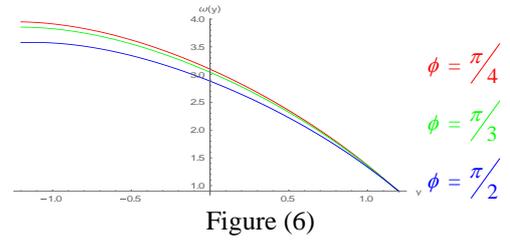
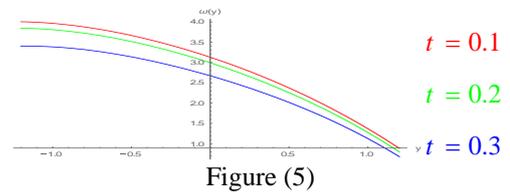
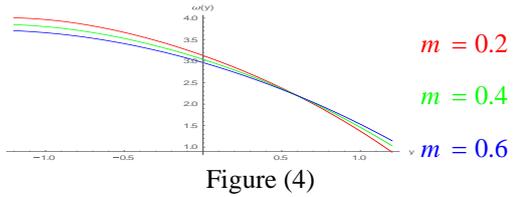
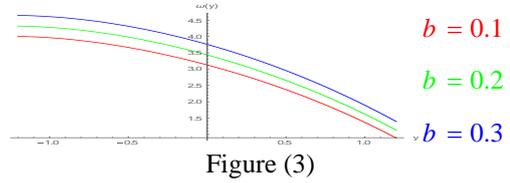
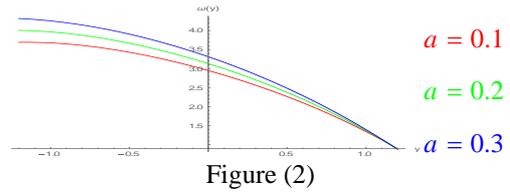


Figure (1-(a-j)): Effect of parameters on velocity profiles  
 $m = 0.2, t = 0.1, \phi = \pi/6, a = 0.2, b = 0.1, M = 1,$   
 $we = 0.6, \eta = 1, n = 0.5, \beta = \pi/3, k = 1, \theta' = 1$

**The characteristics of Temperature**

The expression for temperature versus y is given under the effect of peristaltic transport of carreau fluid is illustrated in figs. (2-12) for the fixed values of (x = 0.3) as temperature is the median kinetic energy of the particles and kinetic energy depends on velocity, therefore increase of a and b lead to temperature enhancement and their graphs are plotted in figs. (2-3) respectively. Figure (4) showed the effect of m on temperature of fluid, it is noticed that an increase in m cause a decreasing in temperature of fluid in the core of channel but it is getting high on the edges of the walls. The impact of t and  $\phi$  are shown in figs. (5-6), it's detected that the temperature is reduced with an increase of these parameters. Figure (7-8) is illustrated the influence of M and k severally, it's discovered that the temperature of fluid will be increased with a rise of M and k. converse manner is noticed for the impact of  $\beta$  and is plotted in Figure (9). Figure (10-11), the effect of  $\theta$  and Br is explained, it is noticed that a rise in these last parameters result in rise in temperature profile, it's due the fact that Brinkman number (Br) is the product of the prandtl number (Pr.) and the Eckert number (Ec.), which is happen because of the viscous dispersion actions and the temperature boost. The effects of carreau parameters (we, n) is represented in Figure (12), it's spotted that the temperature of non-Newtonian fluid is more than the temperature of Newtonian fluid (we=0 or n=1).



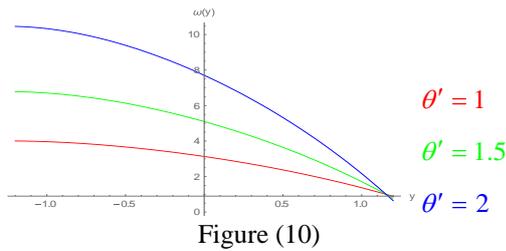


Figure (10)

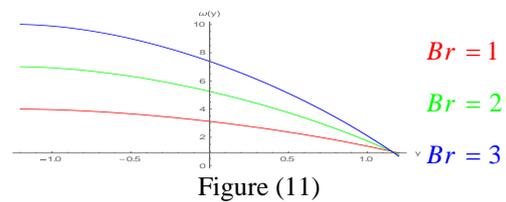


Figure (11)

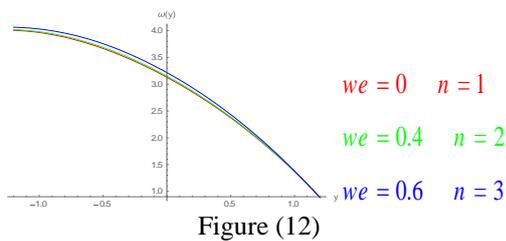


Figure (12)

Figure (2-12)): influence of parameters on profile of Temperature

$$m = 0.2, t = 0.1, \phi = \pi/6, a = 0.2, b = 0.1, M = 1, \\ we = 0.1, n = 0.5, \beta = \pi/3, k = 1, \theta' = 1, Br = 1$$

### Concentration characteristic

The profile of concentration is reverse of profile of temperature and the parameters behaved opposite manner on concentration that a temperature distribution. The effects of  $m, t, \phi, a, b, M, k, we, n, \beta, Br, Sc, Sr$  and  $\theta'$  on concentration versus are considered and illustrated in figs. (13-25). the impact of  $\beta, t, \phi$  are explained in figs. (13-25) severally, it's detected that a rise in these variables causes a rise on value of concentration. Converse manner is noticed for the parameters  $a, b, M, k, Br$  and  $\theta'$  and their graphs are examined in figs. (16-21) respectively, the impact of  $m$  on concentration is observed in Figure (22) which is showed that there is an increase in concentration in the core of channel and walls but the flow will be inflected at the ends of upper wall and so the concentration will be less. The influenced ( $we, n$ ) of carreau's parameters are stated in Figure (23),

we can say that the concentration of carreau fluid is less than Newtonian fluid ( $we=0$  or  $n=1$ ). Figure (24) is showed the effect of  $Sr$  and  $Sc$ , it's famed that a rise of those parameters yield less in concentration of fluid, the reason behind that is the mass diffusion decrease which show decrease in concentration.

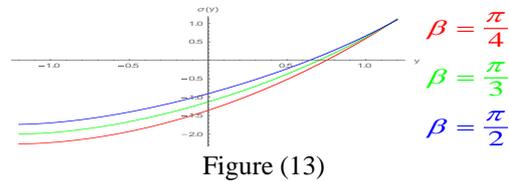


Figure (13)

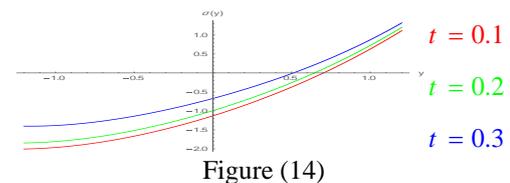


Figure (14)

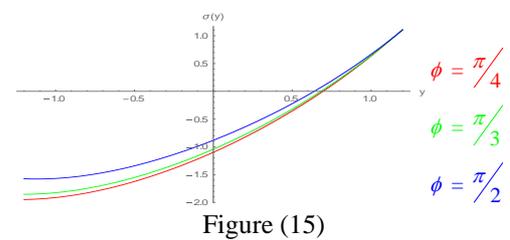


Figure (15)

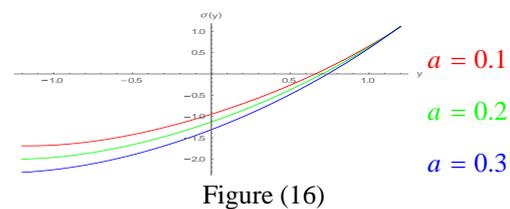


Figure (16)

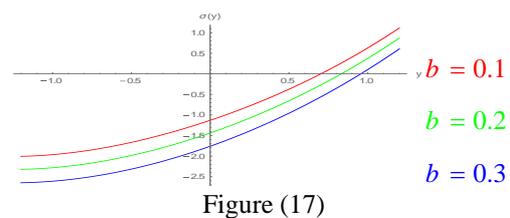


Figure (17)

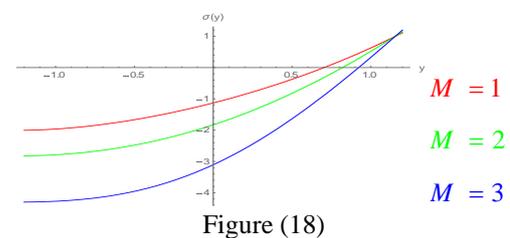


Figure (18)

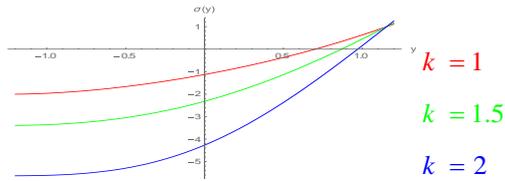


Figure (19)

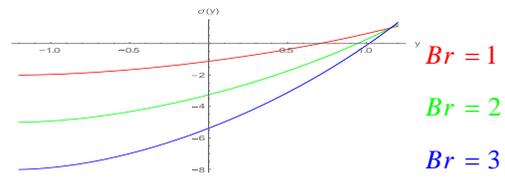


Figure (20)

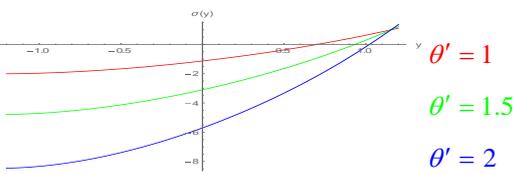


Figure (21)

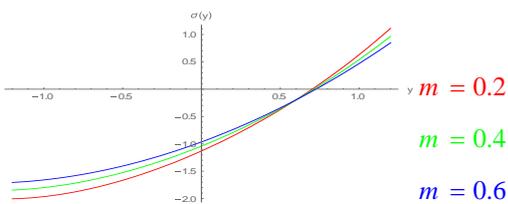


Figure (22)

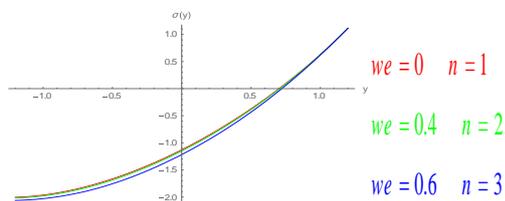


Figure (23)

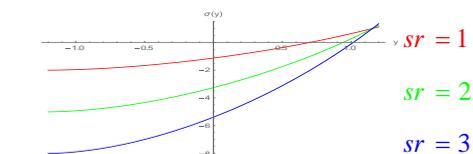


Figure (24)

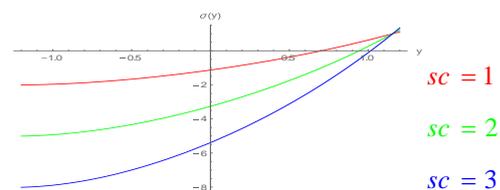


Figure (25)

Figure (13-25)): Effect of parameters on concentration profile

$$m = 0.2, t = 0.1, \phi = \pi/6, a = 0.2, b = 0.1, \\ M = 1, we = 0.1, n = 0.5, \beta = \pi/3, k = 1, \\ \theta' = 1, Br = 1, sc = 1, sr = 1$$

### Conclusions

A mathematical model for peristaltic transport of carreau fluid by using the properly of an incompressible and electrically conducting as well as the viscosity will be used as constant. Combined effects of an inclined magnetic field and heat/ mass transfer through porous material of an inclined tapered asymmetric channel. Perturbation technique is used under the approximations of long wave length and low Reynolds number to determine the analytic solutions for the stream function, temperature and concentration functions. Numerical results and obtained for the axial velocity, temperature and concentration of the fluid. The main notes are observed as follows:

- The axial velocity is increased at all parts of channel at  $\theta'$ , reversed situation for t.
- The axial velocity is increased at the core part of channel with a rise of M, k and decrease at the ends of the walls, converse manner is examined for the effect of  $\beta$ .
- The velocity of the carreau fluid is more than Newtonian fluid when (we=0 or n=1).
- The temperature of the fluid will be increased with an increase of a, b, M, k, Br and  $\theta'$  and will be increased with an increase of  $\beta, t, \phi, m$ .
- The temperature of carreau fluid is more than Newtonian fluid.
- Concentration distribution is reversal case of the temperature analysis.

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