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Research Article

Study of Nuclear Structures for Nd 148,150,152 a Isotopes by Using IBM-1

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Abstract
The nuclear structures of even-even isotopes Nd (A=148,150,152) are studied by using the first Interacting Boson Model (IBM-1). The energy levels of ground state, beta and gamma bands ,energy ratios are calculated. The results showed dynamical symmetry of these isotopes SU(3)-SU(6), SU(5)-SU(6).

الخلاصة
في البحث الحالي استخدم نموذج البوزونات المتفاعله الاول (IBM-1) لحساب مستويات الطاقة للنظير Ndضمن الاعداد الكتليه (A= 148-150-154) وتم حساب مستويات الطاقه للحاله الارضية(ground band)وحزمة بيتا وكاما وكذلك تم حساب نسب الطاقه. واظهرت النتائج التناظرات الديناميكيه للنظائر هي ,SU(3)-SU(6) SU(5)-SU(6)

Introduction

The Interacting Boson Model version one (IBM-1) describes the low-lying nuclear spectra by assuming that an even-even nucleus, consists of inert core plus some valence particle ci. e particles outside the major closed shells at 2, 8, 20, 28, 50, 82, 126and 184. [1]

Many nuclei with N and Z values between magic numbers are constantly deformed in their shape, the deformation arises because of the way valence nucleons regulate themselves in an unfilled shell, in other words the deformation happens only when both proton and neutron shells are partially filled[1].

Furthermore, the valance particles tend to pair together to form bosons one with angular momentum 0 and 2. [1] The pairs with angular momentum I=2 called d-bosons, while the pairs with angular momentum I=0 called S-bosons. [2]

Theoretical Bases

The Interacting Boson Model model (IBM) Hamiltonian operator function according to IBM-1 is written in terms of creation and annihilation operators as follows. [3,4]

$$\hat{H} = \varepsilon \hat{n}_d + a_0(\hat{P}^+ . \hat{P}) + a_1(\hat{I} . \hat{I}) + a_2(\hat{Q} . \hat{Q}) + a_3(\hat{T}_3 . \hat{T}_3) + a_4(\hat{T}_4 . \hat{T}_4) \quad (1)$$

Where ε, a_0, a_1, a_2, a_3 and a_4 are parameters used in IBM-1 to determine the Hamiltonian function, and:

$$\varepsilon = \varepsilon_d - \varepsilon_s \quad (2)$$

Where ε = Boson's energy.

$$\varepsilon_d = \text{d- Boson's energy} \quad (3)$$

$$\varepsilon_s = \text{s- Boson's energy} \quad (4)$$

$$\hat{n}_d = (\hat{d}^+ \times \tilde{\hat{d}}) \equiv d-bosons \quad operator \quad (5)$$

$$\hat{P} = \frac{1}{2}(\hat{\tilde{d}} \times \hat{\tilde{d}}) \frac{1}{2}(\hat{\tilde{S}} \times \hat{\tilde{S}}) \equiv \text{operator of pairing among bosons} \quad (6)$$

$$\hat{I} = \sqrt{10}\left[\hat{d}^+ \times \hat{d}^{\sim}\right]^{(1)} \equiv \quad (7)$$

Angular

momentum operator

$$\hat{Q} = \left[\hat{d}^{+}\times\hat{s}^{\sim}+\hat{s}^{+}\times\hat{d}^{\sim}\right]-\frac{\sqrt{7}}{2}\left[\hat{d}^{+}\times\hat{d}^{\sim}\right]^{(2)} \equiv \quad (8)$$

Quadrupole operator

$$\hat{T}_3 = \left[\hat{d}^{+}\times\hat{d}^{\sim}\right]^{(3)} \equiv \quad (9)$$

Octupole operator

$$\hat{T}_4 = \left[\hat{d}^{+}\times\hat{d}^{\sim}\right]^{(4)} \equiv \quad (10)$$

Hexadecapole operator

Where the operators of creation $\left(\hat{S}^{+},\hat{d}^{+}\right)$ and operators of annihilation $\left(\hat{S}^{\sim},\hat{d}^{\sim}\right)$ are used in fulfilling the following commutatio relations. [5-7].

Results and Discussion

This is done to study the structure of deformed nuclei. These deformed nuclei are; Nd 148– 152, Nd – 148 lying in the limit SU (5) – O (6) while the Nd – 150,154 in the limit SU (3) – O (6) dynamical symmetry. The dynamical symmetry of each isotope was determined according to the following:

1. Energy ratios shown in Table 3.
2. The typical energy spectrum was used also. [4]
3. Energy bands (i. e g, β,γ–bands)

Table 1 shows each isotope which have been studied in this work the best fitted interaction parameters values of equivalent Hamiltonian operators function of equation (1) for the energies in(MeV)units and show dynamical symmetry for each isotope.

Table 2 shows the values of parameters of the Hamiltonian operator functions, which are fitted to the data for special symmetries of these isotopes. Table 3 shows the relation between the experimental and calculated energy ratios as a function of neutron number for (Nd^{148} – Nd^{152}) respectivley. The nuclear structures of even-even nuclei in this mass region A (=148, 150, 152). We have finding the dynamical symmetry of each isotope by comparing the energy ratios with their identical and experimental values as shown in Table 3.

We used the (IBM-1)) Hamiltonian to determain theoretical energy levels compared with the experimental values of of the even – even isotopes $^{148\text{-}150\text{-}152}$Nd.

Table 3 clarify that the energy ratios $E(4_1^+)/E(2_1^+)$, $E(6_1^+)/E(2_1^+)$ and $E(8_1^+)/E(2_1^+)$ for the theoretical and experimental values for Nd(A =148–152) isotopes, there is a good agreement.

Figures 1, 2, and 3 are display the comparisons between theoretical and experimental energy levels for selected isotopes $^{148\text{-}150\text{-}152}$Nd. [8]

Figures 1, 2, and 3 indicate the valuses of bands energies (g, β, γ) for each isotope ($^{148}_{60}Nd_{88}$, $^{150}_{60}Nd_{90}$, $^{152}_{60}Nd_{92}$) rapprochement with the experimental.

From the calculated energy rates $E(4^+_1)/E(2^+_1)$, $E(6^+_1)/E(2^+_1)$ and $E(8^+_1)/E(2^+_1)$, it becomes clear that ($^{148}_{60}Nd_{88}$, $^{150}_{60}Nd_{90}$,) tend to symmetry O(6) and, $^{152}_{60}Nd_{92}$ tend to symmetry SU(3). [9]

Figures A, B, and C show the relation between the energy ratios as a function of number of neutron (N) for the even-even Nd (A=148-152) isotopes.

Conclusions

The Interacting Boson Model, version one (IBM-1), gives a good values for the energy levels as compare with the experimental values. Since the energy levels depends on the overall bosons number so that only the ground state band will appear.

From above it can be seen that when the number of bosons increase the symmetry is translate from O(6) to SU(3). The value of the energy levels is increase when the number of bosons decreases.

Energy levels of even-even isotopes ($^{148}_{60}Nd_{88}$, $^{150}_{60}Nd_{90}$, $^{152}_{60}Nd_{92}$) have been labeled according to the three bands (g, β, γ).

The β band is wider than γ band for dunamical symmetry SU(3) but γ band for isotopes of dynamical symmetry O(6) appears increasind. [10]

Table 1: Theoretical energy levels and enrgy transitions compared with experimental data for chosen even- even isotopes. [11, 12].

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Isotop	I^{π}	Energy level(MeV) Exp()	Energy level(MeV) IBM-1 (pw)	Isotop	I^{π}	Energy level(MeV) Exp()	Energy level(MeV) IBM-1 (pw)	Isotop	I^{π}	Energy level(MeV) EXP(1)	Energy level(MeV) IBM-1 (pw)
	2^+_1	0.3017	0.323		2^+_1	0.1301	0.12		2_1^+	0.0759	0.075
	4_1^+	0.7575	0.688		4_1^+	0.3815	0.397		4_1^+	0.2406	0.237
	6_1^+	1.275	1.095		6_1^+	0.7212	0.825		6_1^+	0.4879	0.47
	8_1^+		1.544		8_1^+	1.1307	1.399		8_1^+	0.81	0.764
	0_2^+	0.7237	0.724		0_2^+	0.6767	0.672		0_2^+	1.139	1.211
	2_2^+	1.171	0.767		2_2^+	0.8514	0.802		2_2^+	1. 251	1.331
	4_2^+	1.8586	1.217		4_2^+	1.1386	1.095		4_2^+		1.521
$_{60}Nd^{148}_{88}$	6_2^+		1.708	$_{60}Nd^{150}_{90}$	6_2^+		1.542	$_{60}Nd^{152}_{92}$	6_2^+		1.774
	8_2^+		2.24		8_2^+		2.132		8_2^+		2.078
	0_3^+	0.9167	1.324		2_3^+	1.0624	1.394		2_3^+		0.93
	2_3^+		1.762		3_1^+		1.536	SU(3)-O(6)	3_1^+		1.065
SU(5)-O(6)	4_3^+		1.831	SU(3)-O(6)	4_3^+		1.703		4_3^+		1.156
	6_3^+		2.405		5_1^+		1.929		5_1^+		1.346
	8_3^+		3.641		6_3^+		2.167		6_3^+		1.43
	2_4^+	1.2486	1.185		7_1^+		2.477		7_1^+		1.691
	3_1^+	1.5124	1.263		8_3^+		2.776		8_3^+		1.752
	4_4^+	1.6875	1.686								
	5_1^+		1.776								
	6_4^+		2.229								
	7_1^+		2.33								
	8_4		2.814								

Table 2: The parameters of Hamiltonian function operator for Nd (A = 148-152) isotopes.

Isotopes	N_{π}	N_{ν}	N_{Tot}	ESP MeV	$\hat{P}.\hat{P}$ MeV	$\hat{I}\hat{I}$ MeV	$\hat{Q}.\hat{Q}$ MeV	$(\hat{T}_3.\hat{T}_3)$ MeV	$(\hat{T}_4.\hat{T}_4)$ MeV	CHI
$^{148}_{60}Nd_{88}$	5	3	8	0.0010	0.0000	0.0048	0.0030	0.0501	.0738	1.3000
$^{150}_{60}Nd_{90}$	5	4	9	0.5500	0.0000	0.0080	-0.0087	0.0311	-0.0312	-1.2400
$^{152}_{60}Nd_{92}$	5	5	10	0.0001	0.0600	0.0009	-0.1300	0.0576	0.0000	-1.1680

Table 3 : show the relation between the experimental and calculated energy ratios as a function of neutron number for (Nd^{148} – Nd^{152}) respectivley. [13]

Isotopes	$E(4_1^+) / E(2_1^+)$		$E(6_1^+) / E(2_1^+)$		$E(8_1^+) / E(2_1^+)$	
	EXP.	IBM-1 (pw)	EXP.	IBM-1 (pw)	EXP.	IBM-1 (pw)
${}^{148}_{60}Nd_{88}$	2.5107	2.1300	4.2260	3.3900	0.0000	4.7801
${}^{150}_{60}Nd_{90}$	2. 9323	3.3033	5.5434	6.8750	8.6910	9.6580
${}^{152}_{60}Nd_{92}$	3.1699	3.1225	6.4281	6.2666	10.8719	10.1866

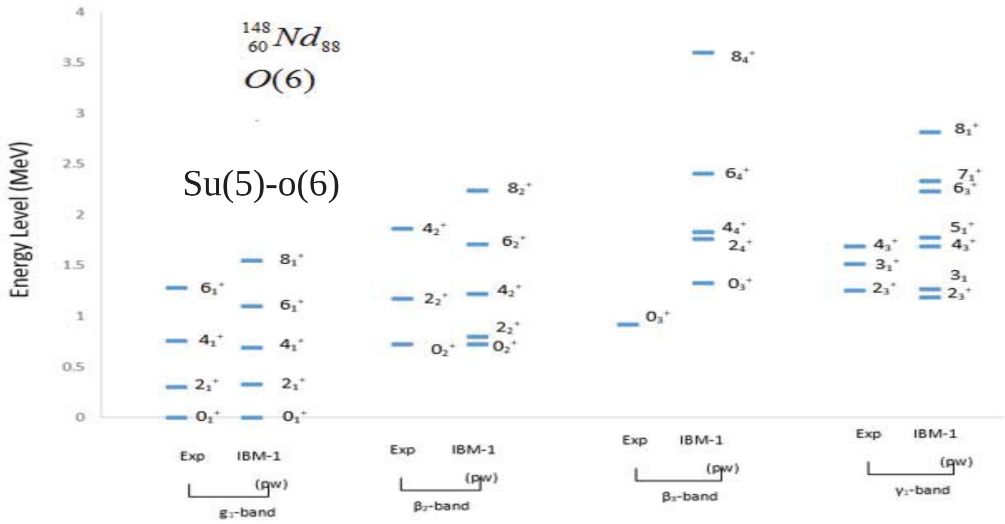


Figure 1: Comparison between calculated IBM (pw) and experimental energy levels states g, β, γin isotope ${}^{148}_{60}Nd_{88}$ of the dynamical symmetry su(5)-o(6).

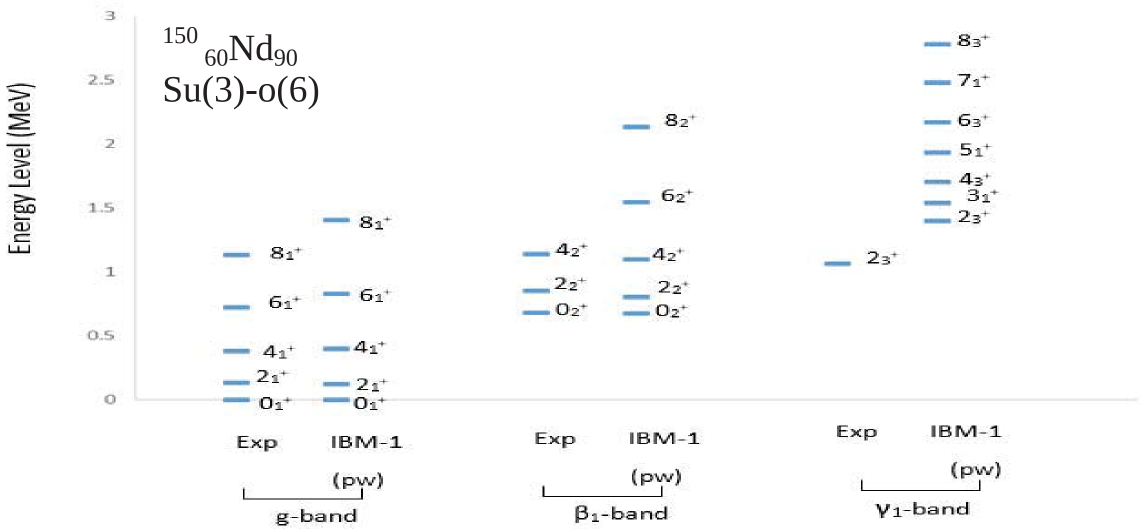


Figure 2: Comparison between calculated IBM (pw) and experimental energy levels states g, β, γin isotope ${}^{150}_{60}Nd_{90}$ of the dynamical symmetry su(3)-o(6).

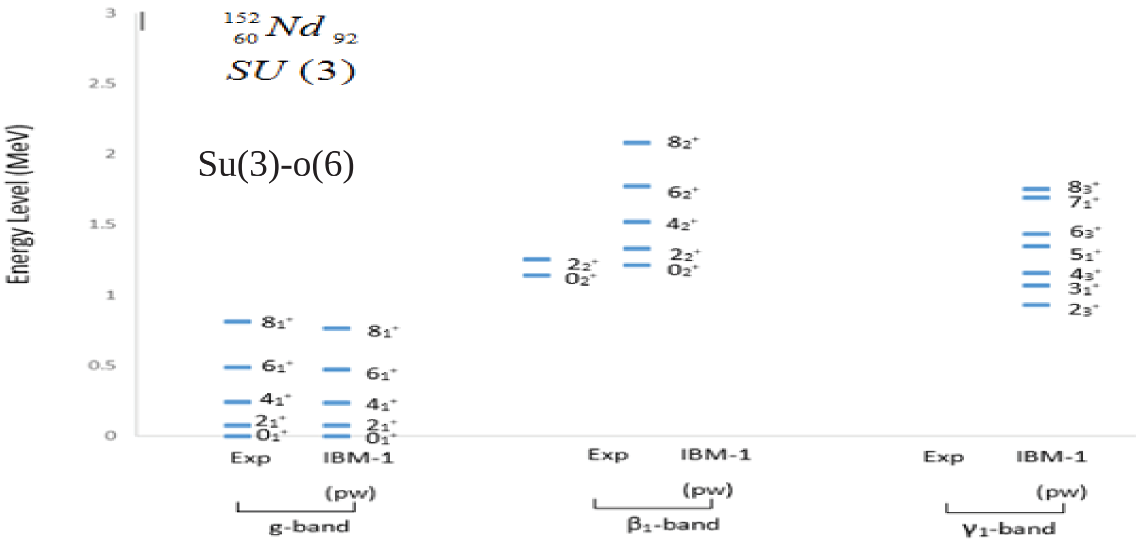
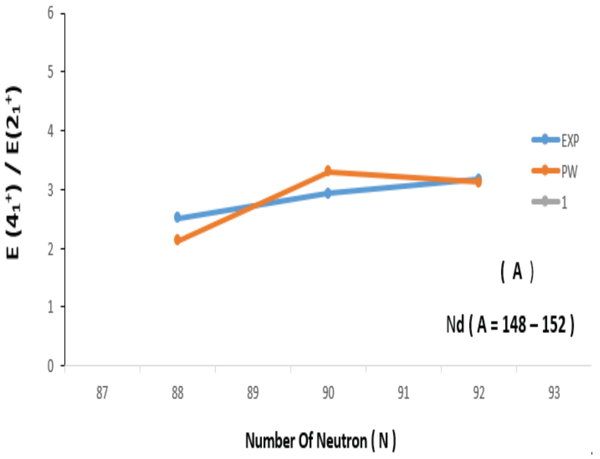
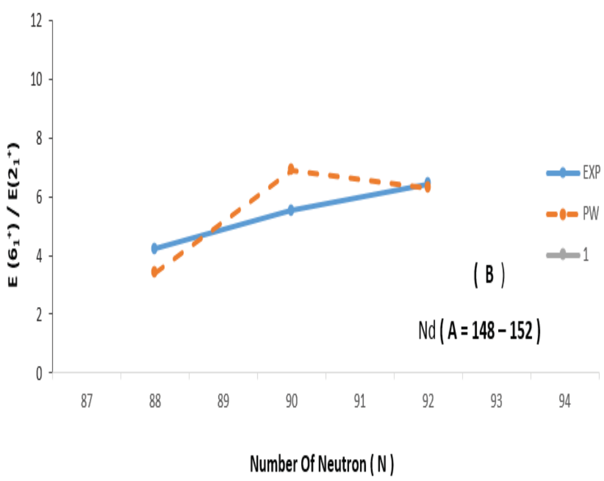
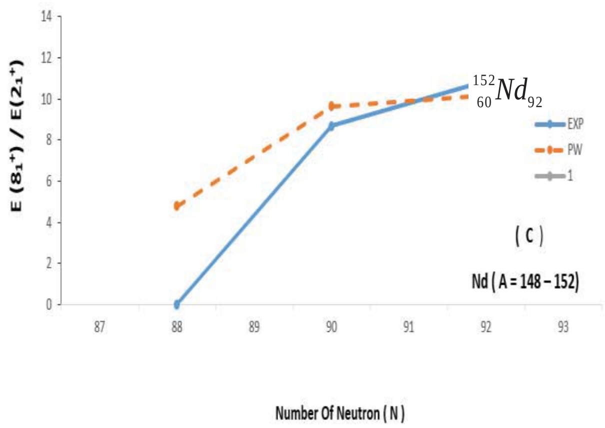


Figure 3: Comparison between calculated IBM (pw) and experimental energy levels states g, β, γin isotope of the dynamical symmetry SU(3)-o(6)







Figures A, B, C: The relation between the energy ratios as a function of number of neutron N for the even-even Nd (A=148-152) isotopes.

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