Research Article

# Comparison of Bayes Estimators for Parameter and Reliability Function for Inverse Rayleigh Distribution by Using Generalized Square Error Loss Function

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**Abstract**

In the current study, we have been derived some Basyian estimators for the parameter and reliability function of the inverse Rayleigh distribution under Generalized squared error loss function. In order to get the best understanding of the behavior of Bayesian analysis, we consider non-informative prior for the scale parameter using Jefferys prior Information as well as informative prior density represented by Gamma distribution. Monte-Carlo simulation have been employed to compare the behavior of different estimates for the scale parameter and reliability function of inverse Rayleigh distribution based on mean squared errors and Integrated mean squared errors, respectively. In the current study, we observed that more occurrence of Bayesian estimate using Generalized squared error loss function using Gamma prior is better than other estimates for all cases.

**Keywords**: Inverse Rayleigh distribution, Bayesian estimator, Generalized Squared error loss Function, Jefferys prior and Gamma prior.

**الخلاصة**

في الدراسة الحالية، تم اشتقاق على بعض المقدرات البيزية لدالة المعولية لتوزيع معكوس رايلي تحت تعميم دالة خساره الخطأ التربيعية. بغية الحصول على اكثر فهما للتحليل البيزي فقد افترضنا عدم وجود معلومات مسبقة عن معلمة الشكل باستخدام دالة جيفري للمعلومات كذلك وجود معلومات مسبقة متمثلة بتوزيع كاما. تم توظيف مونت-كارلو للمحاكاة لمقارنة سلوك مختلف تقديرات معلمة القياس ودالة المعولية لتوزيع معكوس رايلي على اساس متوسط مربعات الخطا ومتوسط مربعات الخطا التكاملي على التوالي. في الدراسة الحالية، لاحظنا ان اداء التقدير البيزي باستخدام تعميم دالة خسارة الخطا التربيعية باستخدام توزيع كاما اكثر دقة من التقديرات الاخرى في جميع الحالات.

## Introduction

The inverse Rayleigh distribution is one of important distributions. The distribution originally derived by Voda since (1972). In (1993) Gharraph derived five measures of location for the inverse Rayleigh distribution [1]. Soliman et al. (2010) studied the estimation and prediction from inverse Rayleigh distribution based on lower record values, Bayes estimator have been developed under squared error and zero one-loss functions [2]. In (2012) Dey discuss the Bayesian estimation of the Parameter and Reliability Function of an Inverse Rayleigh Distribution use different loss function which is Square error, LINEX loss function [3]. In (2013) Sindhu and other researchers studied the Bayes estimation of the parameters of the inverse Rayleigh distribution for left censored data under different loss functions (Symmetric and asymmetric) [4]. Also, In (2013) Prakash discuss the Bayes estimation in the inverse Rayleigh model under two different loss functions (Square error, LINEX loss function) [5]. In (2014) Khan obtained, the Modified inverse Rayleigh distribution is special case of inverse Weibll, which is extension to it [6]. In (2015) the Fan discuss Bayes Estimation for Inverse Rayleigh model under different loss functions squared error loss, LINEX loss and entropy loss functions[7]. In (2015) Rasheed discussed the Comparison of the classical estimators with the Bayes estimators of one Parameter inverse Rayleigh distribution under generalized squared error loss function [8]. In (2016) Rasheed and Aref discussed the Bayesian approach for estimating the scale parameter of Inverse Rayleigh distribution under different loss function [9]. Finally, in (2016) Rasheed and Aref obtained Reliability estimation in inverse Rayleigh distribution using Precautionary loss function [10]. The current study will obtain some Base estimators for Reliability estimation of inverse Rayleigh distribution using generalized square error loss function.

## One Parameter Inverse Rayleigh Distribution

The probability density function (pdf) of the inverse Rayleigh distribution with scale parameter $\theta$ is defined as follows [11]:

$$f(x;\theta)=\frac{2\theta}{x^3}e^{-\frac{\theta}{x^2}}\mathrm{x}\ >\ 0,\theta\ >0 \qquad (1)$$

The cumulative distribution function is:

$$F(x;\theta)=e^{\frac{-\theta}{x^2}};x>0\,,\theta>0 \qquad (2)$$

The Reliability, failure rate and the Cumulative failure rate (Hazard Rate) functions of this distribution are given, respectively, by:

$$R(t;\theta)=1-F(t;\theta)=1-e^{\frac{-\theta}{x^2}} \qquad (3)$$

$$h(t;\theta)=\frac{f(t;\theta)}{R(t;\theta)} \qquad (4)$$

We can say, the function h(t) is a failure function if and only if [12]

1. $h(t)\geq 0$ 2. $\int_0^\infty h(t)dt=\infty$

The relationship between R(t) and h(t) can be defined as

$$h(t)=\frac{f(t)}{R(t)}=-\frac{d}{dt}\ln R(t)$$

By integrating and using lnR(0)=ln(1)=0, yields

$$lnR(t)=-\int_0^t h(u)du$$

$$R(t)=exp\left[-\int_0^t h(u)du\right]$$

$$R(t)=exp[-H(t)]$$

$$H(t;\theta)=-Ln\,R(t)=-Ln\left(1-e^{\frac{-\theta}{x^2}}\right) \qquad (5)$$

Where H(t) is cumulative hazard function.

## *Bayesian estimators under generalized square error loss function*

Al-Nasser and Saleh (2006) introduced a new loss function in estimating the scale parameter and reliability function for Weibull distribution, which is called Generalized squared error loss function, that is defined as follows[13]:

$$L(\hat{\theta},\theta)=(\sum_{j=0}^{k}a_j\theta^j)(\hat{\theta}-\theta)^2,$$
$$k=0,1,2,3,....$$

$$L(\hat{\theta},\theta)=(a_0+a_1\theta+\cdots+a_k\theta^k)(\hat{\theta}-\theta)^2$$

So, the risk function under Generalized squared error loss function which is denoted by $R_{GS}(\hat{\theta},\theta)$, will be:

$$R(\hat{\theta},\theta)=E[L(\hat{\theta}\,,\theta)]=\int_0^\infty L(\hat{\theta}\,,\theta)h(\theta|\underline{x})d\theta$$

The risk can be minimized simply, by setting the first derivative to zero [8]

Hence, $\hat{\theta}$ based on generalized squared error loss function will be:

$$\hat{\theta}=\frac{a_0E(\theta|\underline{x})+a_0E(\theta^2|\underline{x})+\cdots+a_kE(\theta^{K+1}|\underline{x})}{a_0+a_1E(\theta|\underline{x})+\cdots+a_kE(\theta^K|\underline{x})} \qquad (6)$$

## Prior and Posterior Distribution

In the current study, we consider informative as well as non-informative prior density for $\theta$ as follows:

### *(i) Posterior Distribution Using Jeffreys Prior Information*

Assume that $(\theta)$ has non-informative prior density defined as using Jefferys prior information g $(\theta)$, which is given by[14]:

$$g_1\propto\sqrt{I(\theta)}$$

Where, $I(\theta)=-nE\left[\frac{\partial^2\ln f(x;\theta)}{\partial\theta^2}\right]$ is the Fisher information.

Hence,

$$g_1(\theta)=b\sqrt{-nE\left(\frac{\partial^2\ln f(x;\theta)}{\partial\theta^2}\right)}\text{,b is constant}$$

By taking the second derivative of log $f(x;\theta)$ with respect to $\theta$ yields

$$\frac{\partial^2 \ln f(x_{i\,;}\,\theta)}{\partial\,\theta^2} = -\frac{1}{\theta^2}$$

Therefore,

$$E\left(\frac{\partial^2\ \ln f\ (x_i\ ;\ \theta)}{\partial\,\theta^2}\right) = -\frac{1}{\theta^2}$$

$$g_1(\theta) = \frac{b}{\theta}\sqrt{n}, \theta > 0$$

Now, the posterior density function, is defined as:

$$h(\theta|\underline{x}) = \frac{g(\theta)L(\theta; x_1x_2\ldots x_n)}{\int_0^\infty g\,(\theta)L(\theta; x_1x_2\ldots x_n)d\theta} \qquad (7)$$

After simplification, the posterior density functions of $(\theta)$ based on Jefferys prior, will be:

$$h_1(\theta|\underline{x}) = \frac{T^n\,\theta^{n-1}\,e^{-\theta T}}{\Gamma n}, \mathrm{T} = \sum_{i=1}^{n}\frac{1}{{x_i}^2} \qquad (8)$$

It is clear that,
$\theta|\underline{x}\sim\text{Gamma}\ (\ n, \mathrm{T}\ )$,
with $\mathrm{E}(\theta) = \frac{n}{T}, Var\ (\theta) = \frac{n}{T^2}$

Now, based on Jeffreys prior, the m^th moment for $\theta$ is:

$$E(\theta^m|\underline{x}) = \int_0^\infty \theta^m\,\mathrm{h}_1(\theta|\underline{x})\mathrm{d}\theta$$

$$= \int_0^\infty \theta^m \frac{T^n\,\theta^{n-1}\,e^{-\theta T}}{\Gamma n}\,d\theta$$

$$\mathrm{E}(\theta^m|\underline{x}) = \frac{\Gamma n + m}{\Gamma n T^m}$$

$$\hat{\theta}_{J_1} = \frac{a_0\,\frac{n}{T} + a_1\frac{(n+1)n}{T^2}}{a_0 + a_1\,\frac{n}{T}} \qquad (9)$$

$$\hat{\theta}_{J_2} = \frac{a_0\,\frac{n}{T} + a_1\frac{(n+1)n}{T^2} + a_2\left(\frac{(n+2)(n+1)n}{T^3}\right)}{a_0 + a_1\,\frac{n}{T} + a_2\frac{(n+1)n}{T^2}} \qquad (10)$$

***(ii) Posterior Distribution Using Gamma Prior***

The prior predictive prior distribution using Gamma distribution prior is defined as follow:

$$g_2(\theta) = \frac{\beta^\alpha\,\theta^{\alpha-1}e^{-\theta\beta}}{\Gamma\alpha}; \theta > 0\ \alpha > 0, \beta > 0$$

Where, $\beta$ and $\alpha$ are the shape and the scale parameters respectively. Since, using Equation (9), the posterior distribution of $(\theta)$ will be as follows:

$$h_2(\theta|\underline{x}) = \frac{P^{\alpha+n}\,e^{-\theta P}\theta^{\alpha-1+n}}{\Gamma(\alpha+n)} \qquad (11)$$

Where, $P = \sum_{i=1}^{n}\frac{1}{{x_i}^2} + \beta$

Notice that: $\theta|\underline{x}\sim\text{Gamma}(\alpha+n, P)$, with:

$\mathrm{E}(\theta) = \frac{\alpha+n}{P}, Var\ (\theta) = \frac{\alpha+n}{P^2}$

$$E(\theta^m|\underline{x}) = \int_0^\infty \theta^m\,\mathrm{h}_2(\theta|\underline{x})\mathrm{d}\theta$$

$$= \int_0^\infty \theta^m\,\frac{P^{\alpha+n}}{\Gamma(\alpha+n)}\,\theta^{\alpha-1+n}\,e^{-\theta P}\,d\theta$$

$$\mathrm{E}(\theta^m|\underline{x}) = \frac{\Gamma\alpha+n+m}{\Gamma\alpha+n\,P^m}$$

$$\hat{\theta}_{G_1} = \frac{a_0\,\frac{(\alpha+n)}{P} + a_1\frac{(\alpha+n)(\alpha+n+1)}{P^2}}{a_0 + a_1\frac{\alpha+n}{P}} \qquad (12)$$

$$\hat{\theta}_{G_2} = \frac{a_0\,\frac{\alpha+n}{P} + a_1\frac{(\alpha+n)(\alpha+n+1)}{P^2} + a_2\left(\frac{(\alpha+n)(\alpha+n+1)(\alpha+n+2)}{P^3}\right)}{a_0 + a_1\,\frac{\alpha+n}{P} + a_2\frac{(\alpha+n)(\alpha+n+1)}{P^2}} \qquad (13)$$

## Reliability Function

In this subsection, we have derived the Bayesian estimates for reliability function of inverse Rayleigh distribution under Generalized squared error loss function, where

$$\hat{R}(t) = \frac{a_0 E\left(R(t)\middle|\underline{t}\right) + a_0 E\left(R(t)^2\middle|\underline{t}\right) + \cdots + a_k E\left(R(t)^{K+1}\middle|\underline{t}\right)}{a_0 + a_1 E(R(t)|t) + \cdots + a_k E\left(R(t)^K\middle|\underline{t}\right)} \quad (14)$$

### (i) Bayesian Estimator for Reliability function under Jeffreys prior

$$Since\ R(t) = 1 - e^{\frac{-\theta}{t^2}} \quad (15)$$

Now, according to Jeffreys prior,

$$E\left(R(t)\middle|\underline{t}\right) = \int_0^\infty R(t)\ h_1\left(\theta\middle|\underline{t}\right) d\theta$$

$$E\left(R(t)\middle|\underline{t}\right) = 1 - \left(\frac{Tt^2}{t^2T+1}\right)^n \quad (16)$$

$$E\left((R(t))^2\middle|\underline{t}\right) = \int_0^\infty (R(t))^2\ h_1\left(\theta\middle|\underline{t}\right) d\theta$$

$$E\left((R(t))^2\middle|\underline{t}\right) = 1 - 2\left(\frac{Tt^2}{t^2T+1}\right)^n + \left(\frac{Tt^2}{t^2T+2}\right)^n \quad (17)$$

And $E\left((R(t))^3\middle|\underline{t}\right) = \int_0^\infty (R(t))^3\ h_1\left(\theta\middle|\underline{t}\right) d\theta$

$$E\left((R(t))^3\middle|\underline{t}\right) = 2 - 3\left(\frac{Tt^2}{t^2T+1}\right)^n + \left(\frac{Tt^2}{t^2T+2}\right)^n \quad (18)$$

Hence, the Bayesian estimators for the R(t) based on Generalized squared error loss function using Jeffreys prior with the first and second order polynomials, which are denoted by $\hat{R}(t)_{GSJ1}$, $\hat{R}(t)_{GSJ2}$ respectively, are as follows

$$\hat{R}(t)_{GSJ1} = \frac{a_0\left(1 - \left(\frac{Tt^2}{t^2T+1}\right)^n\right) + a_1\left(1 - 2\left(\frac{Tt^2}{t^2T+1}\right)^n + \left(\frac{Tt^2}{t^2T+2}\right)^n\right)}{a_0 + a_1\left(1 - \left(\frac{Tt^2}{t^2T+1}\right)^n\right)} \quad (19)$$

$$\hat{R}(t)_{GSJ2} = \frac{a_0\left(1 - \left(\frac{Tt^2}{t^2T+1}\right)^n\right) + a_1\left(1 - 2\left(\frac{Tt^2}{t^2T+1}\right)^n + \left(\frac{Tt^2}{t^2T+2}\right)^n\right) + a_2\left(2 - 3\left(\frac{Tt^2}{t^2T+1}\right)^n + \left(\frac{Tt^2}{t^2T+2}\right)^n\right)}{a_0 + a_1\left(1 - \left(\frac{Tt^2}{t^2T+1}\right)^n\right) + a_2\left(1 - 2\left(\frac{Tt^2}{t^2T+1}\right)^n + \left(\frac{Tt^2}{t^2T+2}\right)^n\right)} \quad (20)$$

### (ii) Bayes Estimator for Reliability function under Gamma prior

According to Gamma prior, $E\left(R(t)\middle|\underline{t}\right)$ will be

$$E\left(R(t)\middle|\underline{t}\right) = \int_0^\infty R(t)\ h_2\left(\theta\middle|\underline{t}\right) d\theta$$

$$E\left(R(t)\middle|\underline{t}\right) = 1 - \left(\frac{Pt^2}{t^2P+1}\right)^{\alpha+n} \quad (21)$$

$$E\left((R(t))^2\middle|\underline{t}\right) = \int_0^\infty (R(t))^2\ h_2\left(\theta\middle|\underline{t}\right) d\theta$$

$$E\left((R(t))^2\middle|\underline{t}\right) = 1 - 2\left(\frac{Pt^2}{t^2P+1}\right)^{\alpha+n} + \left(\frac{Pt^2}{t^2p+2}\right)^{\alpha+n} \quad (22)$$

And

$$E\left((R(t))^3\middle|\underline{t}\right) = 2 - 3\left(\frac{Pt^2}{t^2P+1}\right)^{\alpha+n} + \left(\frac{Pt^2}{t^2P+2}\right)^{\alpha+n} \quad (23)$$

$$\hat{R}(t)_{GSG1} = \frac{a_0\left(1 - \left(\frac{Pt^2}{Pt^2+1}\right)^{\alpha+n}\right) + a_1\left(1 - 2\left(\frac{Pt^2}{Pt^2+1}\right)^{\alpha+n} + \left(\frac{Pt^2}{Pt^2+2}\right)^{\alpha+n}\right)}{a_0 + a_1\left(1 - \left(\frac{Pt^2}{Pt^2+1}\right)^{\alpha+n}\right)} \quad (24)$$

$$\hat{R}(t)_{GSG2} = \frac{a_0\left(1-\left(\frac{Pt^2}{Pt^2+1}\right)^{\alpha+n}\right)+a_1\left(1-2\left(\frac{Pt^2}{Pt^2+1}\right)^{\alpha+n}+\left(\frac{Pt^2}{Pt^2+2}\right)^{\alpha+n}\right)}{a_0+a_1\left(1-\left(\frac{Pt^2}{Pt^2+1}\right)^{\alpha+n}\right)+a_2\left(1-2\left(\frac{Pt^2}{Pt^2+1}\right)^{\alpha+n}+\left(\frac{Pt^2}{Pt^2+2}\right)^{\alpha+n}\right)} +$$

$$\frac{a_2\left(2-3\left(\frac{Pt^2}{t^2P+1}\right)^{\alpha+n}+\left(\frac{Pt^2}{t^2P+2}\right)^{\alpha+n}\right)}{a_0+a_1\left(1-\left(\frac{Pt^2}{Pt^2+1}\right)^{\alpha+n}\right)+a_2\left(1-2\left(\frac{Pt^2}{Pt^2+1}\right)^{\alpha+n}+\left(\frac{Pt^2}{Pt^2+2}\right)^{\alpha+n}\right)} \quad (25)$$

## Simulation Results

In the current the simulation study, we generated samples of different sizes, $n$ = 10, 25, 50, and 100 from one parameter inverse Rayleigh distribution with three different values of $(\theta = 0.5, 1.5 \text{ and } 3)$. The constants of Generalized squared error loss function are choose as $a_0 = 5000,\ a_1 = 10$ and $a_2 = 0.5$. The values of parameters of Gamma distribution prior are $(\alpha = 0.8, \beta = 1.2, 3)$. The processes have been repeated 5000 times. The expected values and mean squared errors (MSE's) for the Bayes estimates of the parameter $\theta$ are obtained, where

$$MSE(\theta) = \frac{\sum_{i=0}^{R}\left(\hat{\theta}_i - \theta\right)^2}{R}\ ;\ i = 1,2,3,\ldots,R$$

And integral mean squared error (IMSE) for all estimates of the reliability function of Inverse Rayleigh distribution are obtained which is defined as follows:

$$IMSE\left(\hat{R}(t)\right) = \frac{\sum_{j=0}^{n_t} MSE\left(\hat{R}_i\left(t_j\right)\right)}{n_t}$$

Where, i = 1, 2, …, L ; $n_t$ is the random limits of $t_i$

In current paper, we use

$t = 1.5, 1.8, 2.1, 2.4, 2.7, 3$

The results of the simulation study for estimating the scale parameter (θ) were summarized and tabulated in tables (1), (2) and (3) which are contain the expected values and MSE's for different estimates of the scale parameter, while tables (4),

(5) and (6) are contain IMSE's for different estimate of the Reliability function. We have observed that:

1. Table (1), shows that, the Bayesian estimation based on Gamma prior under Generalized Squared error loss function (GSELF) with ($\beta$=3 and $\alpha$=0.8) is the most occurrence in comparing to the others.
2. Table (2), shows that, the Bayes estimator according to Gamma prior under Generalized Squared error loss function (GSELF) with (β=3 and α=0.8) is the best estimator comparing to the other estimator for all sample size expect the sample (10) the performance under Gamma prior with (β=1.2 and α=0.8) is the best estimator.
3. Table (3), shows that, the performance of Bayes estimator with Gamma prior under Generalized Squared error loss function (GSELF) with (β=1.2 and α=0.8) is the best estimator comparing to the other estimator for all sample size.
4. Table (4), shows that, the performance of Bayes estimator with Gamma prior under Generalized Squared error loss function (GSELF) with (β=1.2 and α=0.8) is the best estimator comparing to the other estimator for all sample size expect the sample (10) the performance under Gamma prior with (β=3 and α=0.8) is the best estimator.
5. Table (5) and (6), shows that, the performance of Bayes estimator with Gamma prior under Generalized Squared error loss function (GSELF) with (β=1.2 and α=0.8) is the best estimator comparing to the other estimator for all sample size.
6. Provided that chosen the value of $\beta$ is inversely proportional to the value of $\theta$ in estimation the scale parameters
7. It is clear that, MSE's of all estimates of scale parameter is increasing with increase of the parameter value with all sample sizes.
8. In general, we conclude that, in situation involving estimation of parameter and Reliability of inverse Rayleigh distribution using Generalized squared error loss function using Gamma prior is most occurrence in comparing to the corresponding other estimates.

Table 1: Expected values and MSE's of the different estimators for the Inverse Rayleigh distribution with θ=0.5, A0 = 5000, A1 = 10, A2 = 0.5, α=0.8

| Estimator | | n / Criteria | 10 | 25 | 50 | 100 |
|---|---|---|---|---|---|---|
| $\theta_{J1}$ | | EXP. | 0.5533714 | 0.5218075 | 0.5102229 | 0.5053623 |
| | | MSE | 0.0420948 | 0.0128188 | 0.0056789 | 0.0026506 |
| $\theta_{J2}$ | | EXP. | 0.5533767 | 0.5218090 | 0.5102234 | 0.5053626 |
| | | MSE | 0.0420985 | 0.0128190 | 0.0056790 | 0.0026507 |
| $\hat{\theta}_{G_1}$ | β=1.2 | EXP. | 0.5563744 | 0.5247822 | 0.5119841 | 0.5063035 |
| | | MSE | 0.0362698 | 0.0124200 | 0.0056124 | 0.0026389 |
| | β=3 | EXP. | 0.5050812 | 0.5055112 | 0.5026784 | 0.5017222 |
| | | MSE | 0.0216966 | 0.0101397 | 0.00508069 | 0.00250855 |
| $\hat{\theta}_{G_2}$ | β=1.2 | EXP. | 0.5563788 | 0.5247834 | 0.5119846 | 0.5063038 |
| | | MSE | 0.0362724 | 0.0124202 | 0.0056124 | 0.0026389 |
| | β=3 | EXP. | 0.5050845 | 0.5055123 | 0.5026789 | 0.5017225 |
| | | MSE | 0.0216976 | 0.0101399 | 0.00508073 | 0.00250857 |

Table 2: Expected values and MSE's of the different estimators for the Inverse Rayleigh distribution with θ=1.5, A0 = 5000, A1 = 10, A2 = 0.5, α=0.8

| Estimator | | n / Criteria | 10 | 25 | 50 | 100 |
|---|---|---|---|---|---|---|
| $\theta_{J1}$ | | EXP. | 1.6605250 | 1.56555900 | 1.53073200 | 1.51611700 |
| | | MSE | 0.3793886 | 0.11542730 | 0.05112276 | 0.02385865 |
| $\theta_{J2}$ | | EXP. | 1.6606700 | 1.56559600 | 1.53074600 | 1.51612500 |
| | | MSE | 0.3796858 | 0.11544850 | 0.05112683 | 0.02385961 |
| $\hat{\theta}_{G_1}$ | β=1.2 | EXP. | 1.4708810 | 1.49835300 | 1.49902000 | 1.50066800 |
| | | MSE | 0.1725416 | 0.08656178 | 0.04455720 | 0.02228035 |
| | β=3 | EXP. | 1.16716300 | 1.3521190 | 1.42206800 | 1.46113700 |
| | | MSE | 0.17613910 | 0.0786355 | 0.04201312 | 0.02151709 |
| $\hat{\theta}_{G_2}$ | β=1.2 | EXP. | 1.4709580 | 1.49838300 | 1.4990340 | 1.50067500 |
| | | MSE | 0.1726013 | 0.08657283 | 0.04455989 | 0.02228096 |
| | β=3 | EXP. | 1.1672000 | 1.3521410 | 1.42208100 | 1.46114300 |
| | | MSE | 0.1761270 | 0.0786349 | 0.042013140 | 0.02151714 |

Table 3: Expected values and MSE's of the different estimators for the Inverse Rayleigh distribution with θ=3, A0 = 5000, A1 = 10, A2 = 0.5, α=0.8

| Estimator | n / Crite- | 10 | 25 | 50 | 100 |
|---|---|---|---|---|---|

| | | ria | | | | |
|---|---|---|---|---|---|---|
| $\theta_{J1}$ | | EXP. | 3.3222770 | 3.1315240 | 3.0616580 | 3.0323300 |
| | | MSE | 1.5207390 | 0.4620535 | 0.2045646 | 0.0954521 |
| $\theta_{J2}$ | | EXP. | 3.3234290 | 3.1318160 | 3.0617810 | 3.0323900 |
| | | MSE | 1.5254320 | 0.4623968 | 0.2046310 | 0.0954667 |
| $\hat{\theta}_{G_1}$ | β=1.2 | EXP. | 2.5057740 | 2.7952670 | 2.8937830 | 2.9482420 |
| | | MSE | 0.5943608 | 0.3019409 | 0.1655405 | 0.0856107 |
| | β=3 | EXP. | 1.7478720 | 2.3286370 | 2.6206160 | 2.7995250 |
| | | MSE | 1.6488420 | 0.5743750 | 0.2470325 | 0.1075162 |
| $\hat{\theta}_{G_2}$ | β=1.2 | EXP. | 2.5061360 | 2.7954480 | 2.8938880 | 2.9482920 |
| | | MSE | 0.5943246 | 0.3019750 | 0.1655527 | 0.0856141 |
| | β=3 | EXP. | 1.7479900 | 2.3287460 | 2.6206900 | 2.7995720 |
| | | MSE | 1.6485810 | 0.5742652 | 0.2469937 | 0.1075047 |

Table 4: IMSE's of the Different Estimators for the Inverse Rayleigh distribution θ=0.5, A0 = 5000, A1 = 10, A2 = 0.5 and $R(t)$ =0.0540405, α=0.8

| Estimator \ n | | 10 | 25 | 50 | 100 |
|---|---|---|---|---|---|
| $\theta_{J1}$ | | 0.001679 | 0.000576 | 0.000265 | 0.0001266 |
| $\theta_{J2}$ | | 0.001692 | 0.000581 | 0.000266 | 0.0001264 |
| $\hat{\theta}_{G_1}$ | β=1.2 | 0.001472 | 0.000556 | 0.00026 | 0.0001254 |
| | β=3 | 0.0009735 | 0.0004745 | 0.0002434 | 0.0001224 |
| $\hat{\theta}_{G_2}$ | β=1.2 | 0.001488 | 0.000562 | 0.000263 | 0.0001257 |
| | β=3 | 0.0009653 | 0.0004712 | 0.0002407 | 0.0001202 |

Table 5: IMSE's of the Different Estimators for the Inverse Rayleigh distribution θ=1.5, A0 = 5000, A1 = 10, A2 = 0.5 and $R(t)$ =0.0540405, α=0.8

| Estimator \ n | | 10 | 25 | 50 | 100 |
|---|---|---|---|---|---|
| $\theta_{J1}$ | | 0.0064909 | 0.002557 | 0.0012243 | 0.0005951 |
| $\theta_{J2}$ | | 0.0064888 | 0.0025566 | 0.0012235 | 0.0005945 |
| $\hat{\theta}_{G_1}$ | β=1.2 | 0.0040743 | 0.0021145 | 0.0011193 | 0.0005704 |
| | β=3 | 0.0057931 | 0.0023291 | 0.0011799 | 0.0005884 |
| $\hat{\theta}_{G_2}$ | β=1.2 | 0.0040354 | 0.0020983 | 0.0011103 | 0.0005657 |
| | β=3 | 0.0056619 | 0.0022716 | 0.0011494 | 0.0005727 |

Table 6: IMSE's of the Different Estimators for the Inverse Rayleigh distribution θ=3, A0 = 5000, A1 = 10, A2 = 0.5 and $R(t)$ =0.0540405, α=0.8

| Estimator \ n | | 10 | 25 | 50 | 100 |
|---|---|---|---|---|---|
| $\theta_{J1}$ | | 0.0097372 | 0.0041687 | 0.0020556 | 0.0010152 |
| $\theta_{J2}$ | | 0.0097182 | 0.0041611 | 0.0020514 | 0.0010131 |
| $\hat{\theta}_{G_1}$ | β=1.2 | 0.00948 | 0.0039513 | 0.0020082 | 0.0010035 |
| | β=3 | 0.0292004 | 0.0089456 | 0.0035154 | 0.0014181 |
| $\hat{\theta}_{G_2}$ | β=1.2 | 0.0093714 | 0.0039059 | 0.0019848 | 0.0009917 |
| | β=3 | 0.0289152 | 0.0088241 | 0.0034536 | 0.0013872 |

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