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Mathematical Approximation for Modeling the Photonic Crystal Fibers

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Abstract

In this research, a mathematical approximation for modeling PCFs is derived. The effective refractive index n_{eff} of the clad region obtained by this relation will be used to study the properties of the PCFs such as normalized frequency, effective area, group velocity dispersion and the nonlinear coefficient. All these properties are studied when the holes of the PCFs are filled with air, or with any material like chloroform.
The merit of our mathematical approximation is that, n_{eff} can be easily and directly calculated, then the dispersion profile can be controlled by fine manipulating the (d/Λ) of circular or elliptical air holes in PCF cladding. The results show good agreement with published works. The MATLAB 2010 program is used in this study, which is the most successful program to get appropriate diagrams and results.

Keywords: Photonic Crystal Fibers (PCF), dispersion, refractive index.

Introduction

Photonic crystal fibers (PCFs) which are a new optical fibers with a complex microstructure of various types, can offer advantage in the design of fiber.
Recent years Photonic crystal fibers (PCFs) were an important phenomenon because of there's widespread applications including medical diagnosis , bio imaging ,bio logical sample detection, environment monitoring sensors[1][2] also interesting light guiding mechanisms and many peculiarities (such as high non linearity ,high numerical aperture ,large mode area and endlessly single mode and so on[3][4].
Photonic crystal fibers (PCFs) can be considered as *the state of the art* fibers, because it has unique properties compared with traditional optical fibers (OFs). The properties of the PCFs can be adjusted by manipulation of its main parameters such as; the hole radius (d), pitch size (Λ), air filling factor (d/Λ), Number of rings(Nr), and the type of air or liquid in the holes[3][5].

The features of a PCF are determined by an unusual structure with a spatially periodic microstructure having air array micro holes in the silica cladding around a solid or hollow core. Light in a PCF with a solid core of high refractive index is mainly guided via modified total internal reflection similar to a conventional fiber. Light in a PCF with a hollow core, however, is guided by a so-called photonic band gap (PBG) effect[3]. These unique features come from the fact that optical properties of the guided modes in the core can be easily manipulated by just vary the geometrical parameters air-hole diameter (d) and pitch (Λ) in the cladding[5] as shown in Figure 1. The characteristics of the cladding modes play critical roles in devices that utilize the coupling between the core and the cladding modes[6][7]. In optical communication, dispersion plays an important role as it determines the information carrying capacity of the fiber. Therefore, it becomes important to study the dispersion properties and effective aria of PCF[8].

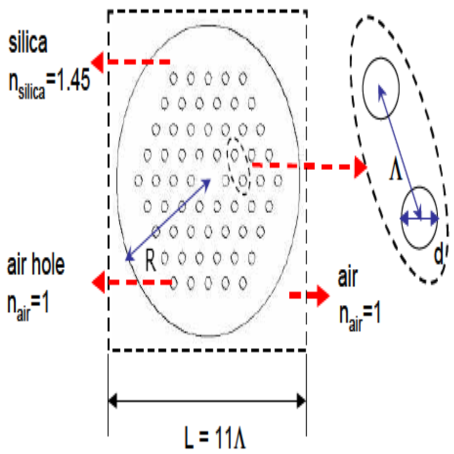

Figure 1: geometrical parameters of photonic crystal fiber (PCF).

Theoretically Background

The aim of this paper is to achieve the Mathematical Approximation for Modeling the Photonic Crystal Fibers to obtain the effective refractive index n_{eff} of the cladding which has circular or ellipse hole as in the following formula[9]:

$$n_{eff} = n_{air} \times p_{air} + n_{silica} \times p_{silica} \quad (1)$$

where p_{air} and p_{silica} are the power fraction of air and silica respectively, that make up the microstructure cladding .

By using Sellmeier formula[9]:

$$n_{silica}^2(\lambda) = 1 + \frac{A_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3\lambda^2}{\lambda^2 - \lambda_3^2} \quad (2)$$

where n_{Silica} is the refractive index of fused silica, λ_j= λ_1, λ_2, λ_3 are an atomic resonance in the fused silica :

A_1=0.6961663 @ λ_1=0.0684043µm,
A_2=0.4079426 @ λ_2=0.1162412 µm,
A_3=0.8974794 @ λ_3= 9.8961600 µm

A_1, A_2, A_3 are the Sellmeier constants. For fused silica,

Let the width of the Cladding (L), which equal to the length of light beam through the cladding (l_{air} air holes and l_{silica} fused silica).

with speed v_{air}, v_{silica} respectively , If the time which the light is required to cross the cladding is (t), then:

$$t = \frac{l_{air}}{v_{air}} + \frac{l_{silica}}{v_{silica}} \quad (3)$$

as $n = \frac{c}{v}$

where n is the refractive index of the medium, C is the light velocity in vacuum.

$$\therefore t = \frac{n_{air} \times l_{air}}{C} + \frac{n_{silica} \times l_{silica}}{C} \quad (4)$$

also,

$$t = \frac{n_{eff} \times L}{C} \quad (5)$$

Where n_{eff} is the effective refractive index of the cladding.

From equation (5, 6) we get:

$$n_{eff} = \frac{n_{air} \times l_{air}}{L} + \frac{n_{silica} \times l_{silica}}{L} \quad (6)$$

Then we reach the formula:

$$(n_{eff} = n_{air} \times p_{air} + n_{silica} \times p_{silica})$$

By trial-and-error adjustment of the effective refractive index of cladding we adding the factor:

$$n_{CF} = \left[n_S - 0.06\left(\frac{d}{(4^{1/\Lambda})\Lambda} - n_{eff}\right)\left(\frac{l-\Lambda}{0.6-\Lambda}\right)\right] \quad (7)$$

to equation (1) and using the adjustment equation of the effective refractive index we get:

$$n_{EFF} = n_{eff} + \left[n_{silica} - 0.06\left(\frac{d}{(4^{1/\Lambda})\Lambda} - n_{eff}\right)\left(\frac{l-\Lambda}{0.6-\Lambda}\right)\right] \quad (8)$$

1. Calculation of the air part and Silica part of the cladding

The cladding width equals the different between the fiber radius and the core radius of the photonic crystal fiber:

$(R_{PCF} - R_{core})$

As, an index-guiding PCF with a pure silica core is a hexagonal pattern of air holes, the effective refractive index of the cladding equal the effective refractive index of apart facing central angle equal 30°

(the surface has 6 equal parts),When we find the total parts of air and silica in the cladding we can applied the equation (1) to get the effective refractive index of the cladding.

1.1 PCF of circular air hole

To compute the total parts of air in the cladding air holes , we must tack a number of light beams, for example(40beams)which make an angle θ where $0 \leq \theta \leq 30$, then by using the circular equation , with the help of quadratic equation we can get the long of air beam through any number of holes like four holes:

$$(P_{air})_{total} = \sum_{Nr=1}^{4} (P_{air} + Pa_{12} + Pa_{23} + Pa_{34} + Pa_{44}) \quad (9)$$

Then the total parts of silica is:

$$(P_{silica})_{total} = [(Nr - 1) \times \Lambda + 5a] - (P_{air})_{total} \quad (10)$$

Then we can compute the effective refractive index of the cladding for different wavelength as λ=[0.5-2]µm by using MATLAB program in this research, which is the most successful program to get

Appropriate diagrams and simulation result :

1- the effective refractive index as a function of wavelength for one ring and d/Λ=(0.2, 0.4, 0.6, 0.8) µm.

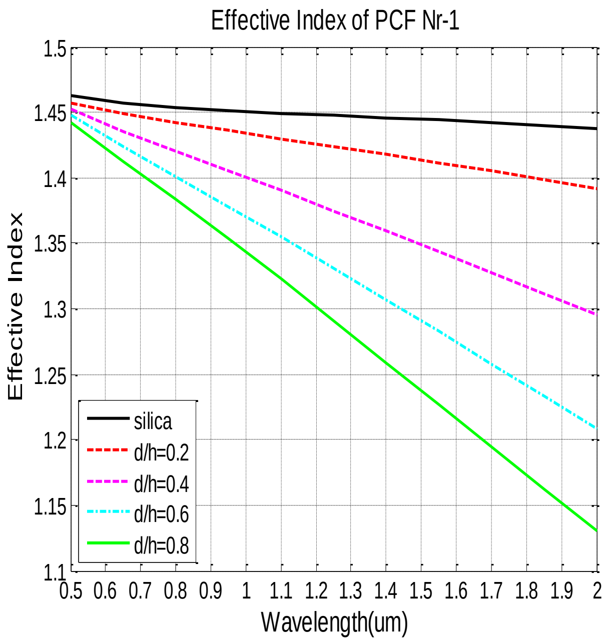

Figure 2: Effective cladding index as a function of wavelength with one ring for different d/Λ.

2- the effective refractive index as a function of wavelength for Nr= 1,2 ,3 and 4 and d/Λ=0.6µm

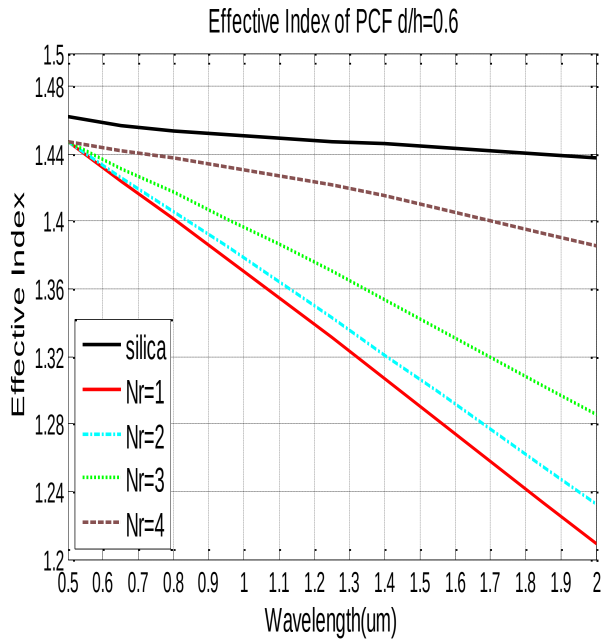

Figure 3: Effective cladding index as a function of wavelength for different rings when d/Λ=0.6 µm.

Chromatic Dispersion

The dispersion $D(\lambda)$ is defined as the change in pulse width per unit distance of propagation (i.e., ps/(nm.km). the total chromatic dispersion is the summation of material dispersion $Dm(\lambda)$ and waveguide dispersion $Dw(\lambda)$. Dispersion can be calculated from following equation:

$$D = \frac{\lambda}{C} \frac{\partial^2 R_e(n_{EFF})}{\partial \lambda^2} \quad (11)$$

Where , λ is the operating wavelength in µm, c is the velocity of the light in a vacuum, Re(n_{EFF}) is the real part of the effective index. by changing the parameters of the cladding (Λ, d, Nr, MNr & d/Λ) the zero dispersion wave length is shifted toward the shortest wave length.

3- Dispersion as a function of wavelength for solid-core PCFs in silica with one ring of holes when d/Λ=(0.2, 0.4, 0.6, 0.8) µm, Figure 4.

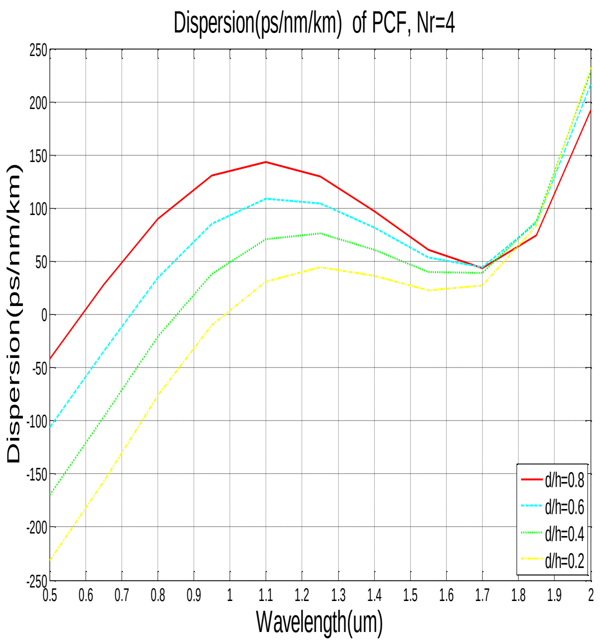

Figure 4: Dispersion as a function of wavelength.

2- Dispersion as a function of wavelength for solid-core PCFs in silica for value of d/Λ=0.486 with various rings of holes Nr=1, 2, 3, 4.

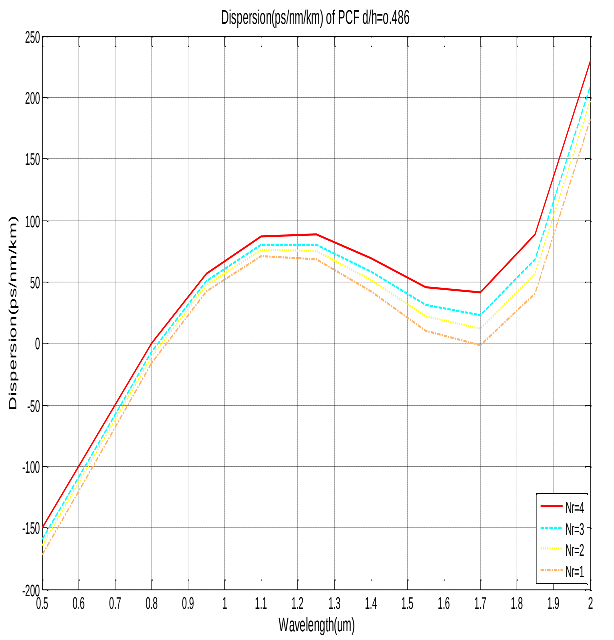

Figure 5: Dispersion as a function of wavelength for solid-core PCFs in silica for value of d/Λ=0.486 with various rings of holes.

3- Zero dispersion λ_{ZD}=0.8(ps/nm/km), when d/Λ=0.577.

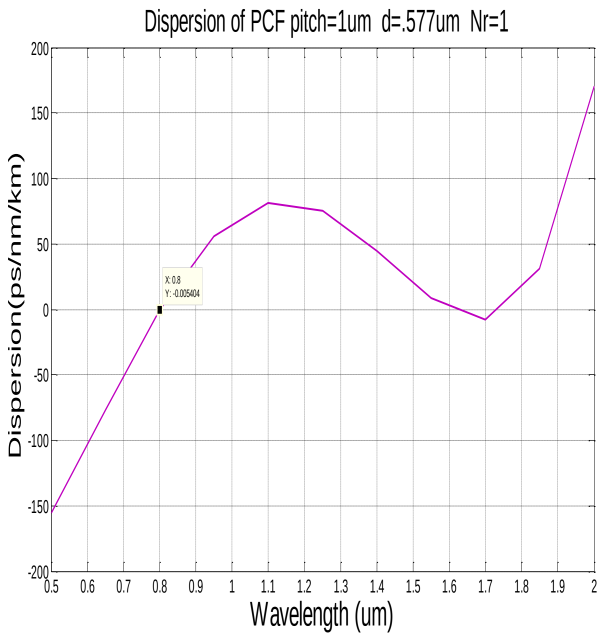

Figure 6: Dispersion as a function of wavelength for different relative air-hole sizes and λ_{ZD}=0.8(ps/nm/km), when d/Λ=0.577.

4- Effective area as a function of wavelength for Nr=1, 2, 3, 4.

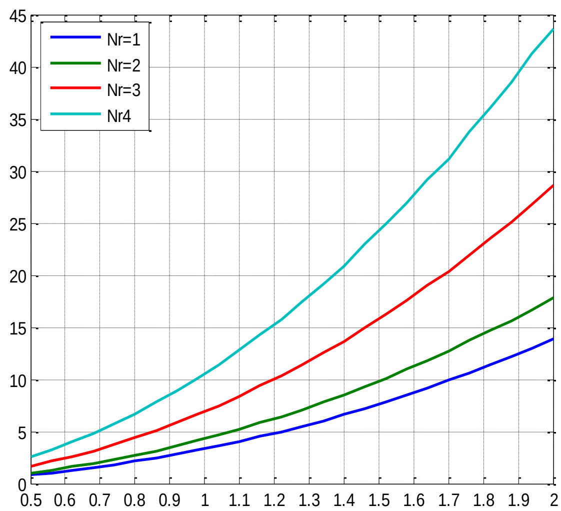

Figure 7: Effective area as a function of wavelength for Nr=1, 2, 3, 4 Pitch=1, a=0.15

Conclusion

The mathematical approximation method is easy to compute effective index of cladding. It can be used for PCFs for both circular and/or elliptical holes, in addition to filling these holes by gas or liquid instead of air. It has been found that the effective index decreases with the increase of wavelength and air hole diameter and increase with the increase of pitch of PCF. Dispersion is calculated for different structural parameters. It has been found that it increases with the increase in short wavelength, air hole diameter, and number of air hole rings, and that it decreases with the increase in pitch. In addition, this method is easy in calculating the effective index for hybrid designs that lead to important non-linear results.

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