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Research Article

# Certain Types of m-Compact Functions

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**Abstract**
In this work we introduce some functions by using cocompact open sets and semi cocompact open sets in m-structure space namely, m-coc-continuous, m-coc*-continuous, m-coc**-continuous, m-coc-compact, m-s-coc-continuous, m-s-coc*-continuous, m-s-coc**-continuous functions the relationships between these concepts have been studied also we give some facts, examples and propositions to illustrate our results.

**Keywords**: m-compact,m-open,m-continuous.

**الخلاصة**
في عملنا هذا قدمنا بعض الدوال باستخدام المجموعات المتراصة المفتوحة من نمط coc باستخدام الفضائات ذات البنية-m اسميناها، المستمرة m-coc، المستمرة m-coc*، المستمرة m-coc**، المتراصة m-coc، المستمرة m-s-coc*، المستمرة m-s-coc**، كذلك درست العلاقة بين هذه المفاهيم وقدمت بعض الامثلة والحقائق والمبرهنات لتعزيز نتائجنا.

## Introduction

In the present paper by m-structure space we mean minimal structure space, ($X\neq\Phi$ and $m_X=\{\Phi,X\}$)[1], every element belong to mx is said to be mx-open set, a function from m-structure space $(X,m_X)$ into an m-structure $(Y,m_Y)$ is said to be m-continuous if the inverse image of every an my-open set in Y is an mx-open set in X [2], If $(X, m_X)$ is an m-structure space and $\{U_\alpha\}_{\alpha\epsilon\mu} \subset m_X$ such that $\{U_\alpha\}_{\alpha\epsilon\mu}$ is a cover to X, then $\{U_\alpha\}_{\alpha\epsilon\mu}$ is called an $m_X$-open cover to X [3]. A subset K of an m-structure space X is said to be m-compact if every $m_x$-open cover of K reducible to a finite subcover [3], an m-structure $m_X$ on a non-empty set X is said to have the property ($\beta$) if the union of any family of subsets belong to $m_X$ belong to $m_X$ and said to have property ($\gamma$) if the intersection of a finite number of $m_X$-open sets is $m_X$-open [1]. In [5] the authors define coc-open sets by the same context we can define $m_x$-open sets by using m-structure spaces.

**Definition 1:**
Let $(X,m_x)$ be an m-structure space, then a subset A of X is said to be an $m_x$-cocompact open set (briefly m-coc-open set) if for every $x \in A$ there exists an mx-open set $U \subset X$ and m-compact subset K such that $x \in U - K \subseteq A$, the complement of $m_X$-coc-open set is called $m_X$-coc-closed set. The family of all mx-coc-open sets in X is denoted by $m^k$.

***Proposition 1:***
Let $(X, m_X)$ be an m-structure space, then $m^k$ forms an m-structure space.

**Proof:**
By definition $\phi \in m^k$, to show that $X \in m^k$, take $U=X$ and $K=\phi$ then $x\in U-K\subseteq X$.

**Remark 1:**
Let X is an m-structure space then $m_x\subseteq m^k$, i.e every $m_x$-open set is $m_x$-coc-open set but the converse may be not true.

***Example 1:***
Let (R, mind) be the indiscrete m-structure space and $Y = R-\{0\}$ is not an m-open in (R, mind), but

for each x ϵ Y there is an mx-open subset R of X and K = {0, x+1} is an m-compact set (since K is a finite set) and x ϵ R – K ⊆ Y. so Y is $m_x$-coc-open set but not an mx-open.

***Definition 2:***
Let f be a function from an m-structure space $(X,m_x)$ into an m-structure space $(Y, m_Y)$, then f is said to be m-coc-continuous function if the inverse image of every $m_Y$-open set U in Y is $m_x$-coc-open in X.
Clearly, every m-continuous function is m-coc-continuous function but the converse may be not true.

***Example 2:***
Let f be the identity function from the indiscrete m-structure space $(R, m_{ind})$, into the discrete m-structure space $(R, m_D)$ so f is m-coc-continuous function but not m-continuous function.

***Proposition 2:***
If f:X→Y is an m-coc-continuous function from m-structure space X into m-structure space Y such that Y has the property (**β**) then, f(m-coc-cl(A)) ⊆ m-cl(f(A))

**Proof**:
Since Y has the property (**β**) then m-cl(f(A)) is $m_Y$-closed in Y but f is m-coc-continuous, then $f^{-1}$(m-cl(f(A)) is m-coc-closed in X containing A, i.e., A ⊆ $f^{-1}$(m-cl(f(A))) m-coc-cl(A) ⊆ m-coc-cl($f^{-1}$(m-cl(f(A)))) f(m-coc-cl(A)) ⊆ m-coc-cl(f($f^{-1}$(m-cl(f(A))))⊆m-cl(f(A)) f(m-coc-cl(A))⊆ m-cl(f(A)).

***Definition 3:***
Let f be a function from an m-structure space $(X, m_x)$ into an m-structure space $(Y, m_Y)$, then f is said to be m-coc*-continuous function if the inverse image of every $m_Y$-coc-open set U in Y is $m_x$-open in X.

***Definition 4:***
Let f be a function from an m-structure space $(X, m_x)$ into an m-structure space $(Y, m_Y)$, then f is said to be m-coc**-continuous function if the inverse image of every $m_Y$-coc-open set U in Y is $m_x$-coc-open in X.

***Propostions 3:***
1. Every m-coc*-continuous function is m-coc-continuous function.
2. Every m-continuous function is m-coc-continuous function.
3. Every m-coc**-continuous function is m-coc-continuous function.
4. Every m-coc*-continuous function is m-coc**-continuous function.
5. Every m-coc-continuous function is m-coc**-continuous function.
6. Every m-coc*-continuous function is m-continuous function.
7. Every m-continuous function is m-coc**-continuous.

**Proof (1):**
Let U be an $m_Y$-open set in Y but every $m_Y$-open set is mY-coc-open set and since f is m-coc*-continuous function then $f^{-1}(U)$ is $m_X$-open which is an $m_X$-coc-open set in X and so f is m-coc-continuous function.
By the same way we can prove the others. The converse of (1) may not be true:-

***Example 3:***
Let IX :$(X, m_x)$→$(X, m_{ind})$ such that $I_x(x)=x$, for each x ϵX then $I_x$ is m-coc-continuous function but not m-coc*-continuous.

***Definition 5:***
A subset A of an m-structure $(X, m_x)$ is said to be an $m_x$-semi-open set in X (briefly $m_x$-s-open set) if A ⊆ $m_x$-cl($m_x$-int(A)).
So, every mx-open set is $m_X$-s-open set but the converse may be not being true.

***Example 4:***
Let R be the set of all real numbers and τu be the usual topological space so it is an m-structure space, then the set [0,1) in (R, τu) is $m_R$-s-open set which is not $m_R$-open set.

***Definition 6:***
A subset A of an m-structure space $(X, m_X)$ is said to be $m_x$-semi cocompact open set (briefly $m_x$-s-coc-open set) if for every x ϵ A there exists an mx-s-open set U ⊆ X and an m-compact subset K such that x ϵ U – K ⊆ A, the complement of $m_x$-s-coc-open set is called $m_x$-s-coc-closed set.

***Definition 7:***
Let f be a function from an m-structure space $(X,m_x)$ into m-structure space $(Y, m_Y)$ then f is

said to be m-s-coc-cotinuous function if the inverse image of mY-open set in Y is an $m_x$-s-coc-open set in X.

***Definition 8:***
Let f be a function from an m-structure space (X, $m_x$) into m-structure space (Y, $m_Y$), then f is said to be an m-s-coc*–continuous function if the inverse image of any $m_Y$-s-coc-open set in Y is $m_x$-open set in X.

***Definition 9:***
Let f be a function from an m-structure space (X, $m_x$) into m-structure space (Y, $m_Y$), then f is said to be an m-s-coc**-continuous function if the inverse image of every $m_Y$-s-coc-open set in Y is $m_x$-s-coc-open set in X.

***Propositions 4:***
1. Every m-continuous function is m-s-coc-continuous function.
2. Every m-coc-continuous function is m-s-coc-continuous function.
3. Every m-s-coc*-continuous function is m-continuous function.
4. Every m-coc*-continuous function is m-s-coc-continuous function.
5. Every m-coc**-continuous function is m-s-coc-continuous function.
6. Every m-s-coc*-continuous function is m-coc*-continuous function.
7. Every m-s-coc*-continuous function is m-coc**-continuous function.

**Proof (1):**
Let f be a continuous function from m-structure space X into m-structure space Y, let U be $m_Y$-open subset of Y then f-1(U) is $m_X$-open in X, but every $m_X$-open set is $m_X$-coc-open, and since every m-coc-open set is m-s-coc-open set then, f-1(U) is $m_X$-s-coc-open set in X, thus f is m-s-coc-continuous function.
By the same way we can prove the others.

***Example 5:***
Let f :(X, $m_{cof}$) →(Y, $m_{ind}$), where X and Y are m-structure spaces, $m_{cof}$ is the cofinite m-structure space, $m_{ind}$ is the indiscrete m-structure space and f(x)=c for each x ϵ X, then
1-the constant function is m-coc-continuous function and so it is m-s-coc-continuous.
2-if X is the indiscrete m-structure space, then f is m-coc-continuous and so it is an m-s-coc-continuous.

***Example 6:***
Let f: (X,$m_X$) →(Y, $m_{ind}$), where (X, $m_X$) & (Y, $m_{ind}$) are m-structure spaces and f(x) = x, for each x ϵ X, then f is m-continuous, since the only $m_y$- open sets in Y are φ and Y, so it is an m-coc-continuous & m-s-coc-continuous.

**Remark 2:**
Every m-coc*-continuous function is m-coc-continuous function but the convers may be not true.

***Example 7:***
Let $I_X$ : (X,$m_X$) →(Y, $m_{ind}$) be a function such that I(x)=x, for each x ϵ X, then $I_X$ is m-coc-continuous function but not m-coc*-continuous function, also it is not an m-s*-coc-continuous function.

***Example 8:***
Let f be a function from an m-structure space (X, $m_D$) into an m-structure space (Y,$m_Y$) where $m_D$ is the discrete m-structure space, then f is continuous since if U ⊂ Y then $f^{-1}$(U) is an open subset of X, but X is the discrete space then $f^{-1}$(U) is $m_X$-open in X so it is $m_X$-coc-open set and $m_X$-s-coc-open set therefore f is m-continuous function and m-s-coc-continuous function.

**Remark 3:**
Every subset of a discrete space X is $m_x$-coc-open set and $m_x$-s-coc-open set. Since every subset of a discrete space is $m_x$-open. On m-coc-compact functions in [4], the author defines coc-compact set, in this work by the same context we can define m-coc-compact set by using m-structure spaces.

***Definition 10:***
If (X, $m_X$) is an m-structure space and $\{U_\alpha\}_{\alpha\epsilon\mu} \subseteq m_X$ such that $\{U_\alpha\}$ is a cover to X, then $\{U_\alpha\}_{\alpha\epsilon\mu}$ is called an $m_X$-coc-open cover to X if every $U_\alpha$ is an $m_X$-coc-open set.

***Definition 11:***
A subset A of an m-structure space X is said to be m-coc-compact set if for every $m_X$-coc-open cover to A has a finite subcover.

**Remarks 4:**
1- Every coc-compact set is an m-coc-compact set to the same topology.
2- every m-coc-compact set is an m-compact set (since, if we suppose that A is an m-coc-compact set, to show that it is m-compact, let $\{U_\alpha\}_{\alpha\epsilon\mu}$ be an $m_X$-open cover to A but every $m_X$-open set is an $m_X$-coc-open set so $\{U_\alpha\}_{\alpha\epsilon\mu}$ is $m_X$-coc-open cover to A which is an $m_x$-coc-compact set so $\{U_\alpha\}_{\alpha\epsilon\mu}$ is reducible to a finite subcover, that is, A is an m-compact).

***Example 9:***
Let (R, $m_{ind}$) be the indiscrete m-structure space, then R is an m-compact space which is not an m-coc-compact space since any $m_X$-open cover to R is either {R,ф} or {R} which is finite so (R, mind) is m-compact but not m-coc-compact, that is, if we take $\{x\}_{x\epsilon R}$ is an $m_X$-coc-open cover to R we can not reduce it to a finite subcover.

***Definition 12:***
A function f from m-structure space (X, $m_X$) into m-structure space (Y, $m_Y$) is said to be m-compact function if $f^{-1}(K)$ is m-compact for every m-compact set K in Y.

***Definition 13:***
A function f from m-structure space (X, $m_X$) into m-structure space (Y, $m_Y$) is said to be m-coc-compact function if $f^{-1}(M)$ is m-coc-compact for every m-compact set M in Y.

**Remark 5:**
Every m-coc-compact function is m-compact function but the converse is not true in general.

***Example 10:***
Let R be the set of all real numbers and let f: (R, $m_{ind}$)→(R, $m_R$) be a function defined by f(x)=c for each x ϵ R, then f is an m-compact function but not an m-coc-compact function.

***Proposition 5:***
Every m-closed subset of an m-compact space is also an m-compact.
**Proof:**
let F be an $m_x$-closed subset of X, & let $\{U_\alpha\}_{\alpha\epsilon\mu}$ be an $m_x$-open cover to F but F is an $m_x$-closed subset of X so F' is $m_x$-open subset of X then X= F' ∪ $\{U_\alpha\}_{\alpha\epsilon\mu}$ but X is m-compact, then X= F' ∪ $\{U_\alpha\}$in but F' ∩ F=ф so F $\subseteq \cup_{i=1}^{n}$ Uαi then F is m-compact

***Propostion 6:***
Let f: (X, $m_X$)→(Y, $m_Y$) be an m-coc-compact function then if A is a closed subset of X, then f|A: (A, $m_A$)→(Y, $m_Y$) is m-compact

**Proof:**
let K be an m-compact subset of Y, then $f^{-1}(K)$ is m-coc-compact subset of X, so $f^{-1}(K)$ is m-compact of X, but A is m-closed in X then $f^{-1}|_A(K)=f^{-1}(K) \cap A$ is an m-compact subset of A, and so $f|_A$ is an m-compact function.

***Propostion 7:***
Let X, Y and G be an m-structure spaces and f1: (X, $m_X$)→(Y, $m_Y$) & f2: (Y, $m_Y$)→(G, $m_G$) be a continuous functions, then
1- If $f_1$ is an m-compact function and $f_2$ is m-coc-compact function then $f_{2o}f_1$ is an m-compact function.
2- If $f_1$ is m-coc-compact function and $f_2$ is m-compact function then $f_{2o}f_1$ is m-coc-compact function.
3- If $f_1$ is m-compact function and $f_2$ is m-compact function then $f_{2o}f_1$ is m-compact.

**Proof:**
1- Let K be an m-compact subset of G, then $f_2^{-1}(K)$ is an m-coc-compact subset of Y and so $f_2^{-1}(K)$ is an m-compact subset of Y, also $f_1$ is an m-compact function then $f_1^{-1}(f2^{-1}(K))$ is an m-compact in X but $f_1^{-1}(f2^{-1}(K))=(f^1{}_o f^2)^{-1}(K)$ therefore $f_{1o}f_2$ is an m-compact function.
2- Let M be an m-compact subset of G, then $f_2^{-1}(M)$ is an m-compact subset of Y, and since $f_1$ is m-coc-compact function then $f_1^{-1}(f_2^{-1}(M))$ is an m-coc-compact in X, but $f_1^{-1}(f_2^{-1}(M))=(f_{2o}f_1)^{-1}(M)$ therefore $f_{2o}f_1$ is an m-coc-compact function.
3- Let N be an m-compact subset of G, then $f_2^{-1}(N)$ is an m-compact subset of Y $f_1$ is m-coc-compact function $f_1^{-1}(f_2^{-1}(N))$ is an m-compact in X, but $f_1^{-1}(f_2^{-1}(N))=(f_{2o}f_1)^{-1}(N)$ therefore $f_{2o}f_1$ is an m-compact function.

## Conclusion
In our work we conclude the following results:
1. Every m-coc*-continuous function is m-coc-continuous function.

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2. Every m-continuous function is m-coc-continuous function.
3. Every m-coc**-continuous function is m-coc-continuous function.
4. Every m-continuous function is m-s-coc-continuous function.
5. Every m-coc-continuous function is m-s-coc-continuous function.
6. Every m-s-coc*-continuous function is m-continuous function.

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