Research Article

On Prime and Semiprime Rings with Symmetric Generalized Biderivations

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Abstract

The propose of this paper is to present some results concerning the symmetric generalized Biderivations when their traces satisfies some certain conditions on an ideal of prime and semiprime rings. We show that a semiprime ring R must have a nontrivial central ideal if it admits appropriate traces of symmetric generalized Biderivations, under similar hypothesis we prove commutativity in prime rings.

Keywords: Prime rings, Semiprime rings, symmetric generalized Biderivations, traces of biadditive maps.

الخلاصة

الهدف من هذه البحث هو تقديم بعض النتائج المتعلقة بالدوال ثنائية المشتقات المُعَمَّمة المتناظرة عندما تحقق دوال الأثر لها بعض الشروط المحددة على مثالي ما في الحلقات الأولية وشبه الأولية. أثبتنا إن الحلقة الأولية يجب ان يكون لها مثالي مركزي إذا تفسح لدوال الأثر المناسبة الخاصة بالدوال ثنائية المشتقات المُعَمَّمة المتناظرة. تحت شروط مشابهة برهنا الأبدالية في الحلقات الأولية.

Introduction

Throughout this paper, R will represent an associative ring and $Z(R)$ will denote the center of R. Recall that R is prime if for any $a,b \in R$, $aRb=\{0\}$ implies either $a=0$ or $b=0$ and semiprime if for any $a \in R$, $aRa=\{0\}$ implies $a=0$. A ring R is said to be a 2-torsion free, if $2x=0$, $x\in R$, implies $x=0$. For $x,y \in R$, the symbol $[x, y]$ will denote the commutator $xy-yx$ and $x \circ z$ denote the anti-commutator $xy + yx$. A mapping $\mathcal{B}$: $R\times R\rightarrow R$ is said to be symmetric if $\mathcal{B}(x, y) = \mathcal{B}(y, x)$ for all $x,y \in R$. A mapping f: $R \longrightarrow R$ defined by $f(x) = \mathcal{B}(x, x)$, where $\mathcal{B}$ is a symmetric mapping will be called the trace of $\mathcal{B}$. It is obvious that, in case $\mathcal{B}$ is a symmetric mapping which is also biadditive (i.e., additive in both arguments), the trace of $\mathcal{B}$ satisfies the relation $f(x+y)=f(x) +2\,\mathcal{B}(x,y)+f(y)$, for all $x,y \in R$. For a symmetric biadditive mapping $\mathcal{B}$, then $\mathcal{B}(0, y) =0$ fulfilled for all $y \in R$. consequently, $\mathcal{B}(-x, y) =- \mathcal{B}(x, y)$ for all pairs $x,y\in R$. Therefore, the trace of any biadditive mapping is an even map.

As usual, an element $c \in R$ for which $f(c) = \mathcal{B}(c, c) =0$ is called constant. Let S be a sub ring of R. A mapping $\mathcal{L}$: $S\longrightarrow R$ is said to be centralizing on S if $[\mathcal{L}(x), x] \in Z(R)$, for all $x \in S$. Furthermore, $\mathcal{L}$ is called commuting whenever $[\mathcal{L}(x), x] =0$, for all $x \in S$ [1]. A symmetric biadditive mapping $D(.,.)$: $R\times R\longrightarrow R$ is called symmetric Biderivation if for all $x,y,z \in R$ $D(xy, z)=D(x, z)\, y + xD(y, z)$ is fulfilled. In [2] G. Maksa introduced the concept of a symmetric Biderivation. While in [3], he showed that symmetric Biderivations are related to general solutions of some functional Equation. The notion of additive commuting mappings is closely connected with notion of Biderivations, that is every commuting additive mapping f:$S\longrightarrow R$ gives rise to Biderivation D: $S\times S\longrightarrow R$ defend by $D(x, y)= [f(x), y]$, for all $x, y \in S$. Vukman present an analogue result of Posner's second theorem, which state that the existence of nonzero bi-derivation D on prime ring R of characteristic different from 2 and 3 such that $[d(x), x]\in Z(R)$ holds for all $x\in R$, where d is the trace of D, forces R to be commutative [4]. N.

Argac introduced the concept of symmetric generalized Biderivation as follows:
A symmetric biadditive mapping $G(.,.): R\times R\longrightarrow R$ is called symmetric generalized Biderivation if there exist symmetric Biderivation D such that $G(xz, y)=G(x, y)z+xD(z, y)$ holds for all $x, y, z \in R$. It's clear that in this case the relation $G(x,yz) = G(x, y)z+ y D(x, z)$ is also satisfied for all $x,y,z \in R$. Argac shows that every generalized Biderivation G on a non-commutative prime ring R is of the form $D(x, y) = \lambda [x, y]$, for all $x, y \in R$ and some $\lambda \in C$, where C is the extended centroid of R (the center of Martindale ring of quotient of R)[5]. In 2007 M. Y. Ceven, and M. A. Öztürk introduce the concept of a (α, τ)-Biderivation as follows: A biadditive mappings $F(.,.): R\times R\longrightarrow R$ is said to be a (α, τ)-Biderivation if both $F(xy, z) = F(x, z)\alpha(y)+ \tau(x) F(y, z)$ and $F(x,yz) = F(x, y)\alpha(z)+ \tau(y) F(x, y)$ holds for all $x,y,z \in R$ [6].
In this paper, we introduce some results which characterize the nontrivial central ideal in semiprime rings. Furthermore, we look for some necessary conditions that force a prime ring to be commutative.

Preliminaries

The following identities my used frequently: For any $x, y, z \in R$.

- $[xz, y] = [x, y] z + x[z, y]$.
- $[x, zy] = [x, z] y + z[x, y]$.
- $(xz) o\, y = x(zoy) - [x, y]z = (xoy)z + x[z, y]$.
- $x\, o\, (zy) = (xoz)y - z[x, y] = z(xoy) + [x, z]y$.

First we review some facts.

Lemma (2.1): [7]
Let R be a prime ring, and $\mathcal{I}$ be a nonzero left ideal of R. If a (σ, τ)-Biderivation $D: R\times R\longrightarrow R$ satisfies that $D(\mathcal{I}, \mathcal{I}) =0$, then $D =0$.

Remarks (2.2): [8]
Let R be a prime ring, $\mathcal{I}$ a nonzero ideal of R. If $a\,\mathcal{I}b=0$, for $a \in R$, it's easy to verify that either $a=0$ or $b=0$.

Lemma (2.3): [9]
Let R be a semiprime ring. If $a,b \in R$ are such that $a\, x\, b=0$, for all $x \in R$, then $ab=ba=0$.

Lemma (2.4): [9]
Let R be a semiprime ring suppose that there exissts $a \in R$ such that $a[x, y] =0$ holds for all pairs $x,y \in R$, then there exists an ideal U of R such that $a \in U\subset Z(R)$.

Lemma (2.5): [10]
Let R be a semiprime ring, $\mathcal{I}$ an ideal of R. If $\mathcal{I}$ is commutative as a ring, then $\mathcal{I} \subset Z(R)$. In addition if R is prime, then R must be commutative.

Lemma (2.6): [11]
Let R be a 2-torsion free prime ring and $\mathcal{I}$ be a nonzero ideal of R. If D is a symmetric Biderivation such that $D(x, x) =0$, all $x \in \mathcal{I}$.then either $D =0$ or R is commutative.

The Main Results

The following theorem provides conditions in order that trace of a symmetric Biderivation commuting on an ideal of R.

Theorem (3.1):
Let R be a 2-torsion free prime ring and U be a nonzero ideal of R. Suppose $G: R\times R\longrightarrow R$ is a symmetric generalized Biderivation with associated Biderivation D satisfies that $D(g(u), u) =0$ for all $u \in U$ where g is the trace of G, then D has a commuting Trace on U.
Proof: By hypothesis, we have:

$$D(g(u), u) =0, \text{ for all } u \in U \tag{1}$$

The linearization of (1), we see:

$$D(g(u), \omega) + D(g(\omega), u) + 2D(G(u, \omega), \omega) + D(G(u, \omega), u) =0, \text{ for all } u, \omega \in U \tag{2}$$

Putting - ω for ω in (2) gives:

$$-D(g(u), \omega) + D(g(\omega), u) +2D(G(u, \omega), \omega) - 2D(G(u, \omega), u) =0, \text{ for all } u, \omega \in U \tag{3}$$

Combining (2) with (3), since R is a 2-torsion free ring, we arrive at:

$$D(g(\omega), u) + 2D(G(u, \omega), \omega) =0, \text{ for all } u, \omega \in U \tag{4}$$

Replacing u by uv in above relation imply that:
$D(g(\omega), u)v +u D(g(\omega), v)+2D(G(u, \omega), \omega) v + 2G(u, \omega)D(v, \omega)+2D(u, \omega) D(v, \omega)+2u D(D(v, \omega), \omega) =0$, for all $u,v, \omega \in U$.
According to (4), the above relation reduces to:

$$uD(g(\omega), v) +2D(u, \omega)D(v, \omega)+2G(u, \omega)D(v, \omega) + 2u D(D(v, \omega), \omega)=0 \tag{1}$$

The substitution u^2 for u in above relation gives for all $u, \omega \in U$:

$u^2D(g(\omega), v) + 2D(u, \omega)\, u\, D(v, \omega)) + 2u\, D(u, \omega)D(v, \omega) + 2G(u, \omega)\, u\, D(v, \omega) + 2uD(u, \omega)D(v, \omega) + 2\, u^2D(D(v, \omega), \omega) = 0,$

In view of (5) and the 2-torsinity free of R, the above relation becomes:

$D(u, \omega)u\, D(v, \omega) + G(u, \omega)\, u\, D(v, \omega) + uD(u, \omega)\, D(v, \omega) - u\, G(u, \omega)D(v, \omega) = 0.$

Equivalently

$\{[G(u, \omega), u] + D(u, \omega)u + uD(u, \omega)\}D(v, \omega) = 0$

That is

$\{[G(u, \omega), u] + D(u^2, \omega)\}D(v, \omega) = 0$

Putting sv for v in the above relation leads to:

$\{[G(u, \omega), u] + D(u^2, \omega)\}s\, D(v, \omega) = 0$, for all $u, \omega \in U$ and $s \in R$.

By primeness of R yields that either $D(v, \omega) = 0$, for all $v, \omega \in U$, consequently by Lemma (2.1) implies that D is zero on R. Or

$$[G(u, \omega), u] + D(u^2, \omega) = 0, \text{ for all } u, \omega \in U \qquad (6)$$

Substituting ωu for ω in the last relation, we find:

$[G(u, \omega), u]u + [\omega, u]\, d(u) + \omega[d(u), u] + D(u^2, \omega)u + \omega D(u^2, u) = 0$, for all $u, \omega \in U$.

Where d is the trace of D. In view of (6), in the above relation reduces to:

$$[\omega, u]\, d(u) + \omega[d(u), u] + \omega D(u^2, u) = 0, \text{ for all } u, \omega \in U. \qquad (7)$$

Putting $d(u)\omega$ instead of ω in (7) and using (7), we get:

$$[d(u), u]\, \omega\, d(u) = 0, \text{ for all } u, \omega \in U \qquad (8)$$

Now, right multiplication of (8) by u gives:

$$[d(u), u]\, \omega\, d(u)u = 0, \text{ for all } u, \omega \in U \qquad (9)$$

The substitution ωu instead of ω in (8) and subtracting the relation so obtained from (9) yields:

$$[d(u), u]\, \omega\, [d(u), u] = 0, \text{ for all } u, \omega \in U.$$

Equivalently

$$[d(u), u]\, U\, [d(u), u] = 0, \text{ for all } u, \omega \in U.$$

Using Remark (2.2) implies that:

$$[d(u), u] = 0, \text{ for all } u \in U.$$

Hence d is commuting on U.

One of the conditions that forces R to have a nontrivial central ideal is given in the following theorem.

Theorem (3.2):

Let R be a 2-torsion free semiprime ring. If R admitting a symmetric generalized Biderivations $G: R\times R \longrightarrow R$ associated with Biderivation D such that their traces g and d respectively satisfies that $d(x)x + x\, g(x) = 0$, for all $x, \in R$, then R contains a central ideal.

Proof*:* In view of our hypothesis, we have:

$$d(x)x + x\, g(x) = 0, \text{ for all } x, \in R \qquad (1)$$

The linearization of (1) and using (1), we see:

$d(x)y + d(y)x + 2D(x, y)x + 2D(x, y)y + x\, g(y) + y\, g(x) + 2xG(x, y) + 2yG(x, y) = 0$, for all $x,y \in R$.

Putting $2x$ for x, then comparing the relation so obtained with the above relation, we get:

$$2D(x, y)x + d(x)y + 2xG(x, y) + y\, g(x) = 0, \text{ for all } x,y \in R \qquad (2)$$

The substitution yx instead of y in(2) gives:

$$2D(x, y)x^2 + 2y\, d(x)x + d(x)yx + 2xG(x, y)x + 2xyd(x) + yx\, g(x) = 0, \text{ for all } x,y \in R \qquad (3)$$

Right multiplication of (2) by x, we get:

$$2D(x, y)x^2 + d(x)yx + 2xG(x, y)x + y\, g(x)x = 0, \text{ for all } x,y \in R \qquad (4)$$

Subtracting the relation (4) from (3) gives:

$2yd(x)x + 2xyd(x) + yx\, g(x) - y\, g(x)x = 0$, for all $x,y \in R$.

According to (1), the last relation reduces to:

$$yd(x)x + 2xyd(x) - yg(x)x = 0, \text{ for all } x,y \in R \qquad (5)$$

Substituting zy for y in (5) gives:

$$zyd(x)x + 2xzyd(x) - zyg(x)x = 0, \text{ for all } x,y,z \in R \qquad (6)$$

Left multiplication of (5) by z then subtracting the relation so obtained from (6), we arrive because of the 2-toritionity free of R at:

$$[z, x]\, y\, d(x) = 0, \text{ for all } x,y,z \in R \qquad (7)$$

Using Lemma (2.3), we find:

$$[z, x]\, d(x)=0, \text{ for all } x,y,z \in R \quad (8)$$

The linearization of the relation (7) with respect x gives:

$[z, \omega]\, y\, d(x) + 2[z, \omega]\, y\, D(\omega, x) + [z, x]\, y\, d(\omega) + 2[z, x]\, y\, D(\omega, x) = 0$

Putting 2ω for ω, comparing the above relation with the relation so obtained, we arrive at:

$[z, \omega]\, y\, d(x) + 2[z, x]\, y\, D(\omega, x)=0,$ for all $x,y,z,\omega \in R$.

The substitution $d(x)\, y\, [z, \omega]$ for y leads because of (8) to:

$[z, \omega]\, d(x)\, y\, [z, \omega]\, d(x) = 0$, for all $x,y,z,\omega \in R$.

By the semiprimeness of R, we have:

$[z, \omega]\, d(x) = 0$, for all $x,z,\omega \in R$.

An application of Lemma (2.4) on the above relation we get a central ideal of R contains $d(x)$.

In case R is prime ring, we have the following corollary.

Corollary (3.3):

Let R be a 2-torsion free prime ring. If R admitting a symmetric generalized Biderivations $G: R\times R \longrightarrow R$ associated with Biderivation D such that their traces g and d respectively satisfies that $d(x)x + x\, g(x)=0,$ for all $x \in R$, then R isi commutative.

Theorem (3.4):

Let R be a 2-torsion free semiprime ring. If R admitting a symmetric generalized Biderivation $G: R\times R \longrightarrow R$ associated with a nonzero Biderivation D such that their traces g and d respectively satisfies that $d(x)x - x\, g(x)=0,$ for all $x \in R$, then R contains a central ideal.

Proof*:*

In view of our hypothesis, we have:

$$d(x)x = x\, g(x), \text{ for all } x, \in R \quad (1)$$

Using similar arguments as used to get (2) from (1), we find:

$$2D(x, y)x + d(x)y = 2xG(x, y) + yg(x) = 0, \text{ for all } x,y \in R \quad (2)$$

Replacing y by xy in (2) leads to:

$2D(x, y)\, x^2 + 2yd(x)x + d(x)yx = 2xG(x, y)x + 2xyd(x) + yxg(x) = 0,$ for all $x,y \in R$.

According to (2), the relation (2) reduces to:

$2yd(x)x = 2xyd(x) + yx\, g(x) - yg(x)x,$ for all $x,y \in R$. Equivalently:

$$2[x, y]\, d(x) + 2y[x, d(x)] + y\, [x, g(x)]=0, \text{ for all } x,y \in R \quad (3)$$

Putting zy for y in (3) gives:

$$2z[x, y]\, d(x) + 2[x, z]\, y\, d(x) + 2zy[x, d(x)] + zy\, [x, g(x)] = 0, \text{ for all } x,y \in R \quad (4)$$

Comparing the two relations (3) and (4) implies because of the 2-toritionity free of R that:

$$[x, z]\, y\, d(x) = 0, \text{ for all } x,y,z \in R.$$

The last relation is similar to relation (7) in theorem (3.2), hence using the same technique as used in the proof of the theorem mentioned above, we get the required result.

Following corollary is the immediate consequence of the above theorem.

Corollary (3.5):

Let R be a 2-torsion free prime ring. If R admitting a symmetric generalized Biderivation $G: R\times R \longrightarrow R$ associated with a nonzero Biderivation D such that their traces g and d respectively satisfies that $d(x)x - x\, g(x)=0,$ for all $x \in R$, then R contains a central ideal.

Theorem (3.6):

Let R be a 2-torsion free prime ring and U be a nonzero ideal of R. If R admitting a symmetric generalized Biderivation $G:R\times R \longrightarrow R$ associated with a Biderivation D such that their traces g and d respectively satisfies that $d(xg)(y)=[x, y],$ for all $x,y \in U$, then R is commutative or D is zero on R.

Proof**:** If $G=0$, then $[x, y]=0,$ for all $x,y \in U$, this means that U is a commutative ideal, consequently R is commutative by Lemma(2.5). Therefore, we shall assume that $G \neq 0$ and suppose:

$$d(xg)(y)=[x, y], \text{ for all } x,y \in U \quad (1)$$

Replacing y by $x+y$ in (1) leads to:

$$d(xg)(x) + d(xg)(y) + 2d(xg)(x, y) = [x, y], \text{ for all } x,y \in U \quad (2)$$

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The above relation reduces because of (1) and the 2-toritionity free of R to:

$$d(xg)(x, y)= 0, \text{ for all } x,y \in U \qquad (3)$$

Putting yx for y in (3) and using (3), we see:
$d(x)\, y\, d(x)= 0,$ for all $x,y \in U$.
Consequently:

$$d(x)y\, R\, d(x)= 0, \text{ for all } x,y \in U \qquad (4)$$

Right multiplication of (4) by y and using the primeness of R, we find:
$d(x)y= 0,$ for all $x,y \in U$.

Equivalently

$$d(x)\, R\, U= 0, \text{ for all } x \in U.$$

Since U is a nonzero ideal, hence $d(x)= 0,$ for all $x \in U$. So an application of Lemma (2.6), we get the requirement of the theorem.

Theorem (3.7):
Let R be a 2-torsion free prime ring and U be a nonzero ideal of R. If R admitting a symmetric generalized Biderivation $G: R\times R \longrightarrow R$ associated with a Biderivation D such that their traces g and d respectively satisfies that $d(xg)(y)= xoy,$ for all $x,y \in U$, then R is commutative or D is zero on R.
Proof**:**
If $G=0$, we have:

$$xoy =0, \text{ for all } x,y \in U \qquad (1)$$

The substitution yz for y in (1) and using (1), we find:
$y\, [z, x] =0,$ for all $x,y,z \in U$.
Equivalently
$U\, R\, [z, x] =0,$ for all $x,y,z \in U$.
Since U is a nonzero ideal, the primeness of R leads to $[z, x] =0,$ for all $x,z \in U$, that is U a commutative ideal, consequently $R$is commutative by Lemma (2.6). Henceforth, we shall assume that $G \neq 0$ and suppose:

$$d(xg)(y)= xoy, \text{ for all } x,y \in U \qquad (2)$$

Substituting $x+y$ instead of y in (2) lead to:
$d(xg)(x) + d(x)\, g(y) +2d(xg)(x, y)= 2x^2 + xoy$, for all $x, y,z \in U$.
The last relation reduces because of (2) and 2-torisionity free of R to:

$$d(xg)(x, y)=0, \text{ for all } x, y \in U \qquad (3)$$

Hence using the same arguments as used in the proof of theorem (3.6), we get the required result.

Theorem (3.7):
Let R be a 2-torsion free prime ring and U be a nonzero ideal of R. If R admitting a symmetric generalized Biderivation $G: R\times R \longrightarrow R$ associated with a Biderivation D such that there traces g and d respectively satisfies that $[d(x), y] = [x, g(y)]$, for all $x,y \in U$, then R is commutative or D is zero on R.
Proof**:** we have:

$$[d(x), y] = [x, g(y)], \text{ for all } x,y \in U \qquad (1)$$

Linearization of (1) with respect to y gives:
$[d(x), y] + [d(x), z] = [x, g(y)] + [x, g(z)] + 2[x, G(y, z)],$ for all $x,y,z \in U$.
In view of (1) and 2-torisionity free of R, the above relation becomes:

$$[x, G(y, z)]=0, \text{ for all } x,y,z \in U \qquad (2)$$

Putting yx for y in (2) and using (2) leads to:

$$[x, y]\, D(x, z) + y[x, D(x, z)]=0, \text{ for all } x,y \in U \qquad (3)$$

The substitution ωy for y, we find:

$$[x, \omega]\, y\, D(x, z) + \omega[x, y]\, D(x, z) + \omega y[x, D(x, z)] =0, \text{ for all } x,y,z,\omega \in U \qquad (4)$$

Combining the relations (3) and (4) implies that:
$[x, \omega]\, y\, D(x, z)=0,$ for all $x,y,z,\omega \in U$.
Using remark (2.2), we have $[x, \omega]=0$ or $D(x, z) =0,$ for all $x,y,\omega \in U$. We obtain that U is the set theoretic union of two proper subgroups viz.
$\mathcal{A} =\{x \in U: [x, \omega] =0, \text{ for all } \omega \in U\}$.
and
$\mathcal{B} =\{x \in U: D(x, z) =0, \text{ for all } z \in U\}$.
But a group cannot be the set-theoretic union of two proper subgroups, hence $U= \mathcal{A}$ or $U= \mathcal{B}$.

If $U=\mathcal{A}$, then $[x, \omega]=0$, for all $x,\omega \in U$, that is U is a commutative ideal of R and by Lemma (2.5) implies that R is commutative. On the other hand, if $U=\mathcal{B}$, then $D(x, z)=0$, for all $x,z \in U$, using Lemma (2.1) yields D is zero on R.
In a similar manner we can obtain the following theorem.

Theorem (3.8):
Let R be a 2-torsion free prime ring and U be a nonzero ideal of R. If R admitting a symmetric generalized Biderivation $G: R\times R\longrightarrow R$ associated with a Biderivation D such that there traces g and d respectively satisfies that $[d(x), y]+[x, g(y)]=0$, for all $x,y \in U$, then R is commutative or D is zero on R.

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