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Image Compression Using Principal Component Analysis

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Abstract

Principal component analysis produced reduction in dimension, therefore in our proposed method used PCA in image lossy compression and obtains the quality performance of reconstructed image. PSNR values increase when the number of PCA components is increased and CR, MSE, and other error parameters decreases when the number of components is increased.

Keywords: Variance, Compression, PCA, Eigenvalue, and Eigenvector.

الخلاصة

أنتجت خوارزمية التحليل الأساسي للمكونات انخفاضا في البعد ، لذلك في الطريقة المقترحة استخدمتنا خوارزمية PCA في انضغاط الصور المفقودة لكي نحصل على جودة أداء الصورة التي أعيد بناؤها. تزيد قيم PSNR عند زيادة عدد مكونات PCA وتقلص CR ومتوسط MSE ومعلمات الخطأ الأخرى عند زيادة عدد المكونات.

Introduction

Principal component analysis (PCA) is representing a data in an eigenvector with eigenvalues which is mean a new coordinate system; therefore PCA is a linear transformation [1]. The efficiently represented faces of images using PCA called Eignface which now become a standard and a common performance benchmark in face recognition [2]. PCA produced dimensionality reduction therefore, it suitable to use for lossy compression, while retaining those characteristics of the dataset which contribute most to its variance [3]. Many algorithms based on various principals leading to the image compression by reducing volume data of image [4]. Lossy Algorithms are based on the image color reduction, but reconstructed image are obviously used for some applications. Color image is converted to gray-scale (intensity) image. The resulted image is reducing the data volume belongs to the most common algorithms [5].

Litreture Review

In 2012, [9] stated that PCA used on wavelet coefficients to maximize edge energy in the reduced dimension images. Large image sets, for a better preservation of image local structures, a pixel and its nearest neighbors are modeled as a vector variable, whose training samples are selected from the local window by Local Pixel Grouping (LPG). In Muresan and Parks [10] proposed a spatially adaptive principal component analysis (PCA) based denoising scheme. Elad and Aharon [11] [12] proposed sparse redundant representation and (clustering-singular value decomposition) K-SVD based denoising algorithm by training a highly over-complete dictionary. Foi *et al.* [13] applied a shape-adaptive discrete cosine transform (DCT) to the neighborhood, which can achieve very sparse representation of the image and hence lead to effective denoising, recently Dabov *et al.* [14] proposed a collaborative image denoising scheme by patch matching and sparse 3D transform. They searched for similar blocks in the image by using block matching and grouped those blocks into a 3D cube.

Materials and Methodologies

Eigenvector and Eigenvalues

PCA is a powerful and widely used linear technique in many applications in data processing. The linear transformation defined of the form [6]:

$$y = Wx \tag{1}$$

The stationary transforming stochastic data $\mathbf{x} \subset R^N$ into the vector $\mathbf{y} \subset R^K$ using the matrix $\mathbf{W} \subset R^{K \times N}$, dimension reduction by $K < N$ in the output space y, but the most important information is compact in the input space $\mathbf{x}$.

$$\mathbf{R_{xx}} = E[\mathbf{xx}^T] \tag{2}$$

Where $\mathbf{x}$ be the random vector of zero mean and $\mathbf{R_{xx}}$ in the correlation matrix of all vectors $\mathbf{x}_i$. The correlation matrix in equation (2) is has two properties such as symmetrical and positive. It means that all eigenvalues of $\mathbf{R_{xx}}$ are real and positive. Eigenvectors is orthogonal and associated with λ_i be denoted by $\mathbf{w}_i$. Therefore the eigenvalues can be sort from the highest value λ_1 to lowest value λ_N and in similar way the eigenvectors $\mathbf{w}_i$ associated with them. According to the eigen-decomposition principal Equation (2) can be reconstructed as follows:

$$R_{xx} = \sum_{k=1}^{N} \lambda_i w_i w_i^T \tag{3}$$

Eigenvalues used to measure the contribution of orthogonal vectors w_i in equation (3). Also, can be seen from equation (3) the association of eigenvectors will be discarded if the magnitudes of eigenvalues are smallest magnitude. Therefore, K is the most important instead of N eigenvalues which is used in eq. (3) only with $K < N$. $\mathbf{W}$ in PCA transformation matrix can be rewritten as $W = [w_1, w_2, \ldots, w_k]^T$. The reconstruction original vector $\mathbf{x}$, is denoted here by $\widehat{\mathbf{x}}$, described by equation (4) [7][8]:

$$\widehat{\mathbf{x}} = W^T y \tag{4}$$

Design the Proposed System

In our proposed image lossy compression can be summarized in the following diagram shown in Figure 1. The proposed system consists of following steps:

1. RGB image conversion to gray scale image.
2. Down-sampling process.
3. PCA process
4. Lossy compression
5. Performance criteria

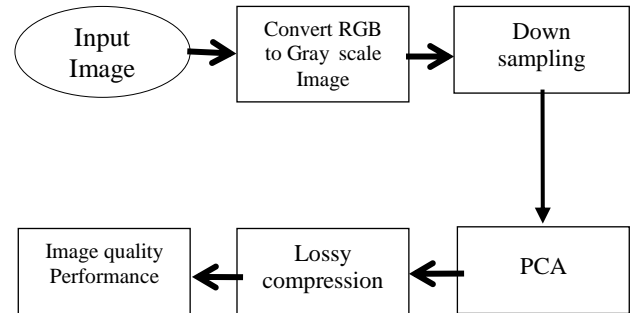All these steps are shown in Figure 1.

Figure 1: PCA lossy compression.

Experimental Results

The size of input image is M rows and N columns; it is represented as a column vector of length MxN. So, the dimension of the input image is MxN. The covariance matrix will be a square matrix of dimensions MxN * MxN Select the maximum eigenvalues and choose the corresponding eigenvectors. The number eigenvectors chosen will represent the new reduced dimensions of the input image. This is the compression part and the user can choose the eigenvectors according to the nature of the application. Larger the number of eigenvectors better will be accuracy of the compressed image.

The following steps of our proposed method:

1. Represent the input data as a column vector.
2. Subtract the mean from the data samples and normalization.
3. Compute the covariance matrix
4. Select the eigenvectors image compression.
5. Represent the input data in reduced dimensions
6. Singular value decomposition
7. Evaluate variances
8. Extract 250 principal components
9. Calculate compression ratio
10. Convert back to original basis

11. Add the row means back on
12. Display results
13. Image quality performance.

Image quality measurement includes:

Compression ratio (CR): can be computed according to the following formula:

$$CR = \frac{size\ of\ uncompressed\ image}{size\ of\ compressed\ image} \qquad (5)$$

Maximum Error (MAXERR): is a metric to measure the maximum absolute squared deviation of the data X, from the approximation, XAPP.

L2RAT: is the ratio of the squared norm XAPP, to the original data X.

The experimental results can be seen in Figure 2 which is illustrated the relationship between the number of eigenvectors and eigenvalues.

Figure 3 illustrates the original color image and its grayscale top to down.

The reconstructed images with different number of PCA components can be seen in Figure 4. The performance qualities of the reconstructed image are shown in Figures 5-9.
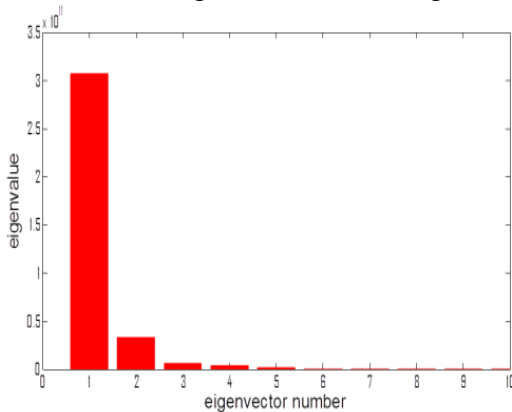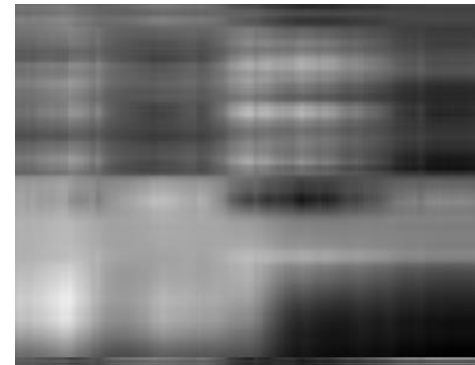

Figure 2: the number of Eigenvectors vs. Eigenvalues.

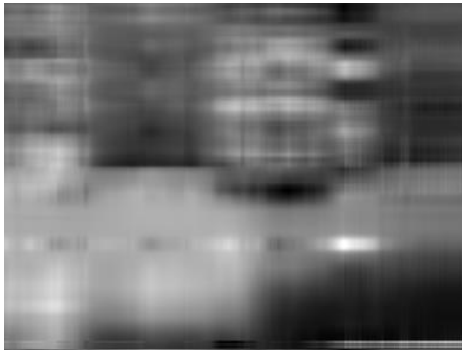Figure 3: The original color image and its grayscale top to down.

170.7000:1 compression
1 PC

102.4000:1 compression
2 PC

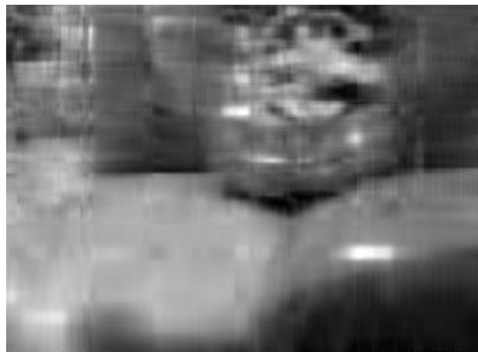
73.1000:1 compression
3 PC

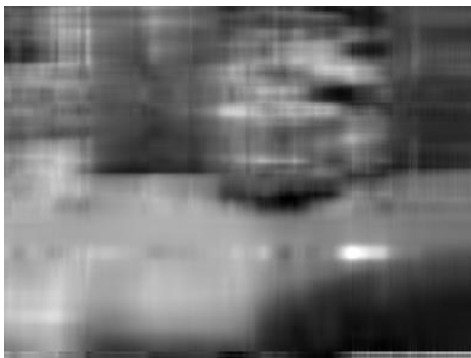143

56.9000:1 compression
4 PC

26.9000:1 compression
9 PC

46.5000:1 compression
5 PC

24.4000:1 compression
10 PC

39.4000:1 compression
6 PC

12.5000:1 compression
20 PC

34.1000:1 compression
7 PC

8.4000:1 compression
30 PC

6.3000:1 compression
40 PC

2.8000:1 compression
90 PC

5.1000:1 compression
50 PC

2.5000:1 compression
100 PC

4.2000:1 compression
60 PC

1.7000:1 compression
150 PC

3.6000:1 compression
70 PC

1.3000:1 compression
200 PC

1.000:1 compression
250 PC

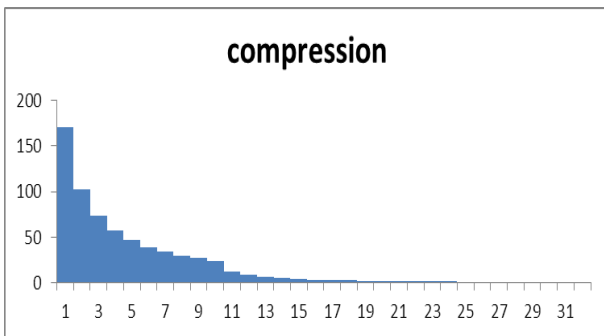Figure 4: Reconstructed images with different principal components.

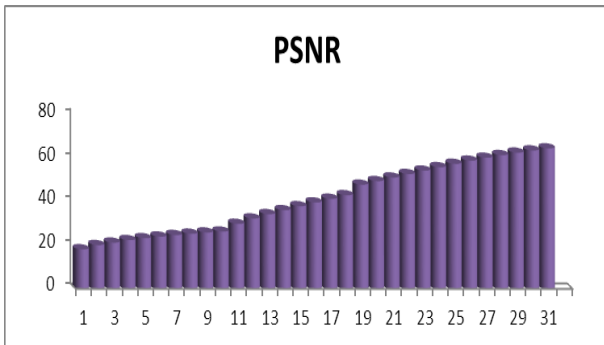Figure 5: Compression ratio values vs. Principal components.

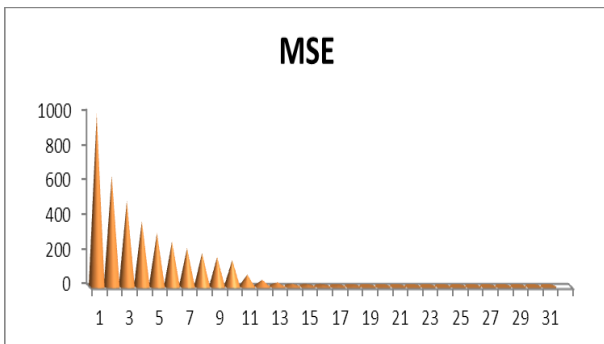Figure 6: PSNR values vs. Principal components.
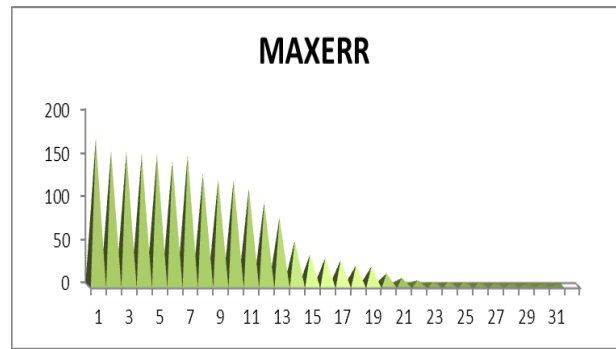

Figure 7: MSE values vs. Principal components.
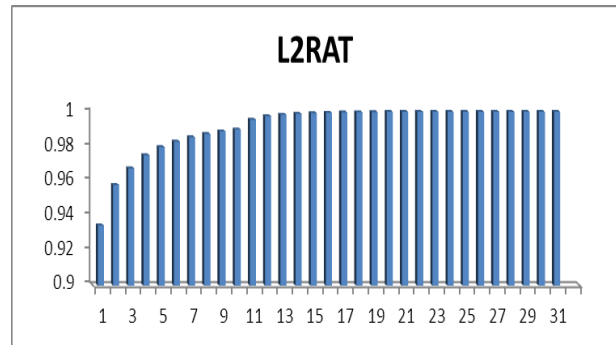

Figure 8: MAXERR values vs. Principal components.

Figure 9: L2RAT values vs. Principal components.

Conclusion

In this paper we used 250 color images with 250 dimensions are converted to gray-scale image, therefore we arrange all the intensity values in a vector. This vector is consists of all the intensity values from the same pixel from each image. We applied PCA algorithm, 250 eigenvectors can be resulted of vector is 250-dimensional. The data will be compressed. However, when the original data is reconstructed, some information is lost. Two reasons effect on loss information; first one is caused by converting color image to gray-scale image, the second reason is caused by converting all pixel values in a vector and determined the best eigenvectors associated with highest value of eigenvalues.

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