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Constructing a New Exponentiated Family Distribution with Reliability Estimation

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Abstract
This paper deals with using one method for transforming two parameters given distribution to another form with three parameters distribution, through using idea of reparameterization with powering the given cumulative distribution function by new parameter, where this work gives a new family through using new parameter which is necessary for generating values of the $r.v$ from given CDF through smoothing the values of the given random variable to obtain new values of $r.v$ using three sets of parameters rather than two. The new model Frechet $p.d.f$ is obtained and also its Cumulative distribution function is found then we apply three methods of estimation (Maximum likelihood, moments estimator and the third method) is nonlinear least square. Different set of initial values of parameters (α, λ, θ) and different samples Size ($n = 25,50,75,100$) was used. The simulation procedure is done using matlab-R2014b, and results are compared using integrated Mean square error.

Keywords: Cumulative distribution function, Maximum likelihood, Moments estimator, Non Linear Least square regression.

الخلاصة
يتناول هذا البحث استخدام الطريقة الاولى لتحويل معلمتين لتوزيع معطى الى شكل توزيع اخر بثلاث معالم من خلال استخدام فكرة اعادة التشكيل وذلك برفع دالة التوزيع التراكمية للتوزيع المعطى لقوى بواسطة معلمة جديدة, حيث يعطي هذا العمل عائلة جديدة من خلال استخدام معلمة جديدة وهو أمر ضروري لتوليد قيم المتغير العشوائي من دالة توزيع تراكمية معينة من خلال صقل قيم المتغير العشوائي نظرا إلى الحصول على قيم جديدة من المتغير العشوائي باستخدام ثلاثة مجموعات من المعلمات بدلا من اثنين. الانموذج الجديد الذي تم الحصول عليه هو دالة كثافة احتمال فريشيت كما تم الحصول على دالة التوزيع التراكمي ومن ثم تطبيق ثلاث طرائق للتقدير (والتي هي الامكان الاعظم، مقدر العزوم، والطريقة الثالثة تعتمد على استخدام طريقة انحدار المربعات الصغرى). تم اختيار مجموعة مختلفة من القيم الابتدائية للمعالم (α, λ, θ) بأحجام عينات مختلفة ($n = 25,50,75,100$). تم إجراء المحاكاة باستخدام ماتلاب-R2014b، وتمت مقارنة النتائج باستخدام متوسط مربعات الخطا التكاملي.

Introduction
Many research work on extending probability distribution through exponentiated or through using certain type of transformation on (CDF) or on reliability function Gupta et al (1998) apply extension to find a new probability density and then Gupta and Kundu (2001) apply transformation to obtain a new Family. Also Nadarajah and Kotz(2006) obtained a new Family called exponentiated gamma, also they Obtained exponentiated Frechet, and studied different properties. Many other researchers, Shirk, D.T and Kakade.C.S (2006) Gutmann. M and Hyvarinen,A. apply estimation through new generated Weibull.Nadaraj ah, S and Gupta,A.K (2007) discuss exponentiated gamma, with all required derivation.

Materials and Methodologies
Theoretical Aspect
We know that the exponentiated distribution is used widely in analyzing failure time data, for situation with constant hazard failure rate. The exponentiated distributions are obtained by different methods [2][4][6]; Firstly, the new

C.D.F [G(z)] is obtained from powering old C.D.F to the parameter (λ) i.e: [2]

$$G(z) = [F(z)]^{\lambda} \quad \lambda > 0 \qquad (1)$$

The new $p.d.f$ $g(z) = G'(z)$.

The second Approach is obtained to obtain new C.D.F [$G(y)$] from [2]

$$G(y) = 1 - [1 - F(y)]^{\lambda} \quad \lambda > 0 \qquad (2)$$

Then:

$$g(y) = G'(y)$$

While the third approach is using transformation to use transformation of $Z = \log(x)$, $x \geq 0$ in the original $p.d.f$ to obtain $g(z)$ from this transformation, here in our research we apply the first type on Frechet distribution (θ, α) to obtain three parameters Frechet $(\theta, \alpha, \lambda)$.

The C.D.F of Frechet distribution is;

$$F(z) = e^{-\left(\frac{\theta}{z}\right)^{\lambda}} \quad z, \theta, \lambda > 0 \qquad (3)$$

While the $p.d.f$ is:

$$f(z;\theta,\lambda) = \lambda\theta^{\lambda} z^{-(\lambda+1)} e^{-\left(\frac{\theta}{z}\right)^{\lambda}} \quad z, \theta, \lambda > 0 \qquad (4)$$

We apply the first approach;

$$G(z) = [F(z)]^{\alpha} = e^{-\left(\frac{\theta}{z}\right)^{\alpha\lambda}} \qquad (5)$$

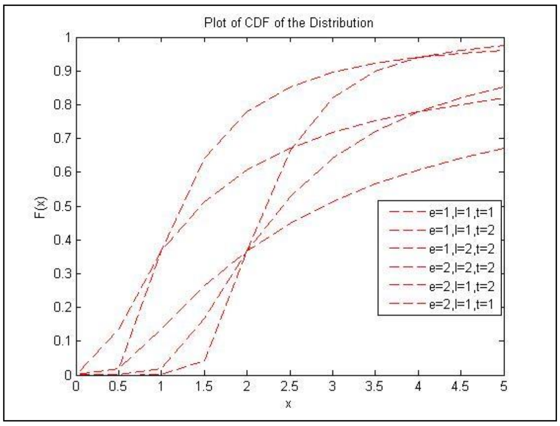

Figure 1: Plot of CDF

Then the new generated $p.d.f$ is:

$$f(z;\theta,\lambda) = \alpha\lambda\theta^{\alpha\lambda} z^{-(\alpha\lambda+1)} e^{-\left(\frac{\theta}{z}\right)^{\alpha\lambda}}$$

$$z, \alpha, \theta, \lambda > 0 \qquad (6)$$

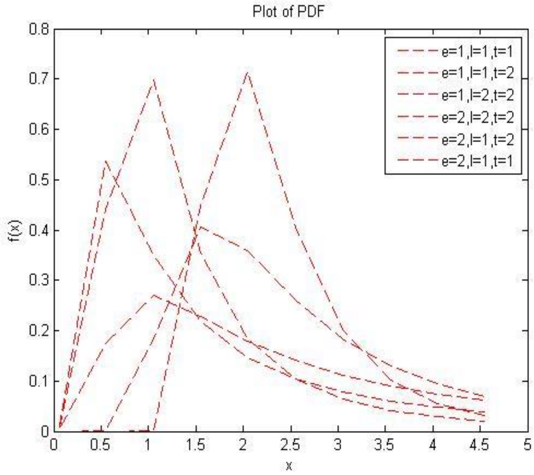

Figure 2: Plot of $p.d.f$

Figure 1, and 2 shows CDF for Frechet distribution and $p.d.f$ respectively, and the reliability function will be:

$$R(z) = 1 - e^{-\left(\frac{\theta}{z}\right)^{\alpha\lambda}} \quad z, \alpha, \theta, \lambda > 0 \qquad (7)$$

Now we work on estimating the three parameters of Frechet distribution where θ is scale parameter and α, λ are shape parameters, by different methods.

Estimation Methods

Three estimation methods are discussed in the following sections [1][5];

Method of moment

The estimators by this method depend on equating μ_r' with m_r' , where;

$$\mu_r' = E(Z^r) \qquad (8)$$

$$m_r' = \frac{\sum_{i=1}^{n} z_i^r}{n} \qquad (9)$$

Then the r[th] moment about origin is derived:

$$\mu_r' = E(Z^r) = \int_0^{\infty} \alpha\lambda\theta^{\alpha\lambda} z^{-(\alpha\lambda+1)} e^{-\left(\frac{\theta}{z}\right)^{\alpha\lambda}} dz$$

$$= \theta^r \Gamma\left(1 - \frac{r}{\alpha\lambda}\right), \quad \alpha, \lambda > 1 \qquad (10)$$

Then

$$\left.\begin{aligned} \mu_1' &= \theta\Gamma(1-\tfrac{1}{\alpha\lambda}) \\ \mu_2' &= \theta^2\Gamma(1-\tfrac{2}{\alpha\lambda}) \\ \mu_3' &= \theta^3\Gamma(1-\tfrac{3}{\alpha\lambda}) \end{aligned}\right\} \quad (11)$$

Then solving $\mu_r' = \frac{\sum_{i=1}^{n} Z_i^r}{n}$ for $r = 1,2,3$, we can find the moments estimators of the three parameters,the reliability estimator will be;

$$\hat{R}_{mom} \cong 1 - e^{-\left(\frac{\hat{\theta}_{mom}}{z}\right)^{\hat{\alpha}_{mom}\hat{\lambda}_{mom}}} \quad (12)$$

Method of Maximum Likelihood

Let $z_1, z_2, ..., z_n$ be a $r.s$ from $p.d.f$ represented by equation (6), then the likelihood function will be;

$$L = \prod_{i=1}^{n} f(z_i;\theta,\lambda) = \alpha^n \lambda^n \theta^{n\alpha\lambda} \prod_{i=1}^{n} z_i^{-(\alpha\lambda+1)} e^{-\sum_{i=1}^{n}\left(\frac{\theta}{z_i}\right)^{\alpha\lambda}} \quad (13)$$

$$\log L = n\log\alpha + n\log\lambda + n\alpha\lambda\log\theta - (\alpha\lambda + 1)\sum_{i=1}^{n}\log z_i - \sum_{i=1}^{n}\left(\frac{\theta}{z_i}\right)^{\alpha\lambda} \quad (14)$$

Then;

$$\frac{\partial \log L}{\partial \alpha} = \frac{n}{\alpha} + n\lambda\log\theta - \lambda\sum_{i=1}^{n}\log z_i - \lambda\sum_{i=1}^{n}\left(\frac{\theta}{z_i}\right)^{\alpha\lambda}\log\left(\frac{\theta}{z_i}\right) \quad (15)$$

$$\frac{\partial \log L}{\partial \lambda} = \frac{n}{\lambda} + n\alpha\log\theta - \alpha\sum_{i=1}^{n}\log z_i - \alpha\sum_{i=1}^{n}\left(\frac{\theta}{z_i}\right)^{\alpha\lambda}\log\left(\frac{\theta}{z_i}\right) \quad (16)$$

$$\frac{\partial \log L}{\partial \theta} = \frac{n\alpha\lambda}{\theta} - \alpha\lambda\theta^{\alpha\lambda-1}\sum_{i=1}^{n}\left(\frac{1}{z_i}\right)^{\alpha\lambda} \quad (17)$$

Which are solved numerically by Newton Raphson algorithm to obtain the values of estimators, and then the reliability estimator will be;

$$\hat{R}_{mle} = 1 - e^{-\left(\frac{\hat{\theta}_{mle}}{z}\right)^{\hat{\alpha}_{mle}\hat{\lambda}_{mle}}} \quad (18)$$

Method of Nonlinear Least Square

Let $z_1, z_2, ..., z_n$ be a $r.s$ from $p.d.f$ represented by equation (5) and ordered increasingly, and the nonparametric estimator of C.D.F can be represented in the following formula;

$$\hat{F}\,(\mathrm{i}) = \frac{i}{n-1} \quad (19)$$

Where $i = 1, ..., n$.

Then C.D.F of the distribution can be represented as follow:

$$\hat{F}(z_i) = e^{-\left(\frac{\theta}{z_i}\right)^{\alpha\lambda}} \quad (20)$$

The above model can be transformed to nonlinear model as follow;

$$\hat{F}(z_i) = e^{-\left(\frac{\theta}{z_i}\right)^{\alpha\lambda}} + \varepsilon_i \quad (21)$$

$$y_i = e^{-\left(\frac{\theta}{x_i}\right)^{\alpha\lambda}} + \varepsilon_i \quad (22)$$

Where:

$$y_i = \hat{F}(z_i)$$
$$z_i = x_i$$

The parameters of the above nonlinear model can be estimated by using nonlinear least square method as follow; [6]

$$Q^2 = \sum_{i=1}^{n}(y_i - e^{-\left(\frac{\theta}{x_i}\right)^{\alpha\lambda}})^2 \quad (23)$$

The estimators of above model reduced to minimization problem of model where the values nonlinear estimates represent the values of (α,θ,λ) that minimized Q^2 which can be found by applying any algorithm of minimization in MATLAB functions, then the reliability estimator will be;

$$\hat{R}_{nls} \cong 1 - e^{-\left(\frac{\hat{\theta}_{nls}}{z}\right)^{\hat{\alpha}_{nls}\hat{\lambda}_{nls}}} \quad (24)$$

Results and Discussion

Simulation Procedure

The estimator of 3 parameters (α,λ,θ) by methods of Maximum likelihood ,Moments and Least Square, are obtained through simulation using different set of initial values of (α,λ,θ) and different set of sample size ($n = 25,50,75,100$) to represent small, moderate and large sample size. Each experiment is repeated ($L = 1000$) times. The comparison is done using statistical measure Mean square error (MSE).

Several values of parameters $\lambda = 2,3,4$, $\theta = 2,3,4$ and $\alpha = 2,3,4$ .as follow;

Table 1: Parameters Values

α	2	2	2	3	3	3	4
λ	2	2	3	3	3	4	4
θ	2	3	3	3	4	4	4

Where the parameters values were taken vertically.

The simulation program was written by using matlab-R2014b program to generate pseudo observations using the formula;

$$z_i = \theta[-\log(u)]^{-\frac{1}{\alpha\lambda}} \quad (25)$$

Where u_i : represent uniform variant.

After the Reliability function was estimated, integrated mean square error (IMSE) was calculated to compare the methods of estimation, Where; [7]

$$IMSE[\hat{R}(t)] = \frac{1}{L}\sum_{i=1}^{L}\left\{\frac{1}{n_t}\sum_{j=1}^{n_t}[R_i(t_j) - \hat{R}_i(t_j)]^2\right\} = \frac{1}{L}\sum_{i=1}^{L} MSE[\hat{R}_i(t)] \quad (26)$$

Where,

n_t:is the number of times chosen to be (10) where (t=0.05-0.55-1.05-1.55-2.05-2.55-3.05-3.55-4.05-4.55)

The results of the simulation study are summarized and tabulated in Tables below of the three estimators for all sample size and (α , λ , θ) values respectively.

Table 2: Different Estimators of Reliability Function when $\alpha = 2, \lambda = 2, \theta = 2$

n	t	real	mom	Mle	nls
25	0.05	1;	1	0.715038	1
	0.55	1	0.999998	0.663225	1
	1.05	0.999998	0.999405	0.64909	0.99996
	1.55	0.937463	0.993832	0.640563	0.937269
	2.05	0.595843	0.966018	0.634438	0.531384
	2.55	0.315048	0.880332	0.629657	0.241206
	3.05	0.168807	0.746528	0.625734	0.119422
	3.55	0.095833	0.613106	0.622409	0.066217
	4.05	0.057736	0.504634	0.619524	0.040241
	4.55	0.036643	0.42245	0.616975	0.026236
50	0.05	1	1	0.673273	1
	0.55	1	0.999998	0.647516	1
	1.05	0.999998	0.999551	0.640552	0.999996
	1.55	0.937463	0.996777	0.636355	0.94412
	2.05	0.595843	0.985605	0.633343	0.575075
	2.55	0.315048	0.929523	0.630991	0.278102
	3.05	0.168807	0.81098	0.629061	0.138145
	3.55	0.095833	0.670506	0.627425	0.074001
	4.05	0.057736	0.543314	0.626005	0.042618
	4.55	0.036643	0.440662	0.624751	0.026093
75	0.05	1	1	0.659496	1
	0.55	1	0.999988	0.642378	1
	1.05	0.999998	0.998686	0.637756	0.999997
	1.55	0.937463	0.99136	0.634972	0.94593
	2.05	0.595843	0.962016	0.632973	0.59358
	2.55	0.315048	0.890228	0.631413	0.299199
	3.05	0.168807	0.777684	0.630133	0.153139
	3.55	0.095833	0.654681	0.629048	0.083753
	4.05	0.057736	0.544653	0.628106	0.048966
	4.55	0.036643	0.454999	0.627274	0.030326
100	0.05	1	1	0.652615	1
	0.55	1	0.998742	0.639803	1
	1.05	0.999998	0.994134	0.636346	0.999994
	1.55	0.937463	0.988703	0.634264	0.939465
	2.05	0.595843	0.977262	0.632769	0.587917
	2.55	0.315048	0.924428	0.631602	0.303128
	3.05	0.168807	0.81345	0.630644	0.15913

	3.55	0.095833	0.67665	0.629832	0.088953
	4.05	0.057736	0.547627	0.629128	0.052959
	4.55	0.036643	0.440515	0.628505	0.0333

The summary of best estimator reliability for Table 2 is shown in Table 3:

Table 3: IMSE value of the reliability function

n	mom	Mle	Nls	best
25	0.173389	0.17514	0.004594	Nls
50	0.172185	0.173933	0.001289	Nls
75	0.167296	0.171531	0.001248	Nls
100	0.157555	0.164485	0.000509	Nls

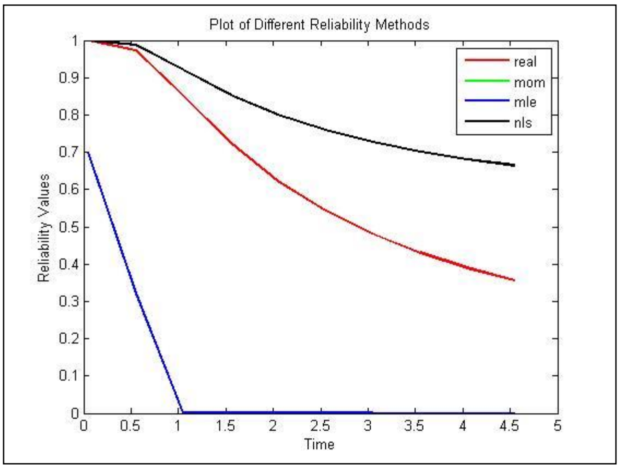

Figure 3: Different estimators forReliability Function Estimators

Table 4: Estimation of Reliability Function when $\alpha = 2, \lambda = 2, \theta = 3$

n	t	real	mom	Mle	nls
25	0.05	1	1	0.69087	1
	0.55	1	1	0.657799	1
	1.05	1	1	0.648831	1
	1.55	0.999999	1	0.643424	0.999911
	2.05	0.98981	1	0.639541	0.986654
	2.55	0.852759	1	0.63651	0.854133
	3.05	0.607815	1	0.634022	0.586976
	3.55	0.399505	1	0.631913	0.366765
	4.05	0.259973	1	0.630083	0.229935
	4.55	0.172205	1	0.628466	0.14889
50	0.05	1	1	0.661114	1
	0.55	1	1	0.644788	1
	1.05	1	1	0.64038	1
	1.55	0.999999	0.999999	0.637724	1
	2.05	0.98981	0.999965	0.635818	0.994384
	2.55	0.852759	0.999779	0.634329	0.876657
	3.05	0.607815	0.999272	0.633108	0.620672
	3.55	0.399505	0.998337	0.632073	0.394755
	4.05	0.259973	0.996958	0.631174	0.246893
	4.55	0.172205	0.995187	0.63038	0.15738
75	0.05	1	1	0.65127	1
	0.55	1	1	0.640444	1
	1.05	1	1	0.637523	1
	1.55	0.999999	1	0.635763	0.999927
	2.05	0.98981	0.999991	0.6345	0.983285
	2.55	0.852759	0.999913	0.633514	0.83469
	3.05	0.607815	0.999624	0.632705	0.588202
	3.55	0.399505	0.998976	0.632019	0.385108
	4.05	0.259973	0.997892	0.631424	0.251352
	4.55	0.172205	0.99637	0.630898	0.167781
1000	0.05	1	1	0.646413	1
	0.55	1	1	0.638313	1
	1.05	1	1	0.636128	1
	1.55	0.999999	0.999985	0.634812	0.999986
	2.05	0.98981	0.99981	0.633868	0.988743
	2.55	0.852759	0.999117	0.63313	0.85278
	3.05	0.607815	0.997465	0.632525	0.59916
	3.55	0.399505	0.994525	0.632012	0.386165
	4.05	0.259973	0.99017	0.631567	0.247479
	4.55	0.172205	0.984463	0.631173	0.162341

Table 5: IMSE value of the reliability function

n	mom	mle	nls	best
25	0.728207	1	0.638048	nls
50	0.175858	0.109994	0.000884	nls
75	0.175432	0.109232	0.000769	nls
100	0.172106	0.107705	0.000556	nls

Table 6: Estimation of Reliability Function when $\alpha = 2, \lambda = 3, \theta = 3$

n	t	real	mom	Mle	nls
25	0.05	1	1	0.690065	1
	0.55	1	1	0.656904	1
	1.05	1	1	0.647913	1
	1.55	1	1	0.642493	1
	2.05	0.999946	1	0.638601	0.999803
	2.55	0.929452	1	0.635561	0.917075
	3.05	0.595694	1	0.633068	0.525889
	3.55	0.305258	1	0.630954	0.233238
	4.05	0.152272	1	0.629119	0.10382

	4.55	0.078875	0.999973	0.627498	0.049253
50	0.05	1	1	0.660616	1
	0.55	1	0.999995	0.644269	1
	1.05	1	0.999786	0.639856	1
	1.55	1	0.999023	0.637197	1
	2.05	0.999946	0.99763	0.635288	0.999849
	2.55	0.929452	0.995545	0.633798	0.928033
	3.05	0.595694	0.992698	0.632575	0.550199
	3.55	0.305258	0.989126	0.631539	0.250685
	4.05	0.152272	0.98496	0.630639	0.114006
	4.55	0.078875	0.980231	0.629844	0.055248
75	0.05	1	1	0.651072	1
	0.55	1	1	0.640236	1
	1.05	1	1	0.637312	1
	1.55	1	0.999999	0.635551	1
	2.05	0.999946	0.99998	0.634286	0.999945
	2.55	0.929452	0.999855	0.633299	0.938109
	3.05	0.595694	0.999488	0.63249	0.593103
	3.55	0.305258	0.998769	0.631803	0.291143
	4.05	0.152272	0.997656	0.631207	0.139829
	4.55	0.078875	0.996047	0.630681	0.07038
100	0.05	1	1	0.646275	1
	0.55	1	1	0.638172	1
	1.05	1	1	0.635986	1
	1.55	1	1	0.634669	1
	2.05	0.999946	0.999984	0.633724	0.999781
	2.55	0.929452	0.999848	0.632986	0.924488
	3.05	0.595694	0.999363	0.632381	0.57741
	3.55	0.305258	0.998284	0.631867	0.287158
	4.05	0.152272	0.996466	0.631422	0.140678
	4.55	0.078875	0.993668	0.631028	0.07227

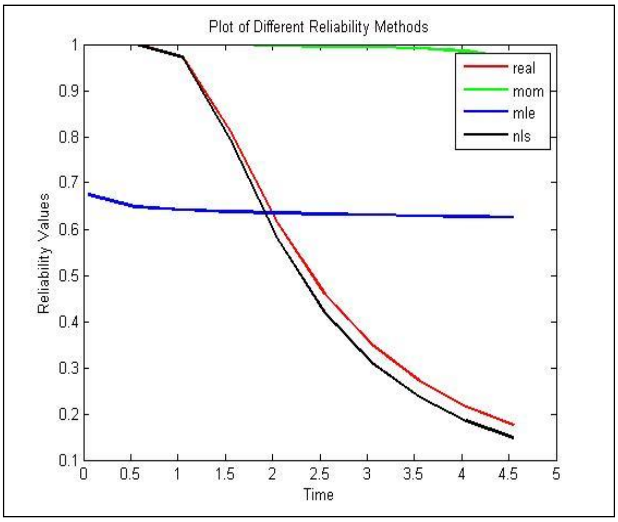

Figure 4: Different estimators for Reliability Function Estimators.

Table 7: IMSE value of the reliability function

n	Mom	mle	nls	best
25	0.221817	0.138655	0.003743	nls
50	0.220504	0.137881	0.001335	nls
75	0.219790	0.136366	0.000418	nls
100	0.213921	0.131838	0.000701	nls

Table 8: Estimation of Reliability Function when $\alpha = 3, \lambda = 3, \theta = 3$

N	t	real	mom	mle	Nls
25	0.05	1	1	0.689738	1
	0.55	1	1	0.656534	1
	1.05	1	1	0.647532	1
	1.55	1	1	0.642105	1
	2.05	1	1	0.638208	1
	2.55	0.986666	1	0.635165	0.984459
	3.05	0.577587	1	0.632669	0.535063
	3.55	0.197324	1	0.630552	0.146154
	4.05	0.064938	1	0.628715	0.042089
	4.55	0.023275	0.999998	0.627092	0.013883
50	0.05	1	1	0.660432	1
	0.55	1	1	0.644078	1
	1.05	1	1	0.639662	1
	1.55	1	1	0.637002	1
	2.05	1	1	0.635092	1
	2.55	0.986666	1	0.633601	0.972527
	3.05	0.577587	1	0.632378	0.518088
	3.55	0.197324	1	0.631341	0.17057
	4.05	0.064938	0.999983	0.630441	0.056853
	4.55	0.023275	0.999421	0.629646	0.021029
75	0.05	1	1	0.650914	1
	0.55	1	1	0.640074	1
	1.05	1	1	0.637149	1
	1.55	1	1	0.635387	1
	2.05	1	1	0.634122	1
	2.55	0.986666	1	0.633135	0.98413
	3.05	0.577587	1	0.632325	0.556662
	3.55	0.197324	1	0.631638	0.175273
	4.05	0.064938	0.999951	0.631042	0.054624
	4.55	0.023275	0.999016	0.630516	0.019022
100	0.05	1	1	0.646188	1
	0.55	1	1	0.638083	1
	1.05	1	1	0.635896	1
	1.55	1	1	0.634579	1
	2.05	1	1	0.633634	1
	2.55	0.986666	1	0.632896	0.985745
	3.05	0.577587	1	0.63229	0.569537
	3.55	0.197324	1	0.631777	0.192545
	4.05	0.064938	0.999969	0.631331	0.064181
	4.55	0.023275	0.999202	0.630937	0.023606

Table 9: IMSE value of the reliability function

n	mom	mle	nls	best
25	0.265123	0.166345	0.00129	nls
50	0.265007	0.165578	0.000679	nls
75	0.264962	0.164026	0.000559	nls
100	0.264922	0.159416	0.000531	nls

Table 10: Estimation of Reliability Function when $\alpha=3,\lambda=3,\theta=4$

N	T	real	mom	Mle	nls
25	0.05	1	1	0.68137	1
	0.55	1	1	0.655013	1
	1.05	1	1	0.647878	1
	1.55	1	1	0.643578	1
	2.05	1	1	0.64049	1
	2.55	1	1	0.638079	1
	3.05	0.99999	1	0.636101	0.999797
	3.55	0.946465	1	0.634423	0.96261
	4.05	0.591074	1	0.632967	0.625177
	4.55	0.269221	1	0.631681	0.279392
50	0.05	1	1	0.65615	1
	0.55	1	1	0.643182	1
	1.05	1	1	0.639682	1
	1.55	1	1	0.637574	1
	2.05	1	1	0.63606	1
	2.55	1	1	0.634878	1
	3.05	0.99999	1	0.633909	0.999972
	3.55	0.946465	1	0.633087	0.956272
	4.05	0.591074	1	0.632374	0.57085
	4.55	0.269221	1	0.631743	0.225138
75	0.05	1	1	0.648034	1
	0.55	1	1	0.639441	1
	1.05	1	1	0.637123	1
	1.55	1	1	0.635726	1
	2.05	1	1	0.634724	1
	2.55	1	1	0.633941	1
	3.05	0.99999	1	0.6333	0.999983
	3.55	0.946465	1	0.632755	0.942012
	4.05	0.591074	1	0.632283	0.552182
	4.55	0.269221	1	0.631865	0.230322
100	0.05	1	1	0.644027	1
	0.55	1	1	0.637602	1
	1.05	1	1	0.635869	1
	1.55	1	1	0.634826	1
	2.05	1	1	0.634076	1
	2.55	1	1	0.633491	1
	3.05	0.99999	1	0.633011	0.99997
	3.55	0.946465	1	0.632605	0.945798
	4.05	0.591074	1	0.632251	0.579204
	4.55	0.269221	1	0.631939	0.255752

Table 11: IMSE value of the reliability function

n	mom	mle	nls	best
25	0.070412	0.115866	0.001754	nls
50	0.070412	0.115177	0.000907	nls
75	0.070412	0.113785	0.000742	nls
100	0.070412	0.109472	0.000148	nls

Table 12: Estimation of Reliability Function when $\alpha=3,\lambda=4,\theta=4$

N	t	real	mom	Mle	nls
25	0.05	1	1	0.680877	1
	0.55	1	1	0.654507	1
	1.05	1	1	0.647369	1
	1.55	1	1	0.643067	1
	2.05	1	1	0.639977	1
	2.55	1	1	0.637565	1
	3.05	1	1	0.635586	1
	3.55	0.984819	1	0.633908	0.988181
	4.05	0.577476	1	0.632452	0.50199
	4.55	0.191925	1	0.631165	0.123702
50	0.05	1	1	0.656084	1
	0.55	1	1	0.643116	1
	1.05	1	1	0.639615	1
	1.55	1	1	0.637507	1
	2.05	1	1	0.635993	1
	2.55	1	1	0.634811	1
	3.05	1	1	0.633842	1
	3.55	0.984819	1	0.63302	0.981466
	4.05	0.577476	1	0.632306	0.555504
	4.55	0.191925	1	0.631676	0.177941
75	0.05	1	1	0.647992	1
	0.55	1	1	0.639399	1
	1.05	1	1	0.63708	1
	1.55	1	1	0.635684	1
	2.05	1	1	0.634681	1
	2.55	1	1	0.633899	1
	3.05	1	1	0.633257	1
	3.55	0.984819	1	0.632712	0.981807
	4.05	0.577476	1	0.63224	0.560164
	4.55	0.191925	1	0.631822	0.181213
100	0.05	1	1	0.643994	1
	0.55	1	1	0.637568	1

	1.05	1	1	0.635835	1
	1.55	1	1	0.634791	1
	2.05	1	1	0.634042	1
	2.55	1	1	0.633457	1
	3.05	1	1	0.632977	1
	3.55	0.984819	1	0.63257	0.982577
	4.05	0.577476	1	0.632217	0.565457
	4.55	0.191925	1	0.631905	0.182077

Table 13: Estimation of Reliability Function when $\alpha = 4, \lambda = 4, \theta = 4$

n	t	real	mom	Mle	nls
25	0.05	1	1	0.680882	1
	0.55	1	1	0.654509	1
	1.05	1	1	0.64737	1
	1.55	1	1	0.643068	1
	2.05	1	1	0.639978	1
	2.55	1	1	0.637566	1
	3.05	1	1	0.635587	1
	3.55	0.998829	1	0.633909	0.995041
	4.05	0.559457	1	0.632452	0.528792
	4.55	0.119517	1	0.631165	0.107031
50	0.05	1	1	0.656031	1
	0.55	1	1	0.643061	1
	1.05	1	1	0.63956	1
	1.55	1	1	0.637452	1
	2.05	1	1	0.635938	1
	2.55	1	1	0.634756	1
	3.05	1	1	0.633786	1
	3.55	0.998829	1	0.632964	0.997774
	4.05	0.559457	1	0.632251	0.538507
	4.55	0.119517	1	0.63162	0.114923
75	0.05	1	1	0.647943	1
	0.55	1	1	0.639348	1
	1.05	1	1	0.63703	1
	1.55	1	1	0.635634	1
	2.05	1	1	0.634631	1
	2.55	1	1	0.633848	1
	3.05	1	1	0.633206	1
	3.55	0.998829	1	0.632662	0.999024
	4.05	0.559457	1	0.632189	0.530657
	4.55	0.119517	1	0.631772	0.100163
100	0.05	1	1	0.643968	1
	0.55	1	1	0.637543	1
	1.05	1	1	0.63581	1
	1.55	1	1	0.634766	1
	2.05	1	1	0.634017	1
	2.55	1	1	0.633432	1
	3.05	1	1	0.632952	1
	3.55	0.998829	1	0.632545	0.998557
	4.05	0.559457	1	0.632192	0.556657
	4.55	0.119517	1	0.63188	0.118175

Table 14: IMSE value of the reliability function

N	mom	mle	nls	best
25	0.096933	0.132917	0.001048	nls
50	0.096933	0.13224	0.000493	nls
75	0.096933	0.130854	0.000446	nls
100	0.096933	0.1267	0.000432	nls

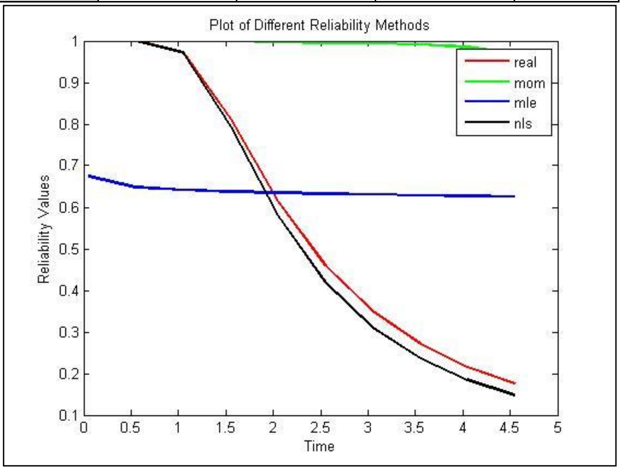

Figure 5: Different estimators for Reliability Function Estimators.

Conclusion

For the initial values ($\alpha = 2, \lambda = 2, \theta = 2$), we find the best estimators of reliability function was (NLS) (non-linear least square) as shown in Table (3), also for another set of initial values ($\alpha = 2, \lambda = 3, \theta = 3$) the best estimator was (NLS), and from the simulation results at ($\alpha = 4, \lambda = 4, \theta = 4 \; and \; n = 25$) we found the best estimator of [R(t)] is ($\hat{R}_{mom}, \hat{R}_{nls}, and \; \hat{R}_{mle} \quad when \; t = 3.55$).

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