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Classification of Arcs in Finite Projective Plane of Order Sixteen

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Abstract

The aim of this research is to classify certain geometric structures, called arcs. The main computing tool is the Computer algebra system GAP. In the plane $PG(2,16)$, an important arcs are called complete when they can't be increased to a larger arc. So far, all arcs up to size eighteen have been classified. Each of these arcs gives rise to an error-correcting code that corrects the maximum possible number of errors for its length.

Keywords: projective plane, arcs, code.

الخلاصة

الهدف من هذا البحث هو تصنيف تشكيل هندسي معين يدعى أقواس. ادوات الحسابات الرئيسيه هي لغة برمجة الأقواس المهمه تدعى كاملة وتلك الأقواس لاتكون متزايدة في $PG(2,q)$. في المستوي GAP الرياضيات قوس أكبر.كل الأقواس الى حجم ثمانية عشر تم تصنيفها.كل هذه الأقواس تعطي تصحيح اخطاء أكبر عدد ممكن من الاخطاء لاطوالها.

Introduction

A projective plane is an incidence structure of points and lines with the following properties:

- Every two points are incident with a unique line;
- Every two lines are incident with a unique point;
- There are four points, no three collinear; see [4].

A Desarguesian projective plane $PG(2,q)$ has as points one-dimensional subspaces and as lines two-dimensional subspaces of a three-dimensional vector space over the finite field F_q of q elements $V(3,q)$. A k-arc in $PG(2,q)$ is a set of k points no three of which are collinear. A k-arc is complete if it is not contained in a $(k+1)$-arc.

The main aims of this paper is to classify arcs of all sizes in projective plane $PG(2,16)$, and classify those arcs which are contained in a conic. Many results of $PG(2,q)$, $q \leq 31$ have been satisfied; see [4],[6],[7],[10],[11]. For more results we are looking at the projective plane of order sixteen. A brief history of the research subject is given as follows. Arcs were first introduced by Bose (1947) in connection with designs in statistics. Further development began with Segre in (1954) showed that every $(q+1)$-arc in $PG(2,q)$ is a conic. An important result is that of Ball, Blokhuis and Mazzocca showing that maximal arcs cannot exist in a plane of odd order. In (1981) Goppa found important applications of curves over finite fields to coding theory. As geometry over a finite field, it has been thoroughly studied in the major treatise of Hirschfeld (1979-1985) and of Hirschfeld –Thas (1991) and Hirschfeld (1998).

Definitions and basic properties

Definition 2.1[4]: Given a homogenous polynomial F in three variables x_0, x_1, x_2 over F_q, a curve $\mathcal{F}$ is the set $\mathcal{F} = v(F) = \{\mathbf{P}(X): F(X) = 0\}$ where $\mathbf{P}(X)$: is the point of $PG(2,q)$:represented by $X = (x_0, x_1, x_2)$. If F has degree two, that is,

$$F = a_0x_0^2 + a_1x_1^2 + a_2x_2^2 + b_2x_0x_1 + b_1x_0x_2 + b_0x_1x_2,$$

then $\mathcal{F}$ is called a quadric. A conic C is a non-singular quadric $\mathcal{F}$.

Definition 2.2[7]: An(n, M) code C over F_q is a subset of $(F_q)^n$ of size M. A linear $[n, k]_q$ code over Galois field F_q is a k-dimensional subspace of $(F_q)^n$ and size $M = q^k$ The vectors in the linear code C are called codewords and we denote them by $x = x_1 x_2 \cdots x_n$,where $x_i \in F_q$.

Theorem 2.3[4]: For any $[n, k, d]_q$ code we have $d \leq n - k + 1$.

Definition 2.4[4]: Let $f(x) = x^n - a_{n-1}x^{n-1} - \cdots - a_0$ be a monic polynomial of degree $n \geq 1$ over F_q.

Its companion matrix $C(f)$ is given by the $n \times n$ matrix

$$C(f) = \begin{pmatrix} 0 & 1\,0 & 0 \\ \vdots & \vdots\,\ddots & \vdots \\ 0 & 0\,0 & 1 \\ a_0 & a_1 \cdots & a_{n-1} \end{pmatrix}$$

Let f be irreducible over F_q and $\alpha \in F_{q^n}$ be a root of f. It is called primitive if the smallest power s of α such that $\alpha^s = 1$ is $(q^n - 1)$; that is, α is a primitive root over F_q^n .

Definition 2.5[4]: Denote by S and S^* two subspaces of $PG(n, K)$, A projectivity $\beta: S \to S^*$ is a bijection given by a matrix T, necessarily non-singular, where $\mathbf{P}(X^*) = \mathbf{P}(X)\beta$ if $tX^* = XT$, with $t \in K \setminus 0$. Write $\beta = M(T)$; then $\beta = M(\lambda T)$ for any λ in K. The group of projectivities of $PG(n, K)$ is denoted by $PGL(n + 1, K)$.

Definition 2.6[4]: A group G acts on a set Λ if there is a map $\Lambda \times G \to \Lambda$ such that given g, g' elements in G and 1 its identity, then

a. $x1 = x$,

b. $(xg)g' = x(g\,g')$ for any x in Λ.

Definition 2.7[4]: The orbit of x in Λ under the action of G is the set $xG = \{xg \mid g \in G\}$.

Definition 2.8[4]: The stabilizer of x in Λ under the action of G is the group $G_x = \{g \in G \mid xg = x\}$.

Definition 2.9[4]: Let K be a k-arc and **P** a point of $PG(2, q) \setminus K$. Then if exactly i bisecants of K pass through **P**, then **P** is said to be a point of index i. The number of these points is denoted by c_i.

Lemma 2.10[4]: The constants c_i of a k–arc K in satisfy the following equations with the summation taken 0 to n for which $c_i \neq 0$:

$\sum c_i = q^2 + q + 1 - k$;

$\sum ic_i = k(k - 1)(q - 1)/2$;

$\sum i(i - 1)c_i = k(k - 1)(k - 2)(k - 3)/8$.

Theorem 2.11[4]: There exists a projective $[n, k, d]_q$-code if and only if there exists an $(n; n - d)$-arc in $PG(k - 1, q)$.

Definition 2.12[4]: n-stigms is a set of n points in $PG(2, q)$, no three of which are collinear,together with the $n(n - 1) / 2$ sides (joins of pairs of points).

Results and Discussion

Construction of Inequivalent k-Arcs

In this section, the algorithm used to classify the k-arcs that contain the standard frame is described.

Let K be a $(k - 1)$-arc, $k \geq 5$, containing the standard frame Υ.

(1) Define C_0^{k-1} to be a set of points not on the bisecants of K; that is, points of index zero. Here $|C_0^{k-1}| = c_0$.

(2) If C_0^{k-1} is not empty, that is, K is not complete, then C_0^{k-1} is separated into orbits by the stabilizer group G_k of K.

(3) A k-arc is constructed by adding one point to K from an orbit.

(4) Let $\left|\frac{K}{2}\right| = n$. Then the values of the constants $c_0, c_1, \ldots, c_n$, are calculated for each k-arc.

(5) Let M^k be the set of all different k-arcs that are constructed from $(k - 1)$-arcs in $PG(2, q)$. Then M^k is partitioned into classes $\{M_i^k\}_{i \in A}$ $c_0, c_1, \ldots, c_n$.

(6) In general, two k-arcs, K and K' are equivalent if there is a projective transformation $\mathfrak{T}$ which transforms the frame Υ to any permutation of four points in K' such that $\mathfrak{T}$ transforms $K' \setminus \Upsilon$ to any permutation of the other $k - 4$ points in K'. Accordingly, any two k-arcs in the same class M_i^k are equivalent if there is a projective transformation between them.

Preliminary to $PG(2,16)$

The field $F_{16} = F_2[X] / \langle X^4 + X + 1\rangle$, where $F_2[X]$ polynomials ring over F_2 and $\langle X^4 + X + 1\rangle$ the principle ideal generated by $X^4 + X + 1$, let $\omega = X + \langle X^4 + X + 1\rangle$, so with $\omega^4 + \omega + 1 = 0$, we have $F_{16} = \{0,1,\omega,\omega^2,\dots,\omega^{14};\omega^{15} = 1\}$. In $PG(2,16)$ the projective plane of order 16, $\theta_1 = 17$, $\theta_2 = 273$, where $\theta_n = |PG(n,q)| = (q^{n+1} - 1)/(q - 1)$ for more informations see [12]; hence we have 273 points, 273 lines, 17 points on each line and 17 lines passing through each point. Let $\mathbf{P_0} = \mathbf{P}(1,0,0)$, and $T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \omega^7 & 1 & 0 \end{pmatrix}$ be a non-singular matrix such that the points of $PG(2,16)$ are generated as following. $\mathbf{P_i} = \mathbf{P}(1,0,0)\,T^i, i = 0,1,\dots,272.$ such that

$$\mathbf{P_0} = \mathbf{P}(1,0,0)\ ,\ \mathbf{P_1} = \mathbf{P}(0,1,0)\ ,\ \dots\ ,\ \mathbf{P_{253}} = \mathbf{P}(1,1,1)\ ,$$

$$\mathbf{P_{254}} = \mathbf{P}(\omega^7,0,1)\ ,\ \mathbf{P_{255}} = \mathbf{P}(\omega^{13},1,0)\ ,\ \dots\ ,\ \mathbf{P_{272}} = \mathbf{P}(1,0,1)\ .$$

Let $\ell_1 = v(Z)$; that is, ℓ_1 is the line passing through points $\mathbf{P}(x,y,z)$ with third coordinate equal to zero. Then ℓ_1 forms the following difference set, with $\mathbf{P_i} = i, i = 0,\dots,272$.

0 1 3 7 15 31 63 90 116 127 136
1 181 194 204 233 238 255

The points $\mathbf{P_i} = i$ and the lines ℓ_i of $PG(2,16)$ can be represented by the following array.

$$\ell_1 = \begin{Bmatrix} 0,1,3,7,15,31,63,90,116,127,136,181,194, \\ 204,233, \\ 238,255 \end{Bmatrix};$$

$$\ell_2 = \begin{Bmatrix} 1,2,4,8,16,32,64,91,117,128,137,182,195, \\ 205,234,239,256 \end{Bmatrix};$$

$$\vdots$$

$$\ell_{273} = \begin{Bmatrix} 272,0,2,6,14,30,62,89,115,126,135,180 \\ ,193,203,232,237,254 \end{Bmatrix}.$$

The unique 4-arc in $PG\ (2,16)$

The Fundamental Theorem of Projective Geometry is applied to the projective plane, the frame Υ is projectively the unique 4-arc in $PG(2,16)$.The frame points in $PG(2,16)$ are the points $0 = \mathbf{P}(1,0,0), 1 = \mathbf{P}(0,1,0), 2 = \mathbf{P}(0,0,1), 253 = \mathbf{P}(1,1,1)$ in numeral form. The stabilizer group of Υ is $\boldsymbol{S_4}$, which can be found by transforming Υ to its 24 permutations. The matrix determining each element of $\boldsymbol{S_4}$ for each permutation $(ijkl)$ of Υ is given by Table 1. The two matrices marked by g_1, g_2 are generators of $\boldsymbol{S_4}$.

Table 1: The stabilizer of the standard frame in $PG(2,16)$

$(ijkl)$	Matrix transformation	$(ijkl)$	Matrix transformation
(1234)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	(3124)	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
(1243)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$	(3142)	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$
(1324)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	(3214)	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
(1342)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	(3241)	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$
(1423)	$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	(3412)	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$
(1432)	$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$	(3421)	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$
(2134)	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = g_1$	(4123)	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
(2143)	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$	(4132)	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
(2314)	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	(4213)	$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
(2341)	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} = g_2$	(4231)	$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
(2413)	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	(4312)	$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$
(2431)	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$	(4321)	$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

The 5-arcs in $PG(2,16)$

The number of points on the sides of a tetrastigm is $l(4,16) = 91$ which is the number of an 4-stigm where $q = 16$. Hence the number of points not on the sides of tetrastigm is

$l^*(4,16) = 273 - 91 = 182$. The projective group $\boldsymbol{S_4}$ of the standard frame ▯ = {0,1,2,253} splits the 182 points not on the bisecants of ▯ into 10 disjoint. This gives the following conclusion.

Theorem 3.4.1. In $PG\,(2,16)$, there are precisely four projectively distinct 5-arcs, as summarized in Table 2, as follows:

Table 2: Inequivalent 5-arcs in PG(2,16)

Symbol	The 5-arc	Stabilizer
A_1	{0,1,2,253,9}	$\boldsymbol{I}$
A_2	{0,1,2,253,12}	$\boldsymbol{Z_2 \times Z_2}$
A_3	{0,1,2,253,24}	$\boldsymbol{Z_2 \times Z_2}$
A_4	{0,1,2,253,101}	$\boldsymbol{A_5}$

Remark 3. 4.2.

1. The values of the constants c_i for any 5-arcs are
   $c_0 = 133$, $c_1 = 120$, $c_2 = 15$
2. The 5-arcs A_2 and A_3 have the same constants c_i and isomorphic stabilizer groups but they are inequivalent.
3. Because of the one-to-one correspondence between the projective line $PG\,(1,16)$ and a conic, for more details see [4]. Let

$$\mathbb{C}^* = v(Y^2 - XZ) = \{\mathbf{P}(t^2, t, 1); \in F_{16} \cup \{\infty = \mathbf{P}(1,0,0)\}\}$$

be a conic. Then the four pentads δ_i as given in [9] correspond to inequivalent four 5-arcs C_i^* on the conic $\mathbb{C}^*$. Each 5-arc $C_i^*, i = 1, \dots, 4$ is equivalent to one of $A_j, j = 1, \dots, 4$. These equivalences and the matrix transformations are given in Table 3, as follows:

Table 3: Transforming C_i^* to A_j

$C_i^* \cong A_j$	Matrix transformation
$C_1^* = \{0,2,253,190,207\} \cong A_3$	$\begin{pmatrix} \omega^5 & 0 & 0 \\ \omega^2 & \omega & 1 \\ \omega & \omega & \omega \end{pmatrix}$
$C_2^* = \{0,2,253,190,215\} \cong A_1$	$\begin{pmatrix} \omega & 1 & \omega^{14} \\ \omega^{10} & \omega^7 & \omega^4 \\ \omega^{12} & 0 & 0 \end{pmatrix}$
$C_3^* = \{0,2,253,190,176\} \cong A_2$	$\begin{pmatrix} \omega & 0 & 0 \\ \omega^4 & \omega^3 & \omega^2 \\ \omega^{10} & \omega^{10} & \omega^{10} \end{pmatrix}$
$C_4^* = \{0,2,253,101,151\} \cong A_4$	$\begin{pmatrix} 1 & 0 & 0 \\ \omega^{10} & \omega^5 & 1 \\ 0 & 0 & \omega^{10} \end{pmatrix}$

Conics Through the Inequivalent 5-Arcs in $PG(2,16)$

There is a unique conic through each 5-arc. Let

$$F = a_0X^2 + a_1Y^2 + a_2Z^2 + a_3XY + a_4XZ + a_5YZ$$

be a form of degree two and $\mathbb{C} = v(F)$ be a conic. Since all five 5-arcs A_i contain the points $\mathbf{P_0}, \mathbf{P_1}, \mathbf{P_2}$ then the form F reduces to

$$XY + a_4^*XZ + a_5^*YZ \;\dots\; (1)$$

Therefore, by substituting $\mathbf{P_{253}}$ and the 5th point of each 5-arc A_i in (1) the following is deduced, then

$$\mathbb{C}_{A_1} = v(XY + \omega^5XZ + \omega^{10}YZ);$$
$$\mathbb{C}_{A_2} = v(XY + \omega^2XZ + \omega^2YZ);$$
$$\mathbb{C}_{A_3} = v(XY + \omega XZ + \omega^4YZ);$$
$$\mathbb{C}_{A_4} = v(XY + \omega^{10}XZ + \omega^5YZ).$$

The 6-arcs in $PG(2,16)$

The number of points on the sides of pentastigm or 5-stigm is $l(5,16) = 140$. Hence the number of points not on the sides of each pentastigm is $l^*(5,16) = 273 - 140 = 133$. So the total number of points not on the sides of the four pentastigms is 532. The action of the stabilizer group of each inequivalent 5-arc on the corresponding set C_0^5 splits the 532 points into orbits. There are five different classes of 6-arcs of type $[c_0, c_1, c_2, c_3]$ and seven different sizes of stabilizer groups. The details about them are given in Table 2. A cell n: $|G|$ in Table 3 means that n is the number of 6-arcs stabilized by the group G

Table 4: Statistics of the constants c_i of 6-arcs

No.	$[c_0, c_1, c_2, c_3]$	n: $|G|$
1	[72,180,0,15]	1: 360
2	[80,156,24,7]	6: 24
3	[84,144,36,3]	98: 4, 32: 3 , 2: 6
4	[86,138,42,1]	216: 1 , 33: 2
5	[87,135,45,0]	131: 1, 13: 5

Theorem 3.6.1. In $PG(2,16)$, there are precisely 61 projectively distinct 6-arcs. The numbers of 6-arcs with their stabilizer group type are given in Table 5, as follows:

Table 5: The stabilizer groups of 6-arcs

Stabilizer	$\boldsymbol{I}$	$\boldsymbol{Z_2}$	$\boldsymbol{Z_3}$	$Z_2 \times Z_2$	$\boldsymbol{S_3}$	$\boldsymbol{Z_5}$	$\boldsymbol{S_4}$	$\boldsymbol{A_6}$
Number	24	5	10	12	2	1	6	1

The eight hexads $\mathrm{E_i}$ as given in [9]correspond to eight inequivalent 6-arcs E_{i}^{*} on the conic $\mathbb{C}^{*}$. Each 6-arc $E_{\mathrm{i}}^{*}, \mathrm{i} = 1, \dots, 8$ is equivalent to one. This gives the following conclusion.

Theorem 3.6.2. In $PG(2,16)$, there are precisely 8 projectively distinct 6-arcs on a conic, as summarized in Table 6, as follows:

Table 6: Inequivalent 6-arcs on the conics

The conic	n: G
$\mathbb{C}_{\mathrm{A}_1}$	2: $\mathbf{S_3}$
$\mathbb{C}_{\mathrm{A}_2}$	3 : $\mathbf{Z_2}$
$\mathbb{C}_{\mathrm{A}_3}$	2: $\mathbf{Z_2}$
$\mathbb{C}_{\mathrm{A}_4}$	1: $\mathbf{Z_5}$

The 7-arcs in $PG(2,16)$

The total number of points not on the sides of the hexastigms or 6-stigms is 5154. The action of the stabilizer group of each inequivalent 6-arc on the corresponding set C_0^6 splits the 5154 points into orbits. There are twelve different classes of 7-arcs of type $[c_0, c_1, c_2, c_3]$ and six different sizes of stabilizer groups. A cell n: $|G|$ denote the number n of 7-arcs with stabilizer group size $|G|$. The constants c_i of 7-arcs are given in Table 7, as follows:

Table 7: Statistics of the constants c_i of 7-arcs

No.	$[c_0, c_1, c_2, c_3]$	n: $\vert G \vert$
1	[41,150,60,15]	74: 5
2	[43,144,66,13]	52: 1 , 46: 2 , 20: 3
3	[44,141,69,12]	184: 1
4	[45,138,72,11]	1066: 1 , 50: 2
5	[46,135,75,10]	1012: 1 , 38: 3
6	[47,132,78,9]	653: 1 , 123: 3 , 10: 3
7	[48,129,81,8]	754: 1
8	[49,126,84,7]	598: 1 , 120: 2
9	[50,123,87,6]	124: 1
10	[51,120,90,5]	136: 1 , 48: 2 , 2: 10
11	[52,117,93,4]	11: 1
12	[53,114,96,3]	14: 1 , 16: 2 , 4: 6

Theorem 3.7.1. In $PG(2,16)$, there are precisely 454 projectively distinct 7-arcs.

The number n of inequivalent 7-arcs with stabilizer group of type G with respect to the constants c_i are given in Table 8, as follows:

Table 8: Statistics of the constants c_i of in equivalent 7-arcs

No.	$[c_0, c_1, c_2, c_3]$	n: G
1	[41,150,60,15]	1: $\boldsymbol{Z_5}$
2	[43,144,66,13]	4: $\boldsymbol{I}$, 5: $\boldsymbol{Z_2}$, 2: $\boldsymbol{Z_3}$
3	[44,141,69,12]	8: $\boldsymbol{I}$
4	[45,138,72,11]	60: $\boldsymbol{I}$, 7: $\boldsymbol{Z_2}$
5	[46,135,75,10]	79: $\boldsymbol{I}$, 3: $\boldsymbol{Z_3}$
6	[47,132,78,9]	58: $\boldsymbol{I}$, 20: $\boldsymbol{Z_2}$, 2: $\boldsymbol{Z_3}$
7	[48,129,81,8]	70: $\boldsymbol{I}$
8	[49,126,84,7]	66: $\boldsymbol{I}$, 18: $\boldsymbol{Z_2}$
9	[50,123,87,6]	12: $\boldsymbol{I}$
10	[51,120,90,5]	17: $\boldsymbol{I}$, 12: $\boldsymbol{Z_2}$, 1: $\boldsymbol{D_5}$
11	[52,117,93,4]	1: $\boldsymbol{I}$
12	[53,114,96,3]	2: $\boldsymbol{I}$, 4: $\boldsymbol{Z_2}$, 2: $\boldsymbol{S_3}$

The ten heptads F_i as given in [9] correspond to ten inequivalent 7-arcs F_{i}^{*} on the conic $\mathbb{C}^{*}$. This gives the following conclusion.

Theorem 3.7.2. In $PG(2,16)$, there are precisely 10 projectively distinct 7-arcs on the conic summarized in Table 9, as follows:

Table 9: Inequivalent 7-arcs on the conic

No.	$[c_0, c_1, c_2, c_3]$	Stabilizer
1	[45,138,72,11]	1: $\boldsymbol{I}$
2	[47,132,78,9]	2: $\boldsymbol{Z_3}$
3	[49,126,84,7]	6: $\boldsymbol{Z_2}$
4	[51,120,90,5]	1: $\boldsymbol{D_5}$

The 8-arcs in $PG(2,16)$

The total number of points not on the sides of the 7-stigms is 21495. The action of the stabilizer group of each inequivalent 7-arc on the corresponding set C_0^7 splits the 21495 points into orbits. There are 62 different classes of 8-arcs of type $[c_0, c_1, c_2, c_3, c_4]$. The minimum and maximum value of each constant c_i for all 8-arcs is as follows:

$$6 \le c_0 \le 36 \ , \ 56 \le c_1 \le 135 \ ,$$
$$72 \le c_2 \le 156 \ , \ 16 \le c_3 \le 38 \ ,$$
$$0 \le c_4 \le 8 \ .$$

Since $c_o \neq 0$ for all 8-arcs so there is no complete 8-arc in $PG(2,16)$. There are eight different sizes of stabilizer groups of the 8-arcs. The details are given in Table 10, as follows:

Table 10: Statistics of the stabilizer groups of 8-arcs

Number of 8-arcs	$\lvert G \rvert$	Number of 8-arcs	$\lvert G \rvert$
19575	1	24	6
1866	2	1	8
15	3	1	8
10	4	3	10

Theorem 3.8.1. In $PG(2,16)$, there are precisely 2633 projectively distinct 8-arcs.

In Table 11, the numbers of inequivalent 8-arcs are listed according to the stabilizer group types

Table 11: Statistics of the inequivalent 8-arcs

Number of 8-arcs	G	Number of 8-arcs	G
2228	$\boldsymbol{I}$	8	$\boldsymbol{S_3}$
368	$\boldsymbol{Z_2}$	1	$\boldsymbol{Z_2} \times \boldsymbol{Z_2}$
2	$\boldsymbol{Z_3}$	1	$\times \boldsymbol{Z_2}$
6	$\boldsymbol{Z_4}$	1	$\boldsymbol{Z_4} \times \boldsymbol{Z_2}$
			$\boldsymbol{D_5}$

Table 13: Statistics of the stabilizer groups of 9-arcs

Number of 9-arcs	$\lvert G \rvert$	Number of 9-arcs	$\lvert G \rvert$
54266	1	19	8
1642	2	4	9
156	3	1	18
38	6		

The eleven octads H_i as given in [9] correspond to eleven inequivalent 8-arcs H_i^* on the conic $\mathbb{C}^*$. This gives the following conclusion.

Theorem 3.8.2. In $PG(2,16)$, there are precisely 11 projectively distinct 8-arcs on a conic, as summarized in Table 12, as follows:

Table 12: Inequivalent 8-arcs on the conic

No.	$[c_0, c_1, c_2, c_3, c_4]$	Stabilizer
1	[18,104,120,16,7]	$1{:}\,\boldsymbol{Z_2} \times \boldsymbol{Z_2} \times \boldsymbol{Z_2}$
2	[22,98,114,30,1]	$4{:}\,\boldsymbol{Z_2}$
3	[22,99,111,33,0]	$2{:}\,\boldsymbol{I}$
4	[26,87,123,39,0]	$1{:}\,\boldsymbol{I}$
5	[28,78,138,18,3]	$2{:}\,\boldsymbol{S_3}$
6	[28,80,132,24,1]	$1{:}\,\boldsymbol{Z_2}$

The 9-arcs in $\boldsymbol{PG(2,16)}$

The total number of points not on the sides of the 8-stigms is 56126. The action of the stabilizer group of each inequivalent 8-arc on the corresponding set $\mathbb{C}_0^8$ splits the 56126 points into orbits. There are 116 different classes of 9-arcs of type $[c_0, c_1, c_2, c_3, c_4]$. The minimum and maximum value of each constant c_i for all 9-arcs is as follows:

$$0 \le c_0 \le 21 \,,\ 0 \le c_1 \le 78 \,,$$
$$99 \le c_2 \le 216 \,,\ 0 \le c_3 \le 93 \,,\ 0 \le c_4 \le 17$$

Since $c_0 = 0$ for some 9-arcs so there is a complete 9-arc in $PG(2,16)$. There are 7 different sizes of stabilizer groups of the 9-arcs. The details are given in Table 13, as follows:

Theorem 3.9.1. In $PG(2,16)$, there are precisely 6014 projectively distinct 9-arcs divided into 608 incomplete arcs and 6 complete arcs.

In Table 14, the numbers of inequivalent 9-arcs are listed according to the stabilizer group types G.

Table 14: Statistics of the inequivalent incomplete 9-arcs

Number of 9-arcs	G	Number of 9-arcs	G
5622	$\boldsymbol{I}$	10	$\boldsymbol{Z_2} \times \boldsymbol{Z_2} \times \boldsymbol{Z_2}$
312	$\boldsymbol{Z_2}$	3	$\boldsymbol{Z_3} \times \boldsymbol{Z_3}$
44	$\boldsymbol{Z_3}$	1	$(\boldsymbol{Z_3} \times \boldsymbol{Z_3}) \rtimes \boldsymbol{Z_2}$
16	$\boldsymbol{S_3}$		

According to the stabilizer group types G, the numbers of complete 9-arcs are listed in Table 15, as follows:

Table 15: Statistics of the inequivalent complete 9-arcs

Number of 9-arc	$\lvert G \rvert$
6	$\boldsymbol{Z_3}$

The 10-arcs in $\boldsymbol{PG(2,16)}$

The total number of points not on the sides of the 9-stigms is 47296. The action of the stabilizer group of each inequivalent 9-arc on the corresponding set $\mathbb{C}_0^9$ splits the 47296 points into orbits. There are 191 different classes of 10-arcs of type $[c_0, c_1, c_2, c_3, c_4, c_5]$. The minimum and maximum value of each constant c_i for all 10-arcs is as follows:

$$0 \le c_0 \le 8,\ 0 \le c_1 \le 42 \,,$$
$$56 \le c_2 \le 120,\ 80 \le c_3 \le 156 \,,$$
$$8 \le c_4 \le 42,\ 0 \le c_5 \le 15.$$

Since $c_0 = 0$ for some 10-arcs so there is a complete 10-arc in $PG(2,16)$. There are 7 dif-

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ferent sizes of stabilizer groups of the 10-arcs. The details are given in Table 16, as follows:

Table 16: Statistics of the stabilizer groups of 10-arcs

Number of 10-arcs	$\lvert G \rvert$	Number of 10-arcs	$\lvert G \rvert$
42407	1	44	6
4607	2	132	8
30	3	9	10
67	4		

Theorem 3.10.1. In $PG(2,16)$, there are precisely 4899 projectively distinct 10-arcs divided into 2955 incomplete arcs and 1944 complete arcs.
In Table 17, the numbers of inequivalent incomplete10-arcs are listed according to the stabilizer group types G.

Table 17: Statistics of the inequivalent incomplete 10-arcs

Number of 10-arcs	G	Number of 10-arcs	G
2642	$\boldsymbol{I}$	6	$\boldsymbol{S_3}$
289	$\boldsymbol{Z_2}$	1	$\boldsymbol{Z_2 \times Z_2 \times Z_2}$
6	$\boldsymbol{Z_3}$	5	$\boldsymbol{D_5}$
6	$\boldsymbol{Z_4}$		

According to the stabilizer group types G, the numbers of complete 10-arcs are listed in Table 18, as follows:

Table 18: Statistics of the inequivalent complete 10-arcs

Number of 10-arcs	G	Number of 10-arcs	G
1503	$\boldsymbol{I}$	12	$\boldsymbol{S_3}$
374	$\boldsymbol{Z_2}$	44	$\boldsymbol{Z_2 \times Z_2 \times Z_2}$
9	$\boldsymbol{Z_4}$	2	$\boldsymbol{D_5}$

The 11-arcs in $PG(2,16)$

The total number of points not on the sides of the 10-stigms is 12280. The action of the stabilizer group of each inequivalent 10-arc on the corresponding set C_0^{10}splits the 12280 points into orbits. There are 23 different classes of 11-arcs of type $[c_0, c_1, c_2, c_3, c_4, c_5]$. The minimum and maximum value of each constant c_i for all 11-arcs is as follows:

$$0 \leq c_0 \leq 7, c_1 = 0,$$
$$30 \leq c_2 \leq 80, 70 \leq c_3 \leq 150,$$
$$60 \leq c_4 \leq 105, 3 \leq c_5 \leq 21.$$

Since $c_0 = 0$ for some 11-arcs so there is a complete 11-arc in $PG(2,16)$. There are six different sizes of stabilizer groups of the 11-arcs. The details are given in Table 19, as follows:

Table 19: Statistics of the stabilizer groups of 11-arcs

Number of 11-arcs	$\lvert G \rvert$	Number of 11-arcs	$\lvert G \rvert$
11172	1	22	5
1047	2	21	6
16	3	2	10

Theorem 3.11.1. In $PG(2,16)$, there are precisely 1171 projectively distinct 11-arcs divided into 1058 incomplete arcs and 113 complete arcs.
In Table 20, the numbers of inequivalent 11-arcs are listed according to the stabilizer group types G.

Table 20: Statistics of the inequivalent incomplete 11-arcs

Number of 11-arcs	G	Number of 11-arcs	G
921	$\boldsymbol{I}$	5	$\boldsymbol{Z_5}$
123	$\boldsymbol{Z_2}$	6	$\boldsymbol{S_3}$
2	$\boldsymbol{Z_3}$	1	$\boldsymbol{D_5}$

According to the stabilizer group types G, the numbers of complete 11-arcs are listed in Table 21, as follows:

Table 21: Statistics of the inequivalent complete 11-arcs

Number of 11-arc	G
80	$\boldsymbol{I}$
33	$\boldsymbol{Z_2}$

The 12-arcs in $PG(2,16)$

The total number of points not on the sides of the 11-stigms is 6640. The action of the stabilizer group of each inequivalent 11-arc on the corresponding set C_0^{11} splits the 6640 points into orbits. There are 8 different classes of 12-arcs of type $[c_0, c_1, c_2, c_3, c_4, c_5, c_6]$ as given below:

$$[0, 0, 0, 126, 72, 54, 9],$$
$$[0, 0, 0, 130, 60, 66, 5],$$
$$[1, 0, 0, 120, 75, 60, 5],$$
$$[6, 0, 0, 60, 180, 0, 15],$$
$$[6, 0, 0, 68, 156, 24, 7],$$
$$[6, 0, 0, 72, 144, 36, 3],$$

$[6,0,0,74,138,42,1]$,
$[6,0,0,75,135,45,0]$.

Since $c_0 = 0$ for some 12-arcs so there is a complete 12-arc in $PG(2,16)$.There are ten different sizes of stabilizer groups of the 12-arcs. The details are given in Table 22, as follows:

Table 22: Statistics of the stabilizer groups of 12-arcs

Number of 12-arcs	$\|G\|$	Number of 12-arcs	$\|G\|$
6168	1	7	5
337	2	64	6
20	3	11	10
12	4	8	18
12	4	1	61

Theorem 3.12.1. In $PG(2,16)$, there are precisely 587 projectively distinct 12-arcs divided into 555 incomplete arcs and 32 complete arcs. In Table 23, the numbers of inequivalent incomplete 12-arcs are listed according to the stabilizer group types G.

Table 23: Statistics of the inequivalent incomplete 12-arcs

Number of 12-arc	G	Number of 12-arc	G
499	$\mathbf{I}$	4	$\boldsymbol{Z_4}$
37	$\boldsymbol{Z_2}$	1	$\boldsymbol{Z_5}$
3	$\boldsymbol{Z_3}$	8	$\boldsymbol{S_3}$
2	$\boldsymbol{Z_2 \times Z_2}$	1	$\boldsymbol{Z_{61}}$

According to the stabilizer group types G, the numbers of complete 12-arcs are
listed in Table 24, as follows:

Table 24: Statistics of the inequivalent complete 12-arcs

Number of 12-arcs	G
8	$\boldsymbol{Z_2}$
2	$\boldsymbol{Z_3}$
14	$\boldsymbol{S_3}$
4	$\boldsymbol{D_5}$
4	$\boldsymbol{(Z_3 \times Z_3) \rtimes Z_2}$

The 13-arcs in $\boldsymbol{PG(2,16)}$

The total number of points not on the sides of the 12-stigms is 3325. The action of the stabilizer group of each inequivalent 12-arc on the corresponding set C_0^{12} splits the 3325 points into orbits. There are only two different classes of 13-arcs of type $[c_0, c_1, c_2, c_3, c_4, c_5, c_6]$ as given below:

$[0,0,0,0,195,0,65]$,
$[5,0,0,0,120,120,15]$.

Since the value of $c_0 = 0$ for some 13-arcs so there is a complete 13-arc in $PG(2,16)$.There are six different sizes of stabilizer groups of the 13-arcs. The details are given in Table 25, as follows:

Table 25: Statistics of the stabilizer groups of 13-arcs

Number of 13-arcs	$\|G\|$	Number of 13-arcs	$\|G\|$
3094	1	6	12
94	2	1	39
28	4	2	60

Theorem 3.13.1. In $PG(2,16)$, there are precisely 260 projectively distinct 13-arcs divided into 259 incomplete arcs and one complete arc. In Table 26, the numbers of incomplete 13-arcs are listed according to their stabilizer group types.

Table 26: Statistics of the inequivalent incomplete 13-arcs

Number of 13-arcs	G	Number of 13-arcs	G
224	$\boldsymbol{I}$	1	$\boldsymbol{A_4}$
30	$\boldsymbol{Z_2}$	1	$\boldsymbol{A_5}$
3	$\boldsymbol{Z_2 \times Z_2}$		

According to the stabilizer group types G, the numbers of complete 13-arcs are
listed in Table 27, as follows:

Table 27: Statistics of the inequivalent complete 13-arcs

Number of 13-arcs	G
1	$\boldsymbol{Z_3 \times Z_{13}}$

Theorem 3.13.2. In $PG(2,16)$, there are precisely 3 projectively distinct 13-arcs on a conic, as summarized in Table 28, as follows:

Table 28: Inequivalent 13-arcs on the conics

The 13-arc	Stabilizer	$[c_0, c_1, c_2, c_3, c_4, c_5, c_6]$	The conic
$\mathcal{J} \cup \{52\}$	$\boldsymbol{A_4}$	$[5,0,0,0,120,120,15]$	$\mathbb{C}_{A_2}$
$\mathcal{J} \cup \{183\}$	$\boldsymbol{Z_2 \times Z_2}$	$[5,0,0,0,120,120,15]$	$\mathbb{C}_{A_2}$
$\mathcal{J} \cup \{213\}$	$\boldsymbol{Z_2 \times Z_2}$	$[5,0,0,0,120,120,15]$	$\mathbb{C}_{A_2}$

Where,
$\mathcal{J} = \{0,1,2,253,12,162,169,149,250,18,226,207\}$.

The 14-arcs in $PG(2,16)$

The total number of points not on the sides of the 13-stigms is 1295. The action of the stabilizer group of each inequivalent 13-arc on the corresponding set $\mathbb{C}_0^{13}$ splits the 1295 points into orbits. There is one class of 14-arcs of type $[c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7]$ as given below:

$$[4,0,0,0,0,168,84,3].$$

Since the value of $c_0 \neq 0$ for all 14-arcs so there is no complete 14-arc in $PG(2,16)$.There are six different sizes of stabilizer groups of the 14-arcs. The details are given in Table 29, as follows:

Table 29: Statistics of the stabilizer groups of 14-arcs

Number of 14-arcs	$\lvert G \rvert$	Number of 14-arcs	$\lvert G \rvert$
1121	1	16	4
123	2	10	6
16	4	9	12

Theorem 3.14.1. In $PG(2,16)$, there are precisely 100 projectively distinct incomplete 14-arcs, as summarized in Table 30, as follows:

Table 30: Statistics of the inequivalent incomplete 14-arcs

Number of 14-arcs	G	Number of 14-arcs	G
76	$\boldsymbol{I}$	4	$\boldsymbol{Z_4}$
16	$\boldsymbol{Z_2}$	1	$\boldsymbol{S_3}$
2	$\boldsymbol{Z_2 \times Z_2}$	1	$\boldsymbol{A_4}$

Theorem 3.14.2. In $PG(2,16)$, there is precisely one projectively 14-arc on a conic, as summarized in Table 31, as follows:

Table 31: 14-arc on the conic

The 14-arc	Stabilizer	$[c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7]$	The conic
$\mathcal{J}_1$	$\boldsymbol{S_3}$	[4,0,0,0,0,168,84,3]	$\mathbb{C}_{A_2}$

Where, $\mathcal{J}_1 = \mathcal{J} \cup \{213,183\}$.

The 15-arcs in $PG(2,16)$

The total number of points not on the sides of the 14-stigms is 400. The action of the stabilizer group of each inequivalent 14-arc on the corresponding set $\mathbb{C}_0^{14}$ splits the 400 points into orbits. There is only one class of 15-arcs of type of $[c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7]$ as given below:

$$[3,0,0,0,0,0,210,45].$$

Since $c_0 \neq 0$ for all 15-arcs so there is no complete 15-arc in $PG(2,16)$. There are four different sizes of stabilizer groups of the 15-arcs. The details are given in Table 32, as follows:

Table 32: Statistics of the stabilizer groups of 15-arcs

Number of 15-arcs	$\lvert G \rvert$	Number of 15-arcs	$\lvert G \rvert$
373	1	25	6
29	2	3	30

Theorem 3.15.1. In $PG(2,16)$, there are precisely 30 projectively distinct incomplete 15-arcs, as summarized in Table 33, as follows:

Table 33: The inequivalent incomplete 15-arcs

Number of 15-arcs	G	Number of 15-arcs	G
20	$\boldsymbol{I}$	5	$\boldsymbol{S_3}$
4	$\boldsymbol{Z_2}$	1	$\boldsymbol{D_{15}}$

Theorem 3.15.2. In $PG(2,16)$, there is precisely one projectively 15-arc on a conic, as summarized in Table 34, as follows:

Table 34: 15-arc on the conic

The 15-arc	Stabilizer	$[c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7]$	The conic
$\mathcal{J}_2$	$\boldsymbol{G_{30}}$	[3,0,0,0,0,0,210,45]	$\mathbb{C}_{A_2}$

Where, $\mathcal{J}_2 = \mathcal{J}_1 \cup \{157\}$.

The 16-arcs in $PG(2,16)$

The total number of points not on the sides of the 15-stigms is 90. The action of the stabilizer group of each inequivalent 15-arc on the corresponding set $\mathbb{C}_0^{15}$splits the 90 points into orbits. There is only one class of 16-arcs of type of $[c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8]$ as given below:

$$[2,0,0,0,0,0,0,240,15].$$

Since $c_0 \neq 0$ for all 16-arcs so there is no complete 16-arc in $PG(2,16)$. There are five different sizes of stabilizer groups of the 16-arcs. The details are given in Table 35, as follows:

Table 35: Statistics of the stabilizer groups of 16-arcs

Number of 16-arcs	$\|G\|$	Number of 16-arcs	$\|G\|$
36	1	4	30
44	2	2	240
4	4		

Theorem 3.16.1. In $PG(2,16)$, there are precisely 9 projectively distinct incomplete 16-arcs, as summarized in Table 36, as follows:

Table 36: The inequivalent 16-arcs

Number of 16-arcs	G	Number of 16-arcs	G
2	$\boldsymbol{I}$	1	$\boldsymbol{G_{30}}$
4	$\boldsymbol{Z_2}$	1	$\boldsymbol{G_{240}}$
1	$\boldsymbol{Z_4}$		

The group $\boldsymbol{G_{240}}$ in Table 36, satisfies the following properties:

- $|G_{240}| = 240$;
- G_{240} contains 15 matrix of order 2 ;
- G_{240} contains 32 matrix of order 3 ;
- G_{240} contains 64 matrix of order 5 ;
- G_{240} contains 128 matrix of order 15 ;
- G_{240} contains an identity matrix .

Theorem 3.16.2. In $PG(2,16)$, there are precisely one projectively 16-arc on a conic, as summarized in Table 37, as follows:

Table 37: 16-arc on the conic

The 16-arc	Stabilizer	$[c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8$	The conic
$\mathcal{J}_3$	$\boldsymbol{G_{30}}$	[2,0,0,0,0,0,0,240,15]	$\mathbb{C}_{A_2}$

Where, $\mathcal{J}_3 = \mathcal{J}_2 \cup \{121\}$.

The 17-arcs in $\boldsymbol{PG(2,16)}$

The total number of points not on the sides of the 16-stigms is 18. The action of the stabilizer group of each inequivalent 16-arc on the corresponding set $\mathbb{C}_0^{16}$ splits the 18 points into orbits. There is only one class of 17-arcs of type of $[c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8]$ as given below:

[1,0,0,0,0,0,0,0,255].

Since $c_0 \neq 0$ for all 17-arcs so there is no complete 17-arc in $PG(2,16)$. There are three different sizes of stabilizer groups of the 17-arcs. The details are given in Table 38, as follows:

Table 38: Statistics of the stabilizer groups of 17-arcs

Number of 17-arcs	$\|G\|$
14	2
3	240
1	4080

Theorem 3.17.1. In $PG(2,16)$, there are precisely three projectively distinct incomplete 17-arcs, as summarized in Table 39, as follows:

Table 39: The inequivalent 17-arcs

Number of 17-arcs	G
1	$\boldsymbol{Z_2}$
1	$\boldsymbol{G_{240}}$
1	$\boldsymbol{PGL(2,16)}$

Theorem 3.17.2. In $PG(2,16)$, there is precisely one projectively 17-arc on a conic, as summarized in Table 40, as follows:

Table 40: 17-arc on the conic

The 17-arc	Stabilizer	$[c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c$	The conic
$\mathcal{J}_4$	$\boldsymbol{PGL(2,16)}$	[1,0,0,0,0,0,0,0,255]	$\mathbb{C}_{A_2}$

Where, $\mathcal{J}_4 = \mathcal{J}_3 \cup \{52\}$.

The 18-arcs in $\boldsymbol{PG(2,16)}$

The total number of points not on the sides of the 17-stigms is three. The action of the stabilizer group of each inequivalent 17-arc on the corresponding set $\mathbb{C}_0^{17}$ splits the three points into orbits. There is only one class of 18-arcs of type of $[c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9]$ as given below:

[0,0,0,0,0,0,0,0,0,225].

Since the value of $c_0 = 0$ for all 18-arcs so all 18-arcs in $PG(2,16)$ are complete. There are two different sizes of stabilizer groups of the 18-arcs. The details are given in Table 41, as follows:

Table 41: Statistics of the stabilizer groups of 18-arcs

Number of 18-arcs	$\|G\|$
1	36
2	4080

Theorem 3.18.1. In $PG(2,16)$, there are precisely two projectively distinct complete 18-arcs, as summarized in Table 42, as follows:

Table 42: The inequivalent 18-arcs

Number of 18-arcs	G
1	$\boldsymbol{G_{36}}$
1	$\boldsymbol{PGL(2,16)}$

The tangent lines $\mathbf{P_{167}P_i}$, for all i in {0,1,2,253,12,162,169,149,250,18,226,207 ,213,183,157,121,52} to a conic $\mathbb{C}_{A_2}$ are concurrent.The point $\mathbf{P}_{167}$ of intersection of the tangents to a conic $\mathbb{C}_{A_2}$ the nucleus. The following figure is shown that.

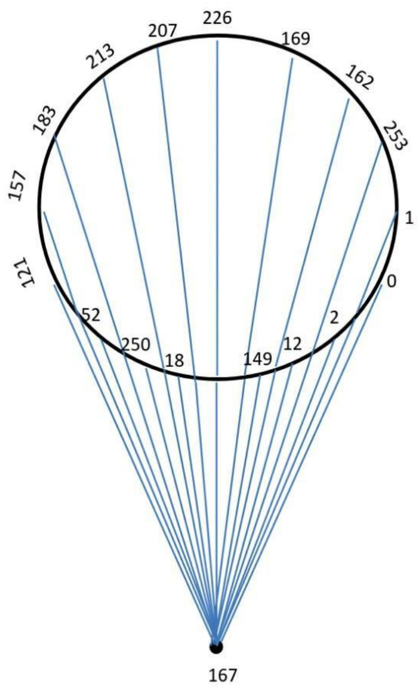

Figure 1: The tangent lines to a conic $\mathbb{C}_{A_2}$

MDS Codes of Dimension Three

According to Theorem 2.11, an $(n; n-d)$-arc in $PG(k-1,q)$ is equivalent to a projective [n; k; d]q-code. Now, if $k=3$; $n-d=2$, and $q=16$, then there is a one-to-one correspondence between n-arcs in $PG(2,16)$ and projective $[n,3,n-2]_{16}$-code C. Since $d(C)$ of the code C is equal to $n-k+1$, thus the projective code C is MDS. In Table 43, the MDS codes corresponding to the n-arcs in $PG(2,16)$ and the parameter e of errors corrected are given.

Table 43: MDS code over PG(2,16)

n-arc	MDS code	e	n-arc	MDS code	e
4-arc	$[4,3,2]_{16}$	0	12-arc	$[12,3,10]_{16}$	4
5-arc	$[5,3,3]_{16}$	1	13-arc	$[13,3,11]_{16}$	5
6-arc	$[6,3,4]_{16}$	1	14-arc	$[14,3,12]_{16}$	5
7-arc	$[7,3,5]_{16}$	2	15-arc	$[15,3,13]_{16}$	6
8-arc	$[8,3,6]_{16}$	2	16-arc	$[16,3,14]_{16}$	6
9-arc	$[9,3,7]_{16}$	3	17-arc	$[17,3,15]_{16}$	7
10-arc	$[10,3,8]_{16}$	3	18-arc	$[18,3,16]_{16}$	7
11-arc	$[11,3,9]_{16}$	4			

Conclusions

No conclusion available.

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