Al-Mustansiriyah Journal of Science
ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 34, Issue 1, 2023 DOI: http://doi.org/10.23851/mjs.v34i1.1215

Research Article **Open Access**

On α-Fuzzy Soft Irreducible Spaces

Majd Hamid Mahmood

Department of Mathematics, Collage of Education, Mustansiriyah University, Baghdad, IRAQ.

Contact: mgmg227@yahoo.com

Article Info

Received
26/08/2022

Accepted
04/12/2022

Published
30/03/2023

ABSTRACT

We define fuzzy soft irreducible sets, α-fuzzy soft irreducible sets in fuzzy soft topological spaces and study the properties including (fuzzy soft continuity; fuzzy soft homeomorphism and fuzzy soft topological properties) on α-fuzzy soft irreducible sets.

KEYWORDS: Fuzzy soft irreducible sets; fuzzy soft connected space; fuzzy soft continuous map.

الخلاصة

سوف نعرف المجاميع الضبابية الميسرة الغير قابلة للاختزال، المجاميع الضبابية الغير قابلة للاختزال- α في الفضاءات الضبابية الميسرة وسوف ندرس الخواص التي تتضمن (الاستمرارية الضبابية الميسرة، التكافؤ الضبابي الميسر والصفات التبولوجية الضبابية الميسرة) على المجاميع الضبابية الغير قابلة للاختزال- α.

INTRODUCTION

The fuzzy sets introduced in 1965 by Zadeh L. A. [12], Soft set introduced in 2001 by Molodtsov D. [8]. Fuzzy soft set introduced and studied in [1], [4], [7], [9] *(simply* $\mathfrak{F}$ *- set).* α-fuzzy soft sets defined in [5] *(simply* $\alpha\mathfrak{F}$ *- set)*. Soft topological space defined in [10]. Fuzzy soft topological space introduced and studied in [2],[6],[11].

In this research, we define fuzzy soft irreducible sets. α-fuzzy soft irreducible sets in fuzzy soft topological spaces and study the properties including: fuzzy soft continuity, fuzzy soft homeomorphism and fuzzy soft topological properties on α-fuzzy soft irreducible sets.

Definition 1.1. [7] For a universal set Ω; $\wp$ set of parameters and soft set$(f,\wp)$. If each soft element $\varkappa$ in $(f,\wp)$ is associated with $\eta\in[0,1]$, then the resulting set is called a fuzzy soft set ($\mathfrak{F}$ - set).

Definitions 1.2. [7]

For a soft set (F,E), F: E→P(Ω) where $F(e_i)\in P(\Omega)$, $\forall e_i \in E$ the set of parameters, and for a family of $\mathfrak{F}$ *- sets* generated by the same soft set (F,E), $\{(f,\wp)_\lambda : \lambda \in \Lambda$, where Λ is an infinite index set},

(1) the $\mathfrak{F}$-union is defined by

$(h,\wp)= \tilde{\cup}_\lambda\ (f,\wp)_\lambda = \{x: x(e_i,F(e_i)^{\kappa i})\}$, where $\kappa i = \{\max \kappa i_\lambda$: ki_λ are the memberships of each soft element in the soft set and $i\in\xi$, ξ is an infinite index set},

(2) the $\mathfrak{F}$-intersection is defined by:

$(h,\wp) = \tilde{\cap}_\lambda\ (f,\wp)_\lambda = \{x : x = (e_i,F(e_i)^{\kappa i})\}$, where $\kappa i=\{\min \kappa i_\lambda$: $ki_\lambda$are the memberships of each soft element in the soft set and $i\in\xi$, ξ is an infinite index set},

(3) $\tilde{A}$ is $\mathfrak{F}$- subset of $\tilde{B}$, $\tilde{A}\tilde{\subseteq}\tilde{B}$ if each $\mathfrak{F}$- element in $\tilde{A}$ is in $\tilde{B}$.

(4) $\tilde{\Phi}$ is the null $\mathfrak{F}$ *- set* where each soft element associated to η=0, $\tilde{\Omega}$ is the universal $\mathfrak{F}$ *- set* where each soft element associated to η=1.

Example 1.3.

For Ω = {a, b}, $\wp$ = {e}. Let $\tilde{G}=(e,\{a^{0.2}, b^{0.3}\})$, $\tilde{L}=(e,\{a^{0.4}, b^{0.5}\})$,
$\tilde{\Phi}=(e,\{a^0, b^0\})$ and $\tilde{\Omega}=(e,\{a^1, b^1\})$
be $\mathfrak{F}$- sets,

$\tilde{G}\,\tilde{\cup}\,\tilde{L}=(e,\{a^{0.2}, b^{0.3}\})\tilde{\cup}(e,\{a^{0.4}, b^{0.5}\})$
$= (e,\{a^{0.4}, b^{0.5}\}) = \tilde{L}$
$\tilde{G}\,\tilde{\cap}\,\tilde{L}=(e,\{a^{0.2}, b^{0.3}\})\tilde{\cap}(e,\{a^{0.4}\ b^{0.5}\})$
$= (e,\{a^{0.2}, b^{0.3}\}) = \tilde{G}$
$\tilde{\Omega}\,\tilde{\cap}\,\tilde{L} = \tilde{L}$, $\tilde{L}\,\tilde{\cup}\,\tilde{\Omega} = \tilde{\Omega}$
$\tilde{\Phi}\,\tilde{\cap}\,\tilde{G} = \tilde{\Phi}$, $\tilde{\Phi}\,\tilde{\cup}\,\tilde{L} = \tilde{L}$
$\tilde{\Phi}\,\tilde{\cap}\,\tilde{\Omega} = \tilde{\Phi}$, $\tilde{\Phi}\,\tilde{\cup}\,\tilde{\Omega} = \tilde{\Omega}$.

Definition 1.4.[11] For a non- empty universal set Ω, ℘ set of parameters, 𝔉 the collection of 𝓕- sets generated from the 𝓕- set $\tilde{\Omega}$ (the non-null universal 𝓕-set), if 𝔉 satisfies the following axioms:

(a) $\tilde{\Phi}$, $\tilde{\Omega}$ are in 𝔉.

(b) The intersection of any two 𝓕 - set belongs to .

(c) The union of members of - sets is in 𝔉 .

Then, is called (𝔉 - topology).

A triple (Ω, ℘) is called 𝔉 - topological space over Ω (simply 𝔉 -Space), the sets of 𝔉 are 𝓕- open sets denoted by 𝔉o - sets and their complements are called 𝔉c- sets.

Example 1.5.

For Ω = {a, b}, ℘ = {e}.

Let $\tilde{N}$ = (e,{ $a^{0.8}$, $b^{0.7}$ }) ,

$\tilde{D}$ = (e,{ $a^{0.5}$, $b^{0.4}$ }) , $\tilde{K}$ = (e ,{ $a^{0.6}$, $b^{0.2}$ })

$\tilde{\Phi}$ = (e, {a^{0}, b^{0} }) , $\tilde{\Omega}$ = (e, {a^{1}, b^{1} })

Let $𝔉_1$ = {$\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{N}$, $\tilde{D}$}, $𝔉_2$ = {$\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{N}$}

$𝔉_3$ = {$\tilde{\Phi}$, $\tilde{\Omega}$}.

Then, $𝔉_1$, $𝔉_2$, $𝔉_3$ are 𝔉 - topologies over Ω.

But $_4$ = {$\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{D}$, $\tilde{K}$} is not a 𝔉 - topology on Ω since $\tilde{D} \tilde{\cap} \tilde{K}$ = (e,{ $a^{0.5}$, $b^{0.2}$ }) ∉ $𝔉_4$.

Definition 1.6. [11]

Let $\tilde{A}$ be a 𝔉 - set in 𝔉 - topological space (Ω, ℘). The 𝔉 - interior of $\tilde{A}$ (or 𝔉 - int ($\tilde{A}$)) is defined by

𝔉 - in($\tilde{A}$) = $\tilde{\cup}$ {$\tilde{G}$: $\tilde{G}$ is 𝔉 o- set and $\tilde{G} \tilde{\subseteq} \tilde{A}$ } .

Definition 1.7. [11]

Let $\tilde{M}$ be a 𝔉 - set in 𝔉 - topological space (Ω ,, ℘) .The 𝔉 - closure of $\tilde{M}$ (or 𝔉- cl($\tilde{M}$)) is defined by

𝔉 - cl($\tilde{M}$) = $\tilde{\cap}$ {$\tilde{C}$: $\tilde{C}$ is 𝔉c - set and $\tilde{M} \tilde{\subseteq} \tilde{C}$} .

Remarks 1.8.

1- 𝔉 - int ($\tilde{A}$)) is the largest 𝓕 - set contained in $\tilde{A}$.

2- 𝔉 - cl ($\tilde{A}$)) is the smallest 𝓕 - set containing $\tilde{A}$.

Examples 1.9.

1- For Ω = {s, d}, ℘ = {e}. Let

$\tilde{C}$ = (e,{ s^{1}, d^{0} }) , $\tilde{D}$ = (e ,{ s^{0} , d^{1} })

$\tilde{\Phi}$ = (e, {s^{0}, d^{0} }), $\tilde{\Omega}$ = (e, {s^{1}, d^{1} }) ,

$𝔉_1$ = {$\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{C}$, $\tilde{D}$}

The sets $\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{C}$ and $\tilde{D}$ are 𝓕o - sets and 𝓕c - sets, $𝔉_1$- in($\tilde{C}$) = $\tilde{C}$, $𝔉_1$ - in($\tilde{D}$) = $\tilde{D}$, $𝔉_1$ - cl($\tilde{C}$) = $\tilde{C}$, $𝔉_1$ - cl($\tilde{D}$) = $\tilde{D}$.

2- For Ω = {s, d}, ℘ = {e}. Let

$\tilde{B}$=(e,{$s^{0.2}$, $d^{0.3}$}) , $\tilde{E}$ = (e ,{ $s^{0.4}$, d^{1} }) ,

$\tilde{F}$=(e,{$s^{0.5}$, d^{1}}) , $\tilde{\Phi}$ = (e , {s^{0}, d^{0} }) ,

$\tilde{\Omega}$=(e, {s^{1}, d^{1}})

For $𝔉_2$ = {$\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{E}$,$\tilde{F}$,$\tilde{B}$}

$𝔉_2$-in($\tilde{F}$) = (e,{$s^{0.2}$, $d^{0.3}$}) $\tilde{\cup}$ (e,{ $s^{0.4}$, d^{1}})$\tilde{\cup}$(e,{$s^{0.5}$, d^{1}})=(e,{ $s^{0.5}$, d^{1}})=$\tilde{F}$

$𝔉_2$-cl($\tilde{F}$) = (e,{$s^{0.5}$,d^{1}}) $\tilde{\cap}$ (e, {s^{1}, d^{1}})=$\tilde{F}$.

α𝔉- IRREDUCIBLE SPACE

In this section we will define and study α𝔉 - irreducible spaces, with examples.

Definition 2.1.

The 𝔉 - set $\tilde{M}$ in 𝔉 - topological space (Ω ,, ℘) is called (α𝔉o-set) if:

$\tilde{M} \tilde{\subseteq}$ 𝔉 - int [𝔉 - cl (- int ($\tilde{M}$))].

And is called (α𝔉c-set) if:

𝔉 - cl [𝔉 - int (𝔉- cl($\tilde{M}$))] $\tilde{\subseteq} \tilde{M}$.

Remarks 2.2.

1- α𝔉 - int ($\tilde{A}$)) is the largest 𝓕 - set contained in $\tilde{A}$.

2- α𝔉 - cl ($\tilde{A}$)) is the smallest 𝓕 - set containing $\tilde{A}$.

3- for α𝔉-set $\tilde{M}$ = {x_{ij} : x_{ij} = (ei , {h_j^{kij}})},

4- $\tilde{M}^c$ = { x_{ij} : x_{ij} = (ei , {h_j^{1-kij}})} , ∀ kij ∈ [0,1].

5- The complement of α𝔉o-set (α𝔉c-set) is α𝔉c-set (α𝔉o- set).

Remark 2.3.

Every 𝔉o- set (𝔉c-set) is α𝔉o-set (α𝔉c-set) but the converse is not true in general.

The contra positive is true also, i.e., if the set is not α𝔉o-set (α𝔉c-set), then it is not 𝔉o-set (𝔉c-set).

Examples 2.4.

For Ω = {a, b}, ℘ = {e} and $\tilde{M}$, $\tilde{N}$, $\tilde{C}$, $\tilde{D}$ are 𝔉 - sets defined as follows:

$\tilde{M}$ = (e, {$a^{0.5}$, $b^{0.6}$ }), $\tilde{N}$ = (e,{ $a^{0.3}$, $b^{0.4}$})

$\tilde{C}$ = (e,{ $a^{0.8}$ $b^{0.7}$ }) , $\tilde{D}$ = (e ,{ $a^{0.5}$ $b^{0.4}$ })

$\tilde{\Phi}$ = (e, {a^{0}, b^{0} }) , $\tilde{\Omega}$ = (e, {a^{1}, b^{1} })

Let 𝔉 = {$\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{M}$, $\tilde{N}$, $\tilde{C}$} be 𝔉 - topology on Ω.

Since $\tilde{C}$ ∈ 𝔉. Then, $\tilde{C}$ is 𝔉o - set so it is α𝔉o-set,

α𝔉 - int($\tilde{C}$) = $\tilde{C}$

[α𝔉 - cl (α𝔉-int($\tilde{C}$))] is the smallest α𝔉c - set containing $\tilde{C}$ which is equal to $\tilde{C}$.

Then, α𝔉 - int [α𝔉 - cl (α𝔉 - int($\tilde{C}$))] which is the largest α𝔉o - set contains $\tilde{C}$ which is equal to $\tilde{C}$. So $\tilde{C}$ is α𝔉o-set, similarly for $\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{M}$ and $\tilde{N}$ are α𝔉o-set,

$\tilde{D}$ is (𝔉c-set) since it's the complement is 𝔉o – set $\tilde{M}$, and $\tilde{D}$ is(α𝔉c-set) since its complement is α𝔉o - set $\tilde{M}$, $\tilde{D}$ is not 𝔉o - set since $\tilde{D}$ ∉ 𝔉 and $\tilde{D}$ is not α𝔉o-set since by definition,

α𝔉 - int ($\tilde{D}$)) = $\tilde{N}$

α𝔉 - cl (α𝔉 - int ($\tilde{D}$)) = α𝔉 - cl ($\tilde{N}$) = $\tilde{D}$

$\alpha\mathfrak{F}$ - int [$\alpha\mathfrak{F}$ - cl (- int ($\widetilde{D}$))] = $\alpha\mathfrak{F}$ - int [$\widetilde{D}$]= $\widetilde{N}$ and $\widetilde{D} \not\subseteq \alpha\mathfrak{F}$ - int [$\alpha\mathfrak{F}$ - cl ($\alpha\mathfrak{F}$ - int ($\widetilde{D}$))] = $\widetilde{N}$.

Definition 2.5.

The $\mathfrak{F}$ - topological space (Ω ,, $\wp$) is called $\alpha\mathfrak{F}$ - irreducible if the intersection of any two non-null $\alpha\mathfrak{F}$o - sets is a non-null set otherwise it will be said to be $\alpha\mathfrak{F}$ - reducible.

Examples 2.6.

1- For $\Omega = \{a, b\}$, $\wp = \{e\}$ and $\widetilde{M}, \widetilde{N}, \widetilde{C}$ are $\mathfrak{F}$ - sets defined as follows:
$\widetilde{M}$=(e,{$a^{0.3}$, $b^{0.5}$ }) , $\widetilde{N}$ =(e,{ $a^{0.2}$, $b^{0.1}$})
$\widetilde{C}$=(e,{ $a^{0.02}$ b^{0} }) , $\widetilde{\Phi}$ =(e , {a^0, b^0 }) ,
$\widetilde{\Omega}$ = (e, {a^1, b^1 }) ,
let $\mathfrak{F} = \{\widetilde{\Phi}, \widetilde{\Omega}, \widetilde{M}, \widetilde{N}, \widetilde{C}\}$ be $\mathfrak{F}$ - topology over Ω.
Since $\widetilde{\Phi}$ $\widetilde{\Omega}$, $\widetilde{M}, \widetilde{N}, \widetilde{C}$ are $\mathfrak{F}$o-sets, then $\widetilde{\Omega}, \widetilde{M}, \widetilde{N}, \widetilde{C}$ are $\alpha\mathfrak{F}$o-set. Since the intersection of any two non-null $\alpha\mathfrak{F}$o - sets is a non-null set.
Then, (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - irreducible space.
2- For $\Omega = \{p, m\}$, $\wp = \{e\}$. Let
3- $\widetilde{U}$ =(e,{ p^1, m^0 }) , $\widetilde{V}$ = (e ,{ p^0 , m^1})
$\widetilde{\Phi}$ =(e, {p^0, m^0 }) , $\widetilde{\Omega}$ =(e , {p^1, m^1 })
With $\mathfrak{F}$ - topology $\mathfrak{F} = \{\widetilde{\Phi}, \widetilde{\Omega}, \widetilde{U}, \widetilde{V}\}$
Since $\widetilde{\Phi}$ $\widetilde{\Omega}$, $\widetilde{U}, \widetilde{V}$ are $\mathfrak{F}$o-sets then they are $\alpha\mathfrak{F}$o-sets. Since $\widetilde{U} \tilde{\cap} \widetilde{V}$ =(e, {p^1, m^0 })$\tilde{\cap}$(e,{p^0, m^1})=(e,{p^0, m^0})=$\widetilde{\Phi}$ then the $\mathfrak{F}$-topological space be $\alpha\mathfrak{F}$ - reducible **.**

Definition 2.7.

In $\mathfrak{F}$ - topological space (Ω, $\wp$), the $\alpha\mathfrak{F}$ - set $\tilde{A}$ is $\alpha\mathfrak{F}$ - dense if $\tilde{A}$ intersect with any non-null $\alpha\mathfrak{F}$o - sets in $\mathfrak{F}$.

Example 2.8.

1- For $\Omega = \{m, n\}$, $\wp = \{c\}$ set of parameters and $\widetilde{Z}, \widetilde{D}, \widetilde{K}$ are $\mathfrak{F}$ - sets are defined as follows:
$\widetilde{Z}$ = (c,{$m^{0.2}$, $n^{0.7}$ }) ,$\widetilde{D}$ = (c ,{ $m^{0.3}$, $n^{0.09}$})
$\widetilde{K}$ = (c,{ $a^{0.4}$ b^1 }) , $\widetilde{\Phi}$ = (c , {a^0, b^0 }) ,
$\widetilde{\Omega}$ = (c, {a^1, b^1 })
Let $\mathfrak{F} = \{\widetilde{\Phi}, \widetilde{\Omega}, \widetilde{Z}, \widetilde{D}, \widetilde{K}\}$ be a $\mathfrak{F}$ - topology over Ω.
Then, $\widetilde{\Phi}, \widetilde{\Omega}, \widetilde{Z}, \widetilde{D}$ and $\widetilde{K}$ are $\alpha\mathfrak{F}$o-sets,
$\widetilde{Z}$ is $\alpha\mathfrak{F}$ - dense set since $\widetilde{Z} \tilde{\cap} \widetilde{D} \neq \widetilde{\Phi}$,
$\widetilde{Z} \tilde{\cap} \widetilde{K} \neq \widetilde{\Phi}$ and $\widetilde{Z} \tilde{\cap} \widetilde{\Omega} \neq \widetilde{\Phi}$.
Similarly, $\widetilde{\Omega}$, is $\alpha\mathfrak{F}$ - dense set.
2- For $\Omega = \{m, n\}$, $\wp = \{e\}$,
Let $\tilde{R}$ = (e,{ m^1, n^0}) , $\tilde{S}$ = (e ,{ m^0, n^1 })
$\widetilde{\Phi}$ = (e,{ m^0, n^0 }) , $\widetilde{\Omega}$ = (e ,{ m^1, n^1})
With $\mathfrak{F}$ - topology $\mathfrak{F} = \{\widetilde{\Phi}, \widetilde{\Omega}, \tilde{R}, \tilde{S}\}$, the elements of $\mathfrak{F}$ are $\alpha\mathfrak{F}$o-sets.
Since $\tilde{R} \tilde{\cap} \tilde{S} = \widetilde{\Phi}$, So $\tilde{R}, \tilde{S}$ are not $\alpha\mathfrak{F}$ - dense set.

Definition 2.9.

The $\mathfrak{F}$-topological space (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - connected if there are no proper, non-null $\alpha\mathfrak{F}$o - separated sets $\widetilde{C}$,$\widetilde{D}$ in $\widetilde{\Omega}$ such that $\widetilde{C} \tilde{\cup} \widetilde{D} = \widetilde{\Omega}$, if (Ω ,$\mathfrak{F}$, $\wp$) is not $\alpha\mathfrak{F}$ – connected, then it is said to be $\alpha\mathfrak{F}$ - disconnected space .

Examples 2.10.

1- For $\Omega = \{m, n, L\}$, $\wp = \{e\}$,
let $\tilde{R}$ = (e,{ m^1, n^0 , L^1 }) ,
$\tilde{T}$=(e,{ m^0, n^1 , L^0 }) ,
$\widetilde{\Phi}$ =(e,{ m^0, n^0 , L^0 })
and $\widetilde{\Omega}$=(e ,{ m^1, n^1 , L^1 }) .
With $\mathfrak{F}$ - topology $\mathfrak{F} = \{\widetilde{\Phi}, \widetilde{\Omega}, \tilde{R}, \tilde{T}\}$, the elements of $\mathfrak{F}$ are $\alpha\mathfrak{F}$o-set.
Since $\tilde{R} \tilde{\cap} \tilde{T} = \widetilde{\Phi}$ so $\tilde{R}$, $\tilde{T}$ are non-null $\alpha\mathfrak{F}$o-separated sets. Then, the space (Ω,$\mathfrak{F}$, $\wp$) is $\alpha\mathfrak{F}$ - disconnected space.
2- For $\Omega = \{m, n\}$, $\wp = \{c\}$ and $\widetilde{M}, \widetilde{N}, \widetilde{C}$ are $\mathfrak{F}$ - sets defined as follows:
$\widetilde{M}$ = (c, {m^0, $n^{0.06}$ }) ,
$\widetilde{N}$ = (c, { $m^{0.07}$, $n^{0.08}$})
$\widetilde{C}$ = (c, { $a^{0.09}$ b^1 }) , $\widetilde{\Phi}$ = (c, {a^0, b^0}) and
$\widetilde{\Omega}$ = (c, {a^1, b^1 }) .
Let $\mathfrak{F} = \{\widetilde{\Phi}, \widetilde{\Omega}, \widetilde{M}, \widetilde{N}, \widetilde{C}\}$ be a $\mathfrak{F}$ - topology on Ω.
Then, $\widetilde{\Phi}, \widetilde{\Omega}, \widetilde{M}, \widetilde{N}$ and $\widetilde{C}$ are $\alpha\mathfrak{F}$o-sets,
since there are no proper, non-null $\alpha\mathfrak{F}$o - separated sets .Then, the $\mathfrak{F}$ - topological space (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - connected.
In the next theorem we provide the equivalents of $\alpha\mathfrak{F}$ - irreducible space.

Theorem 2.11.

For (Ω,$\mathfrak{F}$, $\wp$) space, the next statements are equivalent :
1. The space (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - irreducible.
2. Any $\alpha\mathfrak{F}$o - set in (Ω ,, $\wp$) and non-null is $\alpha\mathfrak{F}$ - dense.
3. Any $\alpha\mathfrak{F}$o - set in (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - connected.

Proof.

(1) $\Leftrightarrow$ (2) Since the $\alpha\mathfrak{F}$ - set $\tilde{A}$ in a $\mathfrak{F}$ - topological space (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - dense if $\tilde{A}$ intersects with any non-null $\alpha\mathfrak{F}$o - sets so the condition (1) is equivalent to (2).
(1) $\Longrightarrow$ (3) Let $\tilde{A}$ be $\alpha\mathfrak{F}$o - set in $\mathfrak{F}$ and suppose $\tilde{A}$ is $\alpha\mathfrak{F}$ - disconnected so there exist two non - null $\alpha\mathfrak{F}$o-sets $\widetilde{M}$, $\widetilde{N}$ in $\mathfrak{F}$ such that $\tilde{A} = \widetilde{M} \tilde{\cup} \widetilde{N}$; $\widetilde{M} \tilde{\cap} \widetilde{N} = \widetilde{\Phi}$, which is contradiction with (1).
(3) $\Longrightarrow$ (1) If (Ω ,$\mathfrak{F}$, $\wp$) is $\alpha\mathfrak{F}$ - reducible space , then the intersection of any two non-null $\alpha\mathfrak{F}$o - sets is a

null set, i.e. If $\widetilde{M}$,$\widetilde{N}$ are two non-null α𝔉o – sets, then $\widetilde{M} \tilde{\cap} \widetilde{N} = \widetilde{\Phi}$, so $\widetilde{M} \tilde{\cup} \widetilde{N}$ is α𝔉-disconnected set which contradicts (3).

Definition 2.12.
Let (Ω ,, ℘) be a 𝔉 - topological space and let $\tilde{A} \tilde{\subseteq} \widetilde{\Omega}$. If the family 𝔉A={$\widetilde{M}^*$: $\widetilde{M}^* = \tilde{A} \tilde{\cap} \widetilde{M}$, $\widetilde{M} \in$ 𝔉} exists and it is 𝔉-topology on A. Then, 𝔉A called the relative 𝔉-topology on $\tilde{A}$ induced by the 𝔉 - topology 𝔉 over Ω. Note that (A, 𝔉A, ℘) is called a 𝔉 - subspace of (Ω,𝔉, ℘) .

Example 2.13.
For Ω = {m, n}, ℘={c} and $\widetilde{M}$, $\widetilde{N}$ are 𝔉-sets defined as follows:
$\widetilde{M}$ = (c, {m^1, $n^{0.6}$}), $\widetilde{N}$=(c,{m^0, $n^{0.4}$})
$\widetilde{\Phi}$ = (c, {a^0, b^0 }), $\widetilde{\Omega}$=(c,{a^1, b^1 })
Let = {$\widetilde{\Phi}$, $\widetilde{\Omega}$, $\widetilde{M}$, $\widetilde{N}$} be a 𝔉 - topology over Ω.
$\tilde{A} \tilde{\subseteq} \widetilde{\Omega}$, $\tilde{A}$ = (c, {$m^{0.1}$, $n^{0.6}$ })
$\widetilde{M}$A= (c, {m^1, $n^{0.6}$ }) $\tilde{\cap}$ (c ,{$m^{0.1}$, $n^{0.6}$ })
= (c, {$m^{0.1}$, $n^{0.6}$ }) = $\tilde{A}$
$\widetilde{N}$A= (c,{m^0, $n^{0.4}$}) $\tilde{\cap}$ (c ,{$m^{0.1}$, $n^{0.6}$})
= (c,{ m^0, $n^{0.4}$}) = $\widetilde{N}$.
Then, 𝔉A={$\widetilde{\Phi}$A , $\widetilde{\Omega}$A, $\widetilde{M}$A , $\widetilde{N}$A}
= {$\widetilde{\Phi}$ A, $\tilde{A}$, $\widetilde{N}$A}
is relative 𝔉 - topology on $\tilde{A}$.

Proposition 2.14.
Let (Ω, ℘) be a 𝔉 - topological space and $\tilde{A} \tilde{\subseteq}$ (Ω,𝔉, ℘). If $\tilde{G}$ α𝔉 - dense in(Ω,𝔉, ℘) ,then $\tilde{G}$ is α𝔉 - dense in $\tilde{A}$.

Proof.
Let $\tilde{G}$ be α𝔉 - dense in (Ω, ℘). To prove $\tilde{G}$ is α𝔉 - dense in $\tilde{A}$, i.e., to prove $\tilde{G} \tilde{\cap} \widetilde{M} \neq \widetilde{\Phi}$, ∀ $\widetilde{M} \tilde{\subseteq} \tilde{A}$.
Let $\widetilde{M} \tilde{\subseteq} \tilde{A}$, $\widetilde{M}$ is α𝔉o - set in 𝔉, $\widetilde{M} \neq \widetilde{\Phi}$,then, $\widetilde{M}^* = \tilde{A} \tilde{\cap} \widetilde{M} = \widetilde{M}$, since given that $\tilde{G}$ is α𝔉 - dense in (Ω,𝔉, ℘).
Thus, $\tilde{G} \tilde{\cap} \widetilde{M} \neq \widetilde{\Phi}$; ∀ $\widetilde{M} \neq \widetilde{\Phi}$, for each $\widetilde{M}$ in 𝔉, $\widetilde{M}^*$= $\widetilde{M}$.
Implying, $\tilde{G} \tilde{\cap} \widetilde{M} \neq \widetilde{\Phi}$; ∀ $\widetilde{M} \neq \widetilde{\Phi}$, for each $\widetilde{M}$ in 𝔉A. So $\tilde{G}$ is α𝔉 - dense in $\tilde{A}$.

Theorem 2.15.
If (Ω ,, ℘) is α𝔉 - irreducible space, then any non-null α𝔉o - set is also α𝔉 - irreducible.

Proof.
Let (Ω, ℘) be α𝔉 - irreducible space, $\tilde{Y}$ be non-null α𝔉o-set and $\tilde{G}$ be a non-null α𝔉o - set in (Y, 𝔉$_Y$, ℘). Since $\tilde{Y}$ is α𝔉o - in (Ω ,, ℘) so $\tilde{G}$ is α𝔉o - in (Ω,𝔉, ℘) by theorem 2.8. and theorem 2.10., then $\tilde{G}$ is α𝔉 - dense in $\tilde{Y}$ and $\tilde{Y}$ is α𝔉 - irreducible set.

Definition 2.16.
For α𝔉 - topological spaces (Ω ,, ℘) and (Ʊ,∂, ℘) , we say a map J: (Ω,𝔉, ℘) → (Ʊ ,∂ , ℘) is α𝔉 - continuous if the inverse image of any α𝔉o - set in ∂ is α𝔉o - set in.
The next theorem shows that α𝔉 - continuous image of α𝔉-irreducible set in is α𝔉 - irreducible set.

Theorem 2.17.
For J: (Ω,𝔉, ℘) →(Ʊ ,∂, ℘) be α𝔉 - map from a 𝔉 - topological space (Ω, 𝔉, ℘) into 𝔉 - topological space (Ʊ,∂, ℘). Then, the α𝔉 - continuous image of α𝔉 - irreducible set in (Ω ,, ℘) is α𝔉 - irreducible set in (Ʊ,∂, ℘).

Proof.
Let $\tilde{A}$ be α𝔉 - irreducible set in (Ω ,𝔉, ℘), $\tilde{G}$,$\tilde{H}$ are two non -null α𝔉o - sets in (Ʊ ,∂ , ℘) such that $\tilde{G} \tilde{\cap} J(\tilde{A}) \neq \widetilde{\Phi}$, $\tilde{H} \tilde{\cap} J(\tilde{A}) \neq \widetilde{\Phi}$, $J^{-1}(\tilde{G})$, $J^{-1}(\tilde{H})$ are two non-null α𝔉o - sets in (Ω ,𝔉, ℘) , since $\tilde{A}$ is α𝔉 - irreducible set .
Then, $J^{-1}(\tilde{G}) \tilde{\cap} \tilde{A} \neq \widetilde{\Phi}$, $J^{-1}(\tilde{H}) \tilde{\cap} \tilde{A} \neq \widetilde{\Phi}$,
$(J^{-1}(\tilde{G}) \tilde{\cap} \tilde{A}) \tilde{\cap} (J^{-1}(\tilde{H}) \tilde{\cap} \tilde{A}) \neq \widetilde{\Phi}$
$(J^{-1}(\tilde{G}) \tilde{\cap} J^{-1}(\tilde{H})) \tilde{\cap} \tilde{A} \neq \widetilde{\Phi}$
$J^{-1}(\tilde{G} \tilde{\cap} \tilde{H}) \tilde{\cap} \tilde{A} \neq \widetilde{\Phi}$ so
$(\tilde{G} \tilde{\cap} \tilde{H}) \tilde{\cap} J(\tilde{A}) \neq \widetilde{\Phi}$
$(\tilde{G} \tilde{\cap} J(\tilde{A})) \tilde{\cap} (\tilde{H} \tilde{\cap} J(\tilde{A})) \neq \widetilde{\Phi}$
where $(\tilde{G} \tilde{\cap} J(\tilde{A}))$, $(\tilde{H} \tilde{\cap} J(\tilde{A}))$ are non -null α𝔉o - sets.
So, $J(\tilde{A})$ is α𝔉 - irreducible set in (Ʊ,∂, ℘).

Definition 2.18.
For two α𝔉 - topological spaces (Ω, ℘) and (Ʊ,∂, ℘) a map J : (Ω ,𝔉, ℘) → (Ʊ ,∂, ℘) is α𝔉c - map if the image of any α𝔉c - set in (Ω, ℘) is α𝔉c - set in (Ʊ,∂, ℘).

Theorem 2.19.
Let J: (Ω,𝔉, ℘) → (Ʊ ,∂, ℘) be an α𝔉c - bijective map from a 𝔉 - topological space (Ω ,𝔉, ℘) into a 𝔉 - topological space (Ʊ ,∂ , ℘), if (Ω ,𝔉, ℘) is α𝔉 - irreducible space , then (Ʊ,∂, ℘) is α𝔉 - irreducible space .

Proof.
Let $\tilde{B}$ be α𝔉 - set in (Ʊ,∂, ℘) to prove (Ʊ,∂, ℘) is α𝔉 - irreducible since J is bijective. Then, there exist $\tilde{A}$ in (Ω, ℘) such that $J(\tilde{A}) = \tilde{B}$ since (Ω,𝔉, ℘) is α𝔉 - irreducible space , then α𝔉- cl($\tilde{A}$) = $\widetilde{\Omega}$,
since J is bijective α𝔉c - map, then J (α𝔉- cl ($\tilde{A}$)) = J ($\widetilde{\Omega}$) = $\widetilde{Ʊ}$, J (α𝔉- cl ($\tilde{A}$))= α𝔉- cl (J ($\tilde{A}$))
α𝔉- cl (J ($\tilde{A}$))= $\widetilde{Ʊ}$⟹ α𝔉- cl ($\tilde{B}$)= $\widetilde{Ʊ}$

Thus $\tilde{B}$ α𝔉- dense set in (Ʊ,∂, ℘)
By theorem 2.8. (Ʊ,∂, ℘) is α𝔉 - irreducible space.

Theorem 2.20.
Let *J*: (Ω,𝔉, ℘) → (Ʊ ,∂, ℘) be an α𝔉- continuous bijective map from a 𝔉 - topological space (Ω ,, ℘) into a 𝔉 - topological space (Ʊ,∂, ℘), if (Ω ,𝔉, ℘) is α𝔉 - reducible space ,then (Ʊ ,∂ , ℘) is α𝔉 - reducible space.

Proof.
Let $\tilde{B}$ be α𝔉 - set in (Ʊ,∂, ℘) to prove that (Ʊ,∂, ℘) is α𝔉 - reducible .
Since *J* is bijective, then there exist $\tilde{A}$ in (Ω ,, ℘) such that *J* ($\tilde{A}$) = $\tilde{B}$.
Since (Ω,𝔉, ℘) is α𝔉 - reducible space which provides (Ω ,𝔉, ℘) is an α𝔉 - disconnected space. Again, *J* is bijective α𝔉 - continuous map and so (Ʊ,∂, ℘) is an α𝔉 - disconnected space, and therefore (Ʊ ,∂ , ℘) is α𝔉 - reducible space .

Definition 2.21.
The map *f* from α𝔉 - space (Ω ,ℑ, ℘) to α𝔉 - space (Ʊ ,∂ , ℘) satisfy :
(1) *f* is α𝔉 - continuous.
(2) *f* is α𝔉 - bijective.
(3) f^{-1} is α𝔉 – continuous (or *f* is α𝔉 - open).
Is called α𝔉 - homeomorphism.
The next corollary show that the α𝔉 - irreducible space is α𝔉 - topological property.

Corollary 2.22.
Let *J* be α𝔉-homeomorphism from α𝔉 - topological space (Ω,𝔉, ℘) onto α𝔉 - topological space (Ʊ ,∂ , ℘) .If (Ω ,𝔉, ℘) is α𝔉 - irreducible space ,then (Ʊ ,∂ , ℘) is α𝔉 - irreducible space .

Proof.
Directly from Definition 2.19. and Theorem 2.20.
The next corollary show that the α𝔉-reducible space is α𝔉 - topological property.

Corollary 2.23.
Let *J* be α𝔉 - homeomorphism from α𝔉 - topological space (Ω, ℘) onto α𝔉 - topological space (Ʊ,∂, ℘) .Then, if (Ω, ℘) is α𝔉 - reducible space, then (Ʊ,∂, ℘) is α𝔉 - reducible space.
Proof. Directly from definition 2.21 and theorem 2.20.

Examples 2.24.
For Ω={m, n, L}, ℘={e} be the set of parameters. Let

$\tilde{O}$=(e,{ $m^{0.1}$, n^0 , $L^{0.3}$ }) ,
$\tilde{B}$=(e,{ m^0, n^1 , L^0 }) ,
$\tilde{\Phi}$ = (e,{ m^0, n^0 , L^0 }) ,
$\tilde{\Omega}$ = (e,{ m^1, n^1 , L^1 }) ,
with 𝔉 - topology 𝔉 = {$\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{O}$, $\tilde{B}$}.
The elements of are α𝔉o-sets.
Since $\tilde{O} \tilde{\sqcap} \tilde{B} = \tilde{\Phi}$ so $\tilde{O}$, $\tilde{B}$ are non-null α𝔉o-separated sets, then the space (Ω,𝔉, ℘) is α𝔉 - disconnected space and so (Ω , , ℘) is α𝔉- reducible space.
Let *J*: (Ω,𝔉, ℘)→(Ʊ,∂, ℘) be α𝔉-homeomorphism , *J* (x ij) = *J* (ei, $\{h_j^{kij}\}$) = (ei, $\{h_j^{1-kij}\}$)}, ∀ kij ∈ [0,1],
where *J* ($\tilde{O}$) =*J* ((e,{$m^{0.1}$, n^0 , $L^{0.3}$ }))
 = (e,{ $m^{0.9}$, n^1 , $L^{0.7}$ }),
J ($\tilde{B}$) = *J* ((e,{ m^0, n^1 , L^0 }))
= (e,{ m^1, n^0 , L^1 }) and
J ($\tilde{\Phi}$) = $\tilde{\Omega}$, *J* ($\tilde{\Omega}$) = $\tilde{\Phi}$.
Since *J* is α𝔉 -open map, then the images are α𝔉o - sets in (Ʊ,∂, ℘) .
Since *J* ($\tilde{O}$) $\tilde{\sqcap}$ *J* ($\tilde{B}$) = $\tilde{\Phi}$, so $\tilde{O}$, $\tilde{B}$ are non-null α𝔉o - separated sets, therefore (Ʊ,∂, ℘) is α𝔉- disconnected space and so is an α𝔉 - reducible space.

CONCLUSIONS

We define 𝔉 -irreducible sets, α𝔉- irreducible sets in 𝔉 - topological spaces, and provide equivalent definitions with examples. We also define α𝔉-continuous and prove that the α𝔉-continuous image of α𝔉-irreducible set is α𝔉- irreducible set. Define α𝔉-close map, α𝔉-homeomorphism and prove that α𝔉-irreducible (α𝔉-reducible) space is α𝔉-topological property.

Disclosure and conflict of interest: The authors declare that they have no conflicts of interest.

REFERENCES

[1] Abd Alrahman, Abd Allah S, Aslam M., Muhammad S.K.,"A study on fuzzy soft set and its operations", Annals of Fuzzy Mathematics and Informatics Volume 6, No. 2, (2013), pp.339-362.
[2] Aras G. C., Bayramov S., "Some results on fuzzy soft topological spaces", Hindawi Publishing Corporation Mathematical Problems in Engineering, Vol. (2013), pp.1-10. https://doi.org/10.1155/2013/835308
[3] Aygunoglu A., Vildan Cetkin , Halis Aygun , "An introduction to fuzzy soft topological spaces",

Hacettepe Journal of Mathematics and Statistics, Vol.43 ,No. 2, (2014), pp.193 - 204 .

[4] Cagman N., Enginoglu S., Citak F.," Fuzzy soft set theory and its applications", IJFS, Vol.8, No.3, (2011), pp.137-147.

[5] Hussain S.," On Weak and Strong Forms of Fuzzy Soft Open Sets", Fuzzy Information and Engineering, Vol. 8, No. 4, (2016), pp.451-463. https://doi.org/10.1016/j.fiae.2017.01.005

[6] Mahanta J., Das P.K.,"Results on fuzzy soft topological spaces", Arunachal Pradesh, INDIA, arXiv:1203.0634v1, (2012), pp.791-109.

[7] Maji P.K., Roy A.R., Biswas R., "Fuzzy soft sets", J.Fuzzy Math., Vol.9, No.3, (2001), pp.589- 602.

[8] Molodtsov D.,"Soft set theory - first results", Computers and Mathematics with Applications, Vol. 37, (1999), pp.19-31. https://doi.org/10.1016/S0898-1221(99)00056-5

[9] Naz M.,Shabir M. ,"Fuzzy soft sets and their algebraic structures",World Applied Sciences Journal, Vol.22,(2013), pp.45-61.

[10] Shabir M., Naz M.," On soft topological spaces", Comput. Math. Appl. Vol. 61, (2011), pp.1786-1799. https://doi.org/10.1016/j.camwa.2011.02.006

[11] Tanya B., Kandemir M. B., " Topological structure of fuzzy soft sets", Computer and Mathematics with Applications Vol. 61, (2011), pp.2952 - 2957. https://doi.org/10.1016/j.camwa.2011.03.056

[12] Zadeh L. A., "Fuzzy sets", Inform. and Control, Vol. 8, (1965), pp.338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

How to Cite

M. H. Mahmood, “ On α-Fuzzy Soft Irreducible Spaces”, *Al-Mustansiriyah Journal of Science*, vol. 34, no. 1, pp. 65–70, Mar. 2023.