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Using Evolving Algorithm with Distance Indicator for Solving Different Non-linear Optimization Problems

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ABSTRACT

In this paper, we have relied on the dominant control system as an important tool in building the group of leaders because it allows leaders to contain less dense areas, avoid local areas and produce a more compact and diverse Pareto front. Nine standard nonlinear functions yielded this result. MaBAT/R2 appears to be more efficient than MOEAD, NSGAII, MPSOD and SPEA2. MATLAB was used to generate all the results of the proposed method and other methods in the same field of work.

KEYWORDS: Many Objective Problems, Bat Algorithm, Inverted Generational Distance.

الخلاصة

في هذه الورقة، اعتمدنا على نظام السيطرة المهيمن كأداة مهمة في بناء مجموعة القادة لأنها تسمح للقادة باحتواء مناطق أقل كثافة، وتجنب المناطق المحلية وإنتاج جبهة باريتو أكثر إحكامًا وتنوعًا. أسفرت تسع وظائف قياسية غير خطية عن هذه النتيجة. يبدو أن (2MaBAT/R) أكثر كفاءة من (2MOEAD, NSGAII, MPSOD and SPEA)، تم استخدام برنامج (MATLAB) لتوليد جميع النتائج الخاصة بالطريقة المقترحة والطرق الاخرى في نفس حقل العمل.

INTRODUCTION

Although the truth of the algorithmic strategy for dealing with combinatorial optimization (CO) has been available for a long time, further application of evolutionary algorithms (EAs) to solve these problems provides a means to deal with large-scale multi-objective optimization.

Often there is not one perfect solution in multi-objective function optimization, but rather a set of optimal Pareto options. Thus, cluster sampling is critical when the co-optimization of an algorithm to generate a comprehensive and varied approximation of the Pareto front is performed [1].

Using the rule of change of weights, a Multi-Purpose Objective Bat Algorithm (MOBAT) introduced to determine the optimal Pareto array for Multipurpose Functions (MO).

The source [2] also presented BAT for multi-objective problem solving, as well as the Multi-objective Bat Algorithm (MOBAT). To verify this, we will develop solutions against a subset of the multi-objective test functions first. We will now use it to address engineering design improvement challenges such as the total and partial steel beam. MOBAT was used for this purpose, it can be described as a successfully biologically inspired algorithm to address problem floor planning in VSLI design in a publication approach [3].

The author in [4] proposed a multi-purpose optimization problem (MOOP) to achieve both of the aforementioned goals. MOOP is solved using a new simple optimization algorithm called BAT Algorithm, which is based on Weight Addition Method (WSM). Therefore, from the literature, we can say here that there is no study before that combined many objective bat algorithm with indicator convergence R2 (MaBAT/R2).

In addition, in another study, a comparison was made between the algorithms for feeding frontal Neural Networks (NN) and then the Gradient Descent (GD) algorithms (Backpropagation and Levenberg Marquardt), and three population-based statistical inference methods were used: the bat algorithm, the Genetic Algorithm (GA), and the

Particle Swarm Optimization (PSO) algorithm for the test. It has been shown that the BAT algorithm is superior to all other algorithms in training to feed-forward Neural Networks (NN) [5]. These results support the use of the best available techniques for further experiments, which greatly contributed to finding the optimal solution.

The advantage of using the bat algorithm is that it allows us to find solutions using population and local search techniques. This work introduced global diversity and rigorous local extraction, both of which are important for exploratory methods. As a result, the Bat algorithm was combined with PSO and local search, in addition to controlling the pulse rate and loudness [6].

MOBAT was used in Many Objective Optimization Problems (MaOPs), which gave us a good balance between diversity and convergence, representing the main issue in MaOPs, by adapting the reference groups approach. Additionally, in 2021, a paper was published entitled using the multipurpose bat algorithm to solve the multipurpose nonlinear programming problem [7]. Moreover, in 2020 [8], a met heuristic hybrid method is proposed to solve multi-objective optimization problems.

We conclude from the above that the main objective of this study is to improve the performance of multi-objective algorithms by developing a new algorithm inspired by bats for multi-objective optimization problems that used a technique to achieve organization and to achieve goals and diversity. Therefore, we proposed a method of increment based on the R2 index distance algorithm to reduce processing efforts in the field of different objective challenges in this paper.

BASIC CONCEPT OF OPTIMIZATION PROBLEM

In this field, we will first address the general form of the issue of multi-objective optimization and a sequence of definitions and important issues related to the core of the subject under study. So the general form of the problem is:

Minimize $F(x) = [f_1(x), f_2(x), \dots, f_k(x)]$

Subject to:

$$\begin{aligned} w_i(x) &\leq 0, i = 1, \dots, k; \\ n_j(x) &= 0, j = 1, \dots, p; \\ x_l &\geq 0, l = 1,2, \dots n \end{aligned} \quad (1)$$

Where $x = [x_1 : x_2, \dots, x_n]^T$ is the vector of decision variables $F_i : R^n \to R;\ i = 1, \dots, k$ are the objective functions and $wi, n_j : R^n \to R,\ i = 1, .., m, and\ j = 1, \dots, p,$ are constraints functions a problem. To describe the objective concept of optimization, we will give some of the following definitions:

Definition (1)[9]**:** (Multi-objective Optimization Problem (MOP)). A MOP is made up of a number of parameters (decision variables), a number of optimization techniques (m), and a number of constraints (m). The determination variables' functions and constraints are functions of the optimization algorithms and requirements. The purpose of optimization is to:

$$\begin{aligned} &Minimize\ y_i = f(x_i) \\ &subj.: to\ e(x) \\ &= (e_1(x);\ e_2(x);\ \dots;\ e_k(x)) \leq 0 \end{aligned} \quad (2)$$

Where X = (x_1, x_2, x_n) and Y = (y_1, y_2, y_m) and x the choice pattern is called the decision vector, the ambition velocity is called the objective vector, the determination space is called the decision sector, and the object space is called the subjective space. The constraints $e(x) \leq 0$ determine the set of feasible solutions.

Definition 2 [9]**:** (Allocative efficiency-optimality) a dimension of choice $x \in X_f$ when it comes to a set, it's said to be completely non $A \subseteq X_f\ iff\ \nexists a \in A : a > x$. If it is evident from the circumstances whichever set A is wanted, the following will simply be omitted. Furthermore, x is described as Allocative Efficiency-Optimal $iff\ x$ is non-dominated regarding X_f.

Definition (3) [9]**:** A set of controller parameters in a scalar $x_1 \epsilon X\ R^n$ is non-dominant when it comes to X, if no $x_2 \in X$ appears in the sense that $f(x_2) < f(x_1)$.

Definition (4) [9]**:** The Allocative efficiency optimal set P* is **characterized** as follows: P*= $\{x_1 \epsilon F : x_1$ is Allocative efficiency optimal$\}$.

USING BAT ALGORITHM TO SOLVE MOP

In this section, we will present the new or improved algorithm based on the characteristic of R2 or based on the influencer R2 that was used well and correctly to choose the optimal value when choosing a leader.

Bats are winged mammals and are known to be able to use echolocation. Approximately 996 unique species of bats have been identified worldwide,

representing about 20% of all well-evolved mammal species [7]. Another improved computation called BAT [10] is based on the swarm concept. Using BAT, one can re-enact some echolocation features of a smaller level bat. The benefits of this approach include ease of use, versatility and simplicity in implementation. Moreover, the approach effectively deals with a wide range of challenges, such as highly non-linear issues. In addition, BAT provides a perfect arrangement that promises quickly and works brilliantly with complex problems. Attempting to follow up are some of the drawbacks of this estimation: conjugation occurs rapidly at first, and the rate of conjugation declines. Furthermore, no scientific study has linked factors to varying rates.
The swarm is responsible for maintaining and re-establishing the perfect Pareto arrangements that have so far been discovered, and which cannot be controlled. The most reasonable arrangement obtained is used in calculating MaBAT/R2. This approach leads people to move in order to find a solution near the best arrangement. Contrasting with Pareto's best suggestions, however, it could not be more objective about space. The Pioneer Choice component is designed to address the research problem under study. The non-dominant and most logical arrangements are recorded in a single volume. The leader selects a piece from among the stacked parts of the space layout and suggests one of the non-dominant options. The random wheel is used to make the appropriate decision, along with the opportunities available to each individual: Below is full details of the proposed algorithm construction step by step based on the R2 optimum value selection component.
The performance measures in this paper are known HV [11] and inverted generations distance (IGD) [12]. Both HV and IGD are able to reflect the focus and diversity of the optimal result set of the algorithms.
Greater similarity to the original PF was indicated by a larger HV value or a smaller IGD number. For many issues, a reference point dominated by true PF is carefully selected to determine HV.

Pseudo-code of the MaBAT/R2

Set $k := 0$ and velocity =0 μ=0.1, $r0 = 0.5, A = 0.6$.
Randomly initialize point P_i $for\ n.population$;
Calculate the fitness values of initial population: $f\ (P\ )$;
Find the non-dominated solutions and initialized the archive with them
WHILE (the termination conditions are not met)
1) BAT Steps
$\mathrm{Q = Qmin + (Qmin - Qmax) * rand}$ (Eq. 2)
$\mathrm{P_{leader1} = Select\ Leader\ (archive)}$
$\mathrm{V_{(t+1)} = V_{(t)} + \left(P_{leader1} - P_{(t)}\right) * Q}$ (Eq. 2.2)
$\mathrm{P_{new} = P_{(t)} + V_{(t+1)}}$ (equation 3)
2)**If** rand > **r**
$\mathrm{P_{leader2} = Select\ Leader(archive)}$
$\mathrm{P_{new}{=}P_{(t)}{+}rand * \left(P_{leader2} - P_{(t)}\right)}$
End
$\boldsymbol{if}$ P_{new} $dominated\ on\ P_{(t)}$ & $(rand < A)$
$\mathrm{P_{(t)}{=}P_{new}}$
End
3)If rand $<\left(\frac{\boldsymbol{1-(k-1)}}{\boldsymbol{Max\ iteration-1}}\right)^{\boldsymbol{1/\mu}}$
$\mathrm{S = Mutation(P_{(t)})}$
if P_{new} $dominated\ on\ P_{(t)}$ & $(rand < A)$
$\mathrm{P_{(t)}{=}S}$
End
End
Find the non-dominated solutions
Update the archive concerning to the obtained non-dominated solutions
If the archive is full
Run the grid mechanism to omit one of the current archive members
Add the new solution to the archive
end if
If any of the new added solutions to the archive is located outside the hypercube
Update the **grids to** cover the new solution(s)
end if
Inc
rease r ***and reduce A***
Set $k := k + 1$;
End While

EXPERIMENTAL RESULTS

Now, we will present the most important results, which proved the superiority of the proposed algorithm MaBAT/R2 over other algorithms using the well-known function DTLZ [13] (The DTLZ suite of benchmark problems. It is unlike the majority of multiobjective test problems in that the problems are scalable to any number of objectives), from which we took only nine functions for comparison and with different sizes in terms of directions, number of target functions and number of repetitions. Especially regarding the problems of irregular Parito Front (PF) patterns.

Inverted generational distance (IGD)

Letting S denote the search result of a MOEA on a specific MOP. Should R be a set of PF representation points that are equally spaced? [1] Can be used to determine S's IGD value in relation to R.

$$IGD = \frac{1}{R}\left(\sum_{n=1}^{R} \min\left(\sqrt[p]{\sum_{n=1}^{R}(d_i^p)}\right)\right) \quad (3)$$

When |R| is the cardinality of R and d (r, S) is the minimum Euclidean distance between r and the points in S. It is important to note that perhaps the elements in R should really be spread evenly, and |R| should be large enough to ensure that the points in R fairly reflect the PF. This ensures that the IGD value of S may accurately assess the solution set's confluence and diversification. S has a lower IGD value, which indicates that it is of higher quality [14].

A set R of indicative points of the PF must be provided in this section to calculate the IGD value of a result set S of a MOEA executing on a MOP.

Hyper volume indicator

The hyperbolic quantity indicator $HV(A)$ calculates the volume of a territory H that is composed of a set of points A and a set of reference points N.

$$HV(A) = \Lambda(\cup\{x \mid a < x < v^*, a \in A\}) \quad (4)$$

As a result, higher indicative values correspond to better solutions. The S metric or the Lebesgue measure are other names for the hyper density indicator. It has a number of appealing attributes that have aided in its adoption and success. It is, in example, the only marker with metric features and the only one that is strictly Pareto monotonic [15]. Because of these characteristics, this indicator has been employed in a variety of applications, including measuring performance and evolutionary programming.

Analysis results

Tests and access points for the best algorithm will be presented using a good statistical test called the Wilcoxon Proficient Placement Test Scale.

WILCOXON MARKED: positional evaluation the Wilcoxon marked positioning test determines the difference between two illustrations [16] and provides an optional territory trial that is influenced by the sizes and indications of these distinctions. The following theories are addressed by this test

$$\begin{aligned} &H0: mean(A) = mean(B)\\ &H1: mean(A) \neq mean(B) \end{aligned} \quad (5)$$

The solutions to the first and second hypothesis are denoted by the letters A and B, correspondingly. Furthermore, this metric determines if one prediction outperforms the other. Let di denote the gap between the presentation scores of two calculations when it comes to dealing with the ith out of n difficulties. Enable R^+ to represent the number of sites for instances where the main computation beats the second. Finally, let R- deal with the number of places for the instances where the next estimate outperforms the previous. Several 0's are equitably spread across the entireties. If any of these totals have an odd number, one of them has been discarded.

$$\begin{aligned} R_+ &= \sum_{d_i > 0} \mathrm{rank}(d_i) + \frac{1}{2}\sum_{d_i = 0} \mathrm{rank}(d_i)\\ R_- &= \sum_{d_i < 0} \mathrm{rank}(d_i) + \frac{1}{2}\sum_{d_i = 0} \mathrm{rank}(d_i) \end{aligned} \quad (5)$$

We utilize MATLAB to find p self-worth in order to contrast the equations at a large degree of alpha = 0.05. Also, rand (di) represent the random number between the interval (0,1).

The invalid hypothesis is rejected when the p-esteem is not exactly the essential part. R+ deals with a high mean estimate that demonstrates predominance over processes of planning using a variety of test setups. This method outperforms all other algorithms in all tests. While $R^+ = \frac{n*(n+1)}{2}$ surpasses all other techniques in all of Adventure.

RESULTS AND DISCUSSION

This section is dedicated to describing and confirming which algorithms are the best in comparison. And the proposed multi-target bat computation (MaBAT/R2) with decay was implemented in Matlab, depending on the problem imposed. The proposed method has been tested with a variety of items, including community size (n), number of iterations, and rate of access reduction β.

The results were applied to fit the proposed methodology for balancing convergence and

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diversity. On the other hand, we compared MaBAT/R2 with two multi-target PSO accounts to get and know its severity and power to reach the optimal solution. MOPSO [11], MOEA/D [10] are two different methods. Each calculation is repeated several times in order to achieve the metrics (IGD) and (HV) for each test work. Tables (1) and (2).

CONVERGENCE GRAPHS

Again, for the data sets, an asymptotic graph was constructed to show the speed of convergence of the fitting values with the number of iterations. 100,000 iterations were run for all data. The graphics below illustrate this methodologically and analytically effectiveness of the proposed algorithm in obtaining the optimal value as quickly as possible. For this reason, these algorithms were used for comparison: MOEA/D, MOPSO, NSGAII, and SPEA2. All seven algorithms have been applied to 100,000 iterations of Hyper Volume (HV) and (IGD) running on them, and their graphs have already been obtained.

The graphs for the Figures 1, 2, and 3 illustrate that the proposed algorithm reaches a faster convergence point. The MODEMR algorithm also reaches a result quickly, but the value obtained by MaBAT/R2 is better than the value obtained by MODEMR.

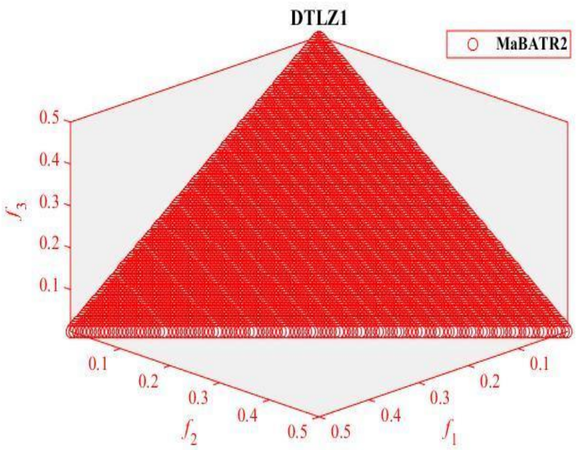

Figure 1. Number of functions VS Fitness Value Graph for DTLZ1, such that N=No. of population, M=No. of objective function, D=dimension.

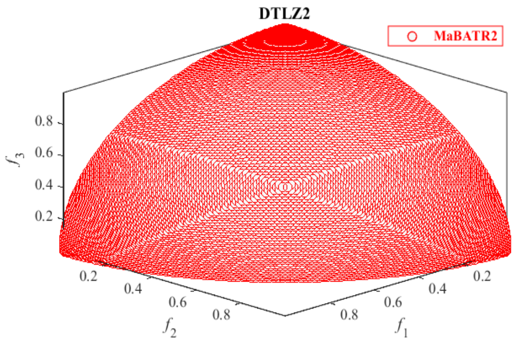

Figure 2. Number of functions VS Fitness Value Graph for DTLZ2, such that N=No. of population, M=No. of objective function, D=dimension.

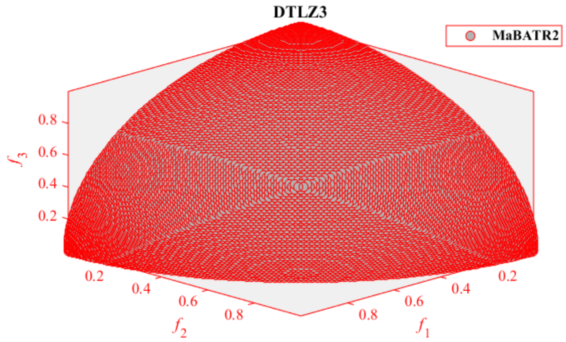

Figure 3. Number of functions VS Fitness Value Graph for DTLZ3, such that M=No. of objective function, D=Dimension.

CONCLUSIONS

Many objective bat algorithms based on deterioration subsystem (MaBAT/R2) are proposed in this paper, in which MOPs is deteriorate into several scalar improvement sub-issues, and each sub-issue is enhanced by just using information from its own few nearby sub-issues in a single run. It is clear from both performance metrics (IGD and HV) that MaBAT/R2 is quite serious and even outflanks the chosen MOBATs. In comparison to the chosen MOBATs, the numbers of Pareto battlefields suggest that MaBAT/ R2 can offer quite well Pareto lines.

Additional tests and examinations of the recommended are performed on a case-by-case basis. Later in the project, we will focus on parametric examinations for a broader range of test concerns, including discrete and blended aim of boosting. We aim to examine the various variations of the Pareto frontline it can generate in order to distinguish the methods for improving this computation to meet a range of difficulties. There are a few effective approaches for creating various Pareto fronts, and combining these procedures with others could considerably improve MaBAT/R2.

Table 1. The mean and standard deviation of the GD value of the proposed algorithms and the six recently comparative algorithms IBEA, BiGE, MOMBIII, MODEMR, KnEA, RVEA and MaBAT/R2 on DTLZ (1-9) for 5, 10, 15 and 20 objective problems, where the best value for each test case is highlighted with a bold background.

Problem	N	M	D	FEs	IBEA	BiGE	MOMBIII	MODEMR	KnEA	RVEA	MaBAT/R2
DTLZ1	150	5	9	150000	1.3095e-1 (1.15e-2) -	6.8661e-2 (1.30e-2) -	4.3205e-2 (1.22e-3) =	7.4069e-2 (2.08e-2) -	1.0114e-1 (2.94e-2) -	5.4514e-2 (2.37e-4) -	4.2967e-2 (2.10e-5)
	200	10	14	200000	1.4519e-1 (5.38e-3) -	4.8500e-2 (2.17e-2) -	1.9021e-1 (6.75e-3) -	2.2591e-1 (9.03e-3) -	8.5311e-2 (5.96e-4) -	1.6463e-1 (1.82e-2) -	1.2284e-3 (1.99e-3)
	250	15	19	250000	1.6128e-1 (9.26e-3) -	2.1209e-2 (5.06e-3) -	1.8270e-1 (1.26e-2) -	1.0389e-1 (1.75e-2) -	1.0010e-1 (6.67e-4) -	1.7451e-1 (1.31e-2) -	4.1108e-3 (2.72e-3)
	300	20	24	300000	2.0682e-1 (6.46e-3) -	8.3912e-3 (4.59e-3) -	2.3354e-1 (7.60e-3) -	2.6353e-1 (2.19e-3) -	1.4777e-1 (3.48e-4) -	2.2063e-1 (1.11e-2) -	2.0028e-4 (1.11e-4)
DTLZ2	150	5	14	150000	1.0626e-1 (1.91e-3) =	1.7552e-1 (9.76e-3) -	1.3635e-1 (6.31e-4) -	1.2658e-1 (5.14e-3) -	1.4002e-1 (4.59e-3) -	1.3516e-1 (1.90e-4) -	1.3636e-1 (2.75e-5)
	200	10	19	200000	3.7959e-1 (7.68e-3) -	3.9734e-1 (9.61e-3) -	5.1893e-1 (1.39e-1) -	3.2632e-1 (2.73e-2) =	3.7217e-1 (2.75e-3) -	3.5449e-1 (2.37e-3) -	3.2484e-1 (3.38e-4)
	250	15	24	250000	4.7548e-1 (7.09e-3) -	5.0092e-1 (1.11e-2) -	6.5156e-1 (9.19e-2) -	4.7518e-1 (1.79e-2) -	4.4223e-1 (5.49e-2) -	4.8500e-1 (7.09e-3) -	4.2505e-1 (3.65e-4)
	300	20	29	300000	5.2117e-1 (1.01e-2) -	5.2866e-1 (9.26e-3) -	7.3735e-1 (4.34e-2) -	5.0365e-1 (1.06e-2) -	4.6910e-1 (4.93e-2) =	5.4603e-1 (8.40e-3) -	5.5173e-1 (6.79e-3)
DTLZ3	150	5	14	150000	5.5877e-1 (5.33e-3) -	2.8701e-1 (6.83e-2) -	1.3570e-1 (1.54e-3) =	4.2453e-1 (2.22e-1) -	2.5969e-1 (1.32e-1) -	8.2283e-2 (5.94e-2) =	1.3552e-1 (7.96e-4)
	200	10	19	200000	6.5750e-1 (8.85e-3) -	8.0714e-2 (1.67e-2) -	7.7595e-1 (2.91e-2) -	9.2244e-1 (2.50e-2) -	3.2335e-1 (1.02e-3) -	5.3100e-1 (1.80e-1) -	7.0652e-3 (1.02e-2)
	250	15	24	250000	7.2121e-1 (1.33e-2) -	6.5181e-2 (1.11e-2) -	8.2924e-1 (1.00e-2) -	9.5999e-1 (6.20e-3) -	4.2224e-1 (1.18e-3) -	7.9239e-1 (1.06e-1) -	5.8071e-4 (1.38e-3)
	300	20	29	300000	7.4634e-1 (1.98e-2) -	3.2021e-3 (1.93e-3) -	8.6704e-1 (1.34e-2) -	9.7073e-1 (1.76e-3) -	5.5596e-1 (1.48e-2) -	7.1768e-1 (9.65e-2) -	4.6332e-4 (4.81e-4)
DTLZ4	150	5	14	150000	1.2991e-1 (8.11e-2) =	1.7122e-1 (9.52e-3) -	1.5227e-1 (5.82e-2) -	1.5454e-1 (8.93e-3) -	1.3813e-1 (5.82e-3) =	1.3757e-1 (2.62e-3) =	1.4400e-1 (4.19e-2)
	200	10	19	200000	3.7388e-1 (1.28e-2) -	3.9477e-1 (2.49e-2) -	4.2371e-1 (3.60e-2) -	4.0731e-1 (1.43e-2) -	3.9495e-1 (6.14e-3) -	4.1605e-1 (4.27e-3) -	3.4864e-1 (4.31e-2)
	250	15	24	250000	4.7677e-1 (8.10e-3) -	4.3268e-1 (1.87e-2) -	5.1074e-1 (8.70e-3) -	5.4929e-1 (4.11e-3) -	4.7925e-1 (3.57e-3) -	5.1835e-1 (2.36e-3) -	4.2317e-1 (5.82e-4)
	300	20	29	300000	5.2320e-1 (8.56e-3) -	4.5429e-1 (1.75e-2) =	5.5593e-1 (4.89e-3) -	5.8632e-1 (6.55e-3) -	5.2006e-1 (1.78e-3) -	5.5332e-1 (2.96e-5) -	5.5216e-1 (6.18e-3)
DTLZ5	150	5	14	150000	2.2986e-2 (5.92e-3) -	1.4416e-2 (4.57e-3) -	1.7996e-1 (2.48e-2) -	6.2321e-1 (4.52e-16) -	1.1366e-2 (8.09e-3) -	3.8491e-2 (8.46e-3) -	7.3600e-3 (7.33e-3)
	200	10	19	200000	3.8386e-2 (6.25e-3) -	3.3790e-1 (2.10e-1) -	5.9344e-1 (1.63e-2) -	6.2321e-1 (4.52e-16) -	1.0311e-2 (1.29e-2) -	5.9807e-1 (1.11e-1) -	3.4541e-3 (3.26e-3)
	250	15	24	250000	4.4151e-2 (1.18e-2) -	3.8178e-1 (2.10e-1) -	5.9322e-1 (2.53e-2) -	6.2321e-1 (4.56e-16) -	9.7536e-3 (5.34e-2) -	6.0099e-1 (9.76e-2) -	1.1890e-3 (1.69e-3)
	300	20	29	300000	4.4331e-2 (1.40e-2) =	7.4101e-2 (8.77e-2) -	6.0969e-1 (1.23e-2) -	6.2321e-1 (4.60e-16) -	5.0234e-2 (1.06e-1) -	6.1370e-1 (4.40e-2) -	8.8106e-4 (9.04e-4)
DTLZ6	150	5	14	150000	4.2271e-2 (1.09e-2) -	5.4617e-1 (9.41e-2) -	2.6715e-1 (2.11e-2) -	6.2321e-1 (4.42e-16) -	2.3882e-2 (1.36e-2) =	3.1850e-2 (4.88e-3) -	2.0438e-2 (9.01e-3)
	200	10	19	200000	1.8904e-1 (4.53e-2) -	5.5268e-1 (6.04e-2) -	5.8521e-1 (3.33e-2) -	6.2321e-1 (4.56e-16) -	1.0541e-1 (9.83e-2) =	6.1704e-1 (9.10e-3) -	5.9005e-2 (4.16e-2)
	250	15	24	250000	2.1788e-1 (6.09e-2) -	5.4958e-1 (8.50e-2) -	5.6230e-1 (4.56e-2) -	6.2321e-1 (4.68e-16) -	1.6826e-1 (8.93e-2) -	6.1396e-1 (1.67e-2) -	6.4730e-2 (3.61e-2)
	300	20	29	300000	1.9908e-1 (5.86e-2) -	5.1526e-1 (9.69e-2) -	5.5728e-1 (4.24e-2) -	6.2321e-1 (4.52e-16) -	1.5637e-1 (1.11e-1) -	6.1839e-1 (9.08e-3) -	2.6315e-2 (2.38e-2)
DTLZ7	150	5	24	150000	1.7844e-1 (5.87e-2) -	2.1550e-1 (7.75e-2) -	4.2208e-1 (7.35e-2) -	5.0066e-1 (4.39e-2) -	1.3651e-1 (6.48e-3)	3.6826e-1 (9.92e-2) -	1.2905e-1 (1.68e-2)
	200	10	29	200000	6.8061e-1 (1.25e-1) -	9.8838e-1 (4.70e-2) -	1.2437e-0 (7.78e-3) -	1.3465e-0 (4.85e-2) -	9.2296e-1 (3.41e-2)	1.2487e-0 (6.79e-2) -	4.2155e-1 (2.13e-2)
	250	15	34	250000	1.3773e-0 (1.27e-1) -	2.0059e-0 (3.03e-2) -	2.0510e-0 (1.72e-2) -	1.9581e-0 (9.00e-2) -	1.5116e-0 (1.33e-1)	2.0640e-0 (2.39e-2) -	3.4960e-1 (1.02e-1)
	300	20	39	300000	1.8215e-0 (1.26e-1) -	2.4544e-0 (2.39e-2) -	2.4184e-0 (9.83e-3) -	2.5497e-0 (2.14e-2) -	1.7321e-0 (1.33e-1)	2.4579e-0 (3.96e-2) -	8.4732e-1 (1.10e-1)
DTLZ8	150	5	50	150000					2.2120e-1 (4.07e-2) =		2.0338e-1 (6.14e-2)
	200	10	100	200000	Non	Non	Non	Non	8.4670e-1 (2.93e-2) -	Non	3.9309e-1 (3.32e-2)
	250	15	150	250000					1.1379e-0 (3.62e-2) -		3.8920e-1 (3.55e-2)
	300	20	200	300000					1.4188e-0 (4.15e-2) -		4.1019e-1 (3.59e-2)
DTLZ9	150	5	50	150000			5.2406e-2 (7.02e-2) =		4.1436e-1 (4.43e-1) -	9.6114e-1 (2.81e-1) -	1.8621e-4 (6.23e-4)
	200	10	100	200000		8.8503e-1 (1.67e-1) -	1.8127e-0 (8.88e-2) -	Non	4.8665e-1 (3.82e-1) -		0.0000e+0 (0.00e+0)
	250	15	150	250000	2.0673e-0 (8.76e-2) -	9.0147e-1 (1.89e-1) -	1.9210e-0 (2.00e-1) -		7.4174e-2 (1.67e-1) -	1.7372e-2 (3.77e-2) =	0.0000e+0 (0.00e+0)
	300	20	200	300000	4.9400e-1 (2.14e-1) -	1.0449e-0 (1.26e-1) -	8.9196e-1 (3.33e-1) -		4.7680e-3 (2.54e-2) =	3.8136e-1 (3.29e-1) -	0.0000e+0 (0.00e+0)
+/-/=					0/27/3	0/30/1	0/29/3	0/27/1	0/30/6	0/28/3	

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Table 2. The mean and standard deviation of the HV value of the proposed algorithms and the six recently comparative algorithms IBEA, BiGE, MOMBIII, MODEMR, KnEA, RVEA and MaBAT/R2 on DTLZ for 5, 10, 15 and 20 objective problems, where the best value for each test case is highlighted with a bold background.

Problem	N	M	D	FEs	IBEA	BiGE	MOMBIII	MODEMR	KnEA	RVEA	MaBAT/R2
	150	5	9	150000	2.1579e-1 (2.86e-2) -	1.1711e-1 (2.83e-2) -	6.3843e-2 (1.97e-3) -	1.0538e-1 (3.09e-2) -	1.7324e-1 (5.79e-2) -	8.3613e-2 (4.07e-4) -	6.3322e-2 (2.21e-5)
	200	10	14	200000	2.6133e-1 (1.96e-2) -	4.8316e-1 (1.39e-1) -	2.6147e-1 (2.13e-2) -	4.3158e-1 (3.90e-2) -	7.1208e-0 (1.08e-1) -	2.1385e-1 (2.88e-2) -	1.1263e-1 (5.54e-4)
	250	15	19	250000	2.9297e-1 (2.00e-2) -	5.4748e-1 (3.01e-1) -	2.4827e-1 (2.69e-2) -	1.3452e-1 (2.91e-2) =	3.8117e-0 (3.63e-0) -	2.2947e-1 (2.40e-2) -	1.2676e-1 (8.46e-4)
DTLZ1	300	20	24	300000	3.1456e-1 (1.45e-2) -	6.3545e-1 (2.46e-1) -	3.0430e-1 (1.56e-2) -	4.6764e-1 (2.57e-2) -	7.2352e-0 (8.67e-0) -	2.7957e-1 (1.74e-2) -	1.9829e-1 (2.10e-3)
	150	5	14	150000	2.0807e-1 (1.28e-3) -	2.3521e-1 (6.00e-3) -	1.9520e-1 (2.81e-4) -	2.0379e-1 (3.43e-3) -	1.9932e-1 (2.83e-3) -	1.9497e-1 (4.80e-5) -	1.9489e-1 (2.59e-6)
	200	10	19	200000	4.5452e-1 (3.67e-3) -	4.7877e-1 (4.46e-3) -	6.3301e-1 (1.49e-1) -	4.5164e-1 (1.89e-2) -	4.5356e-1 (3.95e-3) -	4.5443e-1 (1.61e-3) -	4.3767e-1 (3.29e-4)
	250	15	24	250000	5.4964e-1 (3.22e-3) -	5.8116e-1 (5.66e-3) -	7.7670e-1 (1.08e-1) -	5.6091e-1 (1.44e-2) -	7.1794e-1 (2.31e-1) -	5.7423e-1 (4.43e-3) -	5.2810e-1 (2.80e-4)
DTLZ2	300	20	29	300000	5.9505e-1 (3.16e-3) =	6.1461e-1 (3.54e-3) -	8.8575e-1 (6.83e-2) -	6.2033e-1 (7.19e-3) -	8.8491e-1 (2.80e-1) =	6.2479e-1 (3.66e-3) =	6.2422e-1 (4.62e-3)
	150	5	14	150000	5.9589e-1 (4.95e-3) -	4.8713e-1 (1.71e-1) -	1.9566e-1 (4.39e-4) -	5.9740e-1 (2.80e-1) -	4.3342e-1 (1.74e-1) -	1.7647e-0 (2.03e-0) -	1.9505e-1 (1.89e-4)
	200	10	19	200000	7.1970e-1 (1.13e-2) -	1.4893e-1 (6.89e-0) -	9.5178e-1 (4.21e-2) -	1.2122e-0 (4.73e-2) -	4.1396e-2 (8.01e-1) -	6.6584e-1 (2.22e-1) -	4.3776e-1 (7.05e-4)
	250	15	24	250000	8.1169e-1 (1.71e-2) -	1.8619e-1 (9.04e-0) -	1.0303e-0 (1.38e-2) -	1.2878e-0 (6.17e-3) -	7.4785e-2 (1.19e-2) -	1.0691e-0 (3.04e-1) -	5.2791e-1 (1.62e-3)
DTLZ3	300	20	29	300000	8.4742e-1 (2.86e-2) -	1.8720e-1 (5.57e-0) -	1.1204e-0 (2.24e-2) -	1.3354e-0 (1.42e-3) -	8.5538e-2 (9.84e-1) -	9.3564e-1 (3.45e-1) -	6.2679e-1 (1.66e-2)
	150	5	14	150000	2.2728e-1 (6.64e-2) -	2.3255e-1 (5.38e-3) -	2.1063e-1 (5.71e-2) -	2.2055e-1 (5.60e-3) -	1.9721e-1 (4.36e-3) =	1.9656e-1 (1.33e-3) =	2.0243e-1 (4.13e-2)
	200	10	19	200000	4.5376e-1 (9.29e-3) =	4.9267e-1 (2.76e-2) -	5.1485e-1 (4.57e-2) -	4.8855e-1 (9.14e-3) -	4.5809e-1 (5.21e-3) -	4.9267e-1 (2.59e-3) -	4.6585e-1 (4.56e-2)
	250	15	24	250000	5.5349e-1 (3.60e-3) -	7.7809e-1 (1.49e-2) -	5.9202e-1 (1.17e-2) -	6.2216e-1 (3.78e-3) -	5.3926e-1 (3.87e-3) -	5.9580e-1 (1.95e-3) -	5.2772e-1 (5.19e-4)
DTLZ4	300	20	29	300000	5.9584e-1 (2.07e-3) -	8.1218e-1 (8.25e-3) -	6.2801e-1 (7.63e-3) -	6.5323e-1 (5.91e-3) -	5.7963e-1 (1.53e-3) =	6.2319e-1 (7.56e-5) -	6.2382e-1 (2.23e-3)
	150	5	14	150000	1.8849e-1 (1.64e-2) -	1.0032e-1 (1.63e-2) -	2.7061e-1 (5.00e-3) -	8.9884e-1 (3.59e-2) -	1.6811e-1 (4.75e-2) -	7.2858e-2 (1.14e-2) -	5.6634e-2 (1.24e-2)
	200	10	19	200000	4.7978e-1 (2.41e-1) -	3.3662e-1 (8.61e-2) -	7.1555e-1 (1.45e-2) -	1.6400e-0 (1.17e-1) -	3.4394e-1 (7.25e-2) -	7.1483e-1 (1.25e-1) -	1.2481e-1 (3.61e-2)
	250	15	24	250000	6.4949e-1 (1.75e-1) -	4.1669e-1 (3.25e-2) -	7.1539e-1 (2.25e-2) =	1.7302e-0 (1.28e-1) -	5.4652e-1 (2.14e-1) -	7.1969e-1 (1.01e-1) =	2.2716e-1 (5.99e-2)
DTLZ5	300	20	29	300000	2.9953e-1 (1.62e-1) =	3.9637e-1 (2.88e-2) -	7.3003e-1 (1.10e-2) -	1.8148e-0 (3.06e-1) -	7.8902e-1 (3.47e-1) -	7.3343e-1 (4.02e-2) -	2.3074e-1 (6.57e-2)
	150	5	14	150000	8.2353e-2 (1.61e-2) -	6.7385e-1 (8.36e-2) -	3.1331e-1 (5.15e-3) -	8.3743e-1 (9.93e-2) -	2.8633e-1 (7.83e-2) -	6.1403e-2 (7.39e-3) =	1.1179e-1 (1.84e-2)
	200	10	19	200000	3.4590e-1 (1.72e-2) -	6.7960e-1 (5.35e-2) -	7.0827e-1 (2.95e-2) -	1.2539e-0 (1.99e-1) -	1.1678e-0 (3.28e-1) -	7.3656e-1 (8.15e-3) -	2.5018e-1 (1.06e-1)
	250	15	24	250000	3.5146e-1 (2.96e-2) -	6.7667e-1 (7.62e-2) -	6.8811e-1 (4.03e-2) -	1.5006e-0 (7.53e-1) -	1.5900e-0 (3.45e-1) -	7.3379e-1 (1.50e-2) -	2.2020e-1 (6.40e-2)
DTLZ6	300	20	29	300000	3.5236e-1 (4.21e-2) -	6.4627e-1 (8.64e-2) -	6.8380e-1 (3.73e-2) -	1.2134e-0 (1.54e-1) -	1.5003e-0 (3.95e-1) -	7.3777e-1 (8.15e-3) -	1.9083e-1 (7.53e-3)
	150	5	24	150000	2.9866e-1 (3.51e-2) -	4.2235e-1 (1.02e-1) -	5.4978e-1 (1.51e-1) =	8.9328e-1 (1.06e-1) -	5.6062e-1 (1.46e-2) -	9.0166e-1 (1.50e-1) -	2.7793e-1 (1.11e-2)
	200	10	29	200000	1.1532e-0 (3.27e-1) -	3.0629e-0 (3.85e-1) -	5.5615e-0 (5.08e-2) -	5.3866e-0 (5.21e-1) -	2.4594e-0 (2.80e-1) -	5.1838e-0 (6.68e-1) -	9.4318e-1 (2.15e-2)
	250	15	34	250000	3.0003e-0 (9.14e-1) -	1.0083e-1 (5.16e-1) -	1.1001e-1 (2.25e-1) -	9.7764e-0 (6.90e-1) -	3.7625e-0 (7.28e-1) -	1.0680e-1 (4.26e-1) -	2.2694e+0 (1.97e-1)
DTLZ7	300	20	39	300000	5.6229e-0 (1.68e-0) -	1.4931e-1 (3.62e-1) -	1.4870e-1 (1.11e-1) -	1.5629e-1 (1.80e-1) -	3.3721e-0 (4.32e-1) -	1.4900e-1 (5.73e-1) -	3.2242e+0 (6.21e-1)
	150	5	50	150000					2.6793e-1 (4.14e-2) =		2.4844e-1 (6.19e-2)
	200	10	100	200000	Non	Non	Non	Non	8.9074e-1 (3.94e-2) -	Non	5.4102e-1 (3.99e-2)
	250	15	150	250000					1.1777e-0 (4.46e-2) -		6.2401e-1 (3.33e-2)
DTLZ8	300	20	200	300000					1.4650e-0 (4.84e-2) -		7.0367e-1 (2.95e-2)
	150	5	50	150000			6.2969e-1 (5.01e-3) =		1.1823e-0 (5.21e-1) -	1.4127e-0 (2.83e-1) =	4.9685e-1 (9.33e-2)
	200	10	100	200000		3.5635e-0 (2.10e-0) -	1.9979e-0 (1.02e-1) -	Non	9.3883e-0 (5.16e-1) -		1.6384e+0 (1.82e-1)
	250	15	150	250000	2.2339e-0 (1.33e-1) -	8.9054e-0 (1.12e-0) -	2.3968e-0 (1.94e-1) -		1.5394e-1 (4.53e-1) -	6.0172e-0 (1.05e-0) -	2.0085e+0 (2.96e-1)
DTLZ9	300	20	200	300000	5.0188e-0 (1.05e-0) -	1.4692e-1 (1.26e-0) -	4.2373e-0 (8.05e-1) -		1.9929e-1 (4.77e-1) -	5.2702e-0 (8.41e-1) -	2.1655e+0 (2.38e-1)
+/-/=					0/27/3	0/31/0	0/29/3	0/27/1	0/32/4	0/26/5	

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