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Theoretical Approaches Parallel Identical Machines with Multi-Objective Functions

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ABSTRACT

In this study, we propose multi-objective functions which consist of the sum of completion time, tardiness time and earliness time where C_i denoted the completion time of job (i), $T_i=max\{C_i-d_i,0\}$, denotes the tardiness of job (i), $E_i=max\{d_i.C_i,0\}$ be denoted the earliness of job (i).This problem is defined by $P_2\ //\sum_{j=1}^{2}\sum_{i}^{n}(C_{ji}\ +T_{ji}+E_{ji})$. In this paper, we will present some theoretical analysis discussion, and prove when we have a problem with scheduling n jobs on two identical parallel machines (IPMSP).

KEYWORDS: Multi-objective functions, identical parallel machines, completion time, tardiness, earliness, dominance rules

الخلاصة

في هذه الدراسة، نقترح وظائف متعددة الأهداف تتكون من مجموع وقت الإكمال، ووقت التأخير، ووقت التبكير حيث يشير C_i إلى وقت إكمال الوظيفة$\{(C_i-d_i،0)\}$ $T_i = max$ ، يشير إلى تأخر الوظيفة$\{0)،d_i-C_i\}$ $E_i = max$ ، أن تدل على أبكر الوظيفة (i). يتم تعريف هذه المشكلة بواسطة $//\sum_{j=1}^{2}\sum_{i=1}^{n}(C_{ji}\ +T_{ji}+E_{ji})P_2$. في هذه الورقة، سوف نقدم بعض مناقشات التحليل النظري، ونثبت، عندما نواجه مشكلة في جدولة عدد من الوظائف على جهازين متوازيين متطابقين IPMSP).)

INTRODUCTION

Scheduling n of jobs on identical parallel machines m be stated as follows. Every one of n occupations i (numbered (i= 1,…,n) is to be handle on one of two indistinguishable equal machines numbered (m=1,2) on the machine can deal with more than one occupation at a time, each occupation i, (i=1,…,n) opens up for handling at time zero requires a positive whole number handling time P_i on the machine to which it is allocated, acquisition of occupations is not permitted. This problem is defined by $P_2//\sum_{j=1}^{2}\sum_{i}^{n}(C_{ji}\ +T_{ji}+E_{ji})$ to find an assignment of the jobs to the machines and to decide the sequencing of jobs on the machines. In this paper, we offer a theoretical analysis, discussion, and proof, we give a related review of the literature on the identical parallel machine problem, we describe our problem by $P_2//\sum_{j=1}^{2}\sum_{i}^{n}(C_{ji}\ +T_{ji}+E_{ji})$, and we will introduce some theorems and their proof for our problem. The end we will take special cases and prove our mathematical.

LITERATURE OF PARALLEL MACHINE PROBLEM

The parallel machine climate has been read up for a long time because of its significance to the scholarly community and industry.

The upside of utilizing heuristics is that as far as possible the pursuit by decreasing the number of choices. Garey and Johnson in [1] showed that the issue of booking indistinguishable parallel machines to limit makespan is NP-hard in any event, for two machines. Alidaee and Rosa in [2] featured an instance of indistinguishable parallel machines in limiting the complete weighted lateness utilizing the changed due date (MDD) heuristic. Azizoglu and Kirca in [3] considered the NP-difficult issue of planning position on indistinguishable parallel machines to limit absolute lateness, they introduced properties that portray the design of an ideal timetable, and they proposed a branch and bound calculation that consolidates the properties alongside an effective lower bouncing plan, they established that ideal arrangements can be acquired in sensible times for issues with up to 15 positions. A mathematical model is introduced to exhibit the methodology. Yalaoui and Chu in [4] they mindful of the issue of planning n free positions on m indistinguishable equal machines for the goal of limiting all out lateness of the positions, and created predominance properties and lower limits, and fostered a branch and bound calculation utilizing these properties and lower limits as well as upper limits acquired from a heuristic calculation. Computational trials are performed on haphazardly produced test issues and results introduced that the calculation tackles issues with moderate sizes in a sensible measure of calculation time. Mokotoff in [5] dissected an indistinguishable parallel machine issue including makespan minimization with direct programming relaxations and productive guidelines and Dominance rules are significant in creating bits of knowledge and new methodologies. Shim and Kim in [6] zeroed in on the issue of booking (n) autonomous positions on (m) identical parallel machines for the goal of limiting all out lateness of the positions and creating strength properties and lower limits and fostering a branch and bound methods utilizing these properties and lower limits as well as upper limits acquired from a heuristic calculation. Nessah *et al*. in [7] tended to an indistinguishable parallel machine issue with the discharge of an absolute weighted culmination time objective capacity; the creators likewise fostered a few strength properties. Tanaka and Araki in [8] fostered a branch and bound calculation to take care of indistinguishable parallel machine issues with complete lateness objective capacities. Chiang *et al*. in [9] tended to a planning issue roused by booking of dissemination activities in the wafer creation office, in the objective issue, occupations show up at the cluster machines at various time moments, and just positions having a place with a similar family can be handled together. Selvi in [10] worked on multi-objective improvement issues on indistinguishable parallel machine planning utilizing hereditary calculations, their exploration attempted to tackle booking issues including same equal machines, the spot the objective was once to streamline the multi-objective planning issues the utilization of hereditary calculations. Wang and Leung in [11] tended to resemble cluster handling with indistinguishable handling timework on machines with various abilities to limit the makespan. German *et al*. in [12] read up the makespan minimization for indistinguishable parallel machines, the issue included a task number of occupations (n) to a bunch of indistinguishable parallel machines (m), when the goal is to limit the makespan (greatest consummation season of the keep going position on the last machine of the framework), to such an extent that the issue is meant by ($P_m//C_{max}$), and writers fostered a calculation to observe the ideal planning answer for the parallel machine Schedule issue the ILP and LPT calculation used to create the underlying arrangement then the created calculation used to work on the answer for limiting the makespan [12], indistinguishable equal cluster handling machine with tow objective of limiting makespan and most extreme lateness.

Chachan and Hameed in [13] concentrated on the issue of planning various items (n-occupations) on one (single) machine with the multi-standards objective capacity, these capacities are (finish time, lateness, earliness, and late work) which figured out as $1//\sum_{j=1}^{n}(C_j + T_j + E_j + V_j)$, the branch and bound (BAB) techniques are utilized as the primary strategy for tackling the issue.

Kramer *et al*. in [14] read up the makespan Minimization for indistinguishable parallel machines, the issue included a task number of occupations (n) to a bunch of indistinguishable

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equal machines (m), when the goal is to limit the makespan.

FORMULATION OF THE PROBLEM

In this section, we considered that we have two identical parallel machines such that each job has a specific time. After that, we will study some theorems, analyze them and prove them according to mathematical operations so that we can know the scheduling of these jobs on the two machines in a good and faster way to get the optimal solution. This problem is defined by $P_2//\sum_{j=1}^{2}\sum_{i=1}^{n}(C_{ji}+T_{ji}+E_{ji})$. The basic problem indicated by (p) and can express as follows:

$min\ Z = min\sum_{j=1}^{2}\sum_{i=1}^{n}(C_{ji}+T_{ji}+E_{ji})$

subject to:

$C_{ji} = P_{ji}$, $i = 1,2,\dots,n, j = 1,2$
$C_{ji} = C_{ji-1} + P_{ji}$, $i = 1,2,\dots,n, j = 1,2$
$T_{ji} \geq C_{ji} - d_i$, $i = 1,2,\dots,n, j = 1,2$
$T_{ji} \geq 0$, $i = 1,2,\dots,n, j = 1,2$
$E_{ji} \geq d_i - C_{ji}$, $i = 1,2,\dots,n, j = 1,2$
$E_{ji} \geq 0$, $i = 1,2,\dots,n, j = 1,2$ $\}\dots(p)$

Special Cases and Dominance Rules (DR's) of (p)-problem

Now, we introduce some special cases for the problem $P_2//\sum_{j=1}^{2}\sum_{i=1}^{n}(C_{ij}+T_{ij}+E_{ij})$ to these are proved in theorem (1).

Let S_{1K}=($\beta_1\ ik\ \beta_2$) and S_{2K}=($\beta_1\ ki\ \beta_2$) be two sequences where β_1 and β_2 are disjoint subsequences of the n-2 operations that remain, the jobs (i) and (k) are adjacent jobs on the same machine with

$P_{ij} \leq P_{kj}$, and, let (t) be completion time of β_1.

Let F_{ikj}(t) be functions value where $F_{ikj}(t)) = \sum_{j=1}^{2}\sum_{i=1}^{n}(C_{ij}+T_{ij}+E_{ij})$ for the subsequences of the two jobs (i) and(k) when (i) precedes (k) and F'_{kij}(t) is function value for the subsequences of two jobs (i) and(k) when (k) precedes (i). Now, we investigator (i) and (k) when (k) precedes (i). Now, we investigate the modifications in

$\Delta_{ikj}(t) = F_{ikj}(t) - F'_{kij}(t)$, with the following cases $\Delta_{ikj}(t) \leq 0$ it means that

1) If $\Delta_{ikj}(t) < 0$ then, at time t, job i should come before job k.
2) If $\Delta_{ikj}(t) > 0$ then, at time t, job k should come before job i.
3) If $\Delta_{ikj}(t) = 0$, then, there is no difference in scheduling (i) or (k) first.

Theorem (1): For the $P_2//\sum_{j=1}^{2}\sum_{i=1}^{n}(C_{ij}+T_{ij}+E_{ij})$ problem if $P_{ij} \leq P_{kj}$ and $d_i \leq d_k$ where the jobs (i) and (k) are two adjacent jobs on the same machine, then the job (i) precedes the job (k) in at least on optimal sequences.

S_{1k}

β_1	i	k	β_2

S_{2k}

β_1	k	i	β_2

As can be seen from Figure 1, there are (7) case examples:

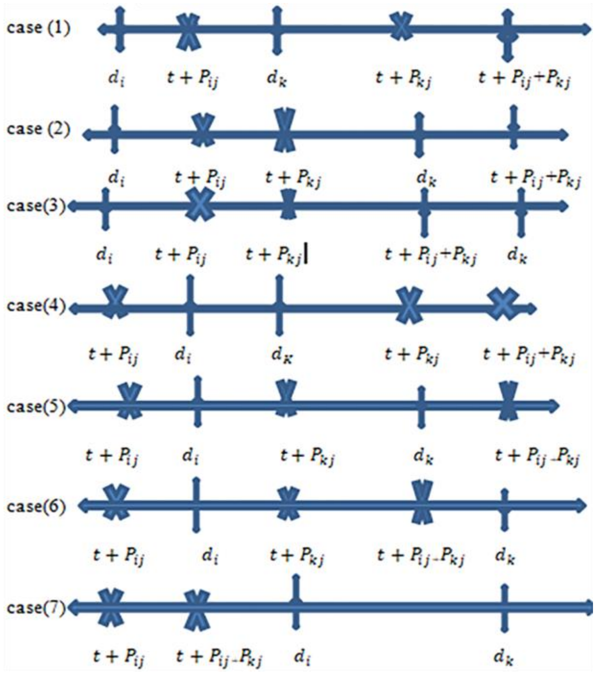

Figure 1. Cases for the Theorem (1).

Case 1: If $d_i \leq t + P_{ij}$, $d_k \leq t + P_{kj}$, jobs (i) and (k) are always tardy then for any two adjacent jobs (i, k) on machine j, j=1,2 then $E_{ij} = E_{kj} = 0$ (see Figure 1 case (1))

Proof:

$\Delta_{ikj}(t) = F_{ikj}$(t) $-F'_{kij}$(t) =
[$(t+P_{ij})$+$(t+P_{ij}-d_i)$+0+$(t+P_{ij}+P_{kj})$+($t+P_{ij}+P_{kj}-d_i$) +($t+P_{ij}+P_{kj}-d_k$) +0)
-[$(t+P_{ij})$+$(t+P_{ij}-d_k)$+0+($t+P_{ij}+P_{kj}$)+$(t+P_{ij}+P_{kj}-d_i)$+0)]
=[$4t+4P_{ij}+2P_{kj}-d_i-d_k$)–[$4t+4P_{kj}+2P_{ij}-d_i-d_k$]=

$2P_{ij}$-$2P_{kj}$ ≤ 0

Since $P_{ij} < P_{kj}$, then $i \to k$

<u>Case 2</u>: If $d_i \leq t+P_{ij}$, $t+P_{ij} \leq d_k \leq t+P_{kj}$, , (i) is always tardy and (k) is tardy if not scheduled first, then E_{ij}=0, T_{Kj}=0 (see Figure 1 case (2))

Proof:

$\Delta_{ikj}(t) = F_{ikj}$(t) $-F'_{kij}$(t) =
[([($t+P_{ij}$)+$(t+P_{ij}-d_i)$+0 +($t+P_{ij}+P_{kj}$)+($t+P_{ij}+P_{kj}$ - d_k) +o)]-[$(t+P_{ij})$+0- d_k-t-P_{kj} + ($t+P_{ij}+P_{kj}$)+$(t+P_{ij}+P_{kj}-d_i)$+0)]
=[$4t+4P_{ij}+2P_{kj}-d_i-d_k$)]– [$2t+2P_{kj}+2P_{ij}-d_i+d_k$]= [$2t+2P_{ij}-2d_k$] ≤ 0

Since $t+P_{ij} < d_k$, then $i \to k$

<u>Case 3</u>: If $d_i \leq t+P_{ij}$, $t+P_{ij}+P_{kj} \leq d_k$,then (i) is always tardy and the (k) is always early. (see Figure 1 case (3)).

Proof: since (i) is always tardy and (k) is always early then $E_{ij}=0=T_{Kj}$

$\Delta_{ikj}(t) =$ [([($t+P_{ij}$)+$(t+P_{ij}-d_i)$+0 +($t+P_{ij}+P_{kj}$)+($d_k-t-P_{ij}-P_{kj}$]-[$(t+P_{ij})$+0- d_k-t-P_{kj} + ($t+P_{ij}+P_{kj}$)+$(t+P_{ij}+P_{kj}-d_i)$+0)]
=[$2t+2P_{ij}-d_i+d_k$)]–[$2t+2P_{ij}+2P_{kj}-d_i-d_k$)]
$=-2P_{kj}\leq 0$

Since P_{kj}>0, then $i \to k$

<u>Case 4</u>: If $t+P_{ij} \leq d_i \leq d_k \leq t+P_{kj}$, (i.e. the job (i) is always tardy and (k) is tardy if not scheduled firstly.
(See Figure 1 case (4))

Proof:

$\Delta_{ikj}(t) = F_{ikj}$(t) $-F'_{kij}$(t)=
[$(t+P_{ij})$+ 0+d_i - $t-P_{ij}$ + ($t+P_{ij}+P_{kj}$)+($t+P_{ij}+P_{kj}-d_k$) +0]
-[[$(t+P_{kj})$+0+d_i+$t+P_{kj}-d_k$)+0+$(t+P_{ij}+P_{kj})$+($t+P_{ij}+P_{kj}-d_i$) +0]
=[$2t+2P_{ij}+2P_{kj}-d_k+d_i$] $-$ [$4t+2P_{ij}+4P_{kj}-d_i-d_k$]=
-2($t+P_{kj}-d_i$)≤ 0, Since $t+P_{kj} \geq d_i$,then $i \to k$

<u>Case 5</u>: If $t+P_{ij} \leq d_i$, $t+P_{ij} \leq d_k \leq t+P_{ij}+P_{kj}$, (i.e. each of the two jobs (i) and (k) are tardy if they not scheduled first). (see Figure 1 case (5))

Proof:

$\Delta_{ikj}(t) = F_{ikj}$(t) $-F'_{kij}$(t) =
[($t+P_{ij}$)+ 0+(d_i-$t-P_{ij}$) + ($t+P_{ij}+P_{kj}$)+($t+P_{ij}+P_{kj}-d_k$) +0]
-[$(t+P_{kj})$+0+(d_k-t-P_{kj}) + ($t+P_{ij}+P_{kj}$)+$(t+P_{ij}+P_{kj}-d_i)$+0)]
=[$2t+2P_{ij}+2P_{kj}+d_i-d_k$)]–[$2t+2P_{ij}+2P_{kj}-d_i+d_k$)]
= $2d_i-2d_k \leq 0$

Since $d_k \geq d_i$, then $i \to k$

<u>Case 6</u>: If $t+P_{ij} \leq d_i \leq t+P_{ij}+P_{kj}, \leq d_k$, (i.e. the job (i) is tardy if not scheduled first, the job (k) is always early) (see Figure 1) case (6))

Proof:

$\Delta_{ikj}(t) = F_{ikj}$(t) $-F'_{kij}$(t) =
[($t+P_{ij}$)+ 0+(d_i - $t-P_{ij}$) + $(t+P_{ij}+P_{kj})$ + 0+($d_k-t-P_{ij}-P_{kj}$]
-[$(t+P_{kj})$+0+(d_k-t-P_{kj}) + ($t+P_{ij}+P_{kj}$)+$(t+P_{ij}+P_{kj}-d_i)$+0)]=[d_i+d_k)]–[$2t+2P_{ij}+2P_{kj}-d_i+d_k$]

$= 2t + 2P_{ij}+2P_{kj} + 2d_i \leq 0$, since $d_i \leq t + P_{ij}+P_{kj}$ then $i \rightarrow k$

Case (7): If $t + P_{ij}+P_{kj} \leq d_i \leq d_k$, (i.e. both (i) and (k) are early) (see Figure 1 case (7))

Proof:

$\Delta_{ikj}(t) = F_{ikj}(t) - F'_{kij}(t)=$
$[(t + P_{ij})+ 0+(d_i - t - P_{ij}) + (t + P_{ij}+P_{kj}) + 0+(d_k - t-P_{ij} - P_{kj}]$-
$[(t + P_{kj})+0+(d_k - t - P_{kj}) + (t + P_{ij} + P_{kj})+(d_i- t - P_{ij}+P_{kj})]$
$=[d_i + d_k]$- $[d_i + d_k)]=0$

Theorem (2):

For the $P_2/d_{ik} =d /\sum_{j=1}^{2}\sum_{i=1}^{n}(C_{ij} + T_{ij} + E_{ij})$ problem. If $P_{ij} \leq P_{kj}$, where the jobs (i) and (k) are adjacent jobs on the same machine, then job (i) should precede job (k) for at least one sequence with optimal value (SPT rule).

Proof:

Let $S_{1K}=(\beta_1\ ik\ \beta_2)$ and $S_{2K}=(\beta_1\ ki\ \beta_2)$ be two sequences where β_1 and β_2 are disjoint and let (t) be the completion time of β_1
, we will examine the value of changes $\Delta_{ikj}(t) = F_{ikj}(t) - F'_{kij}(t)$

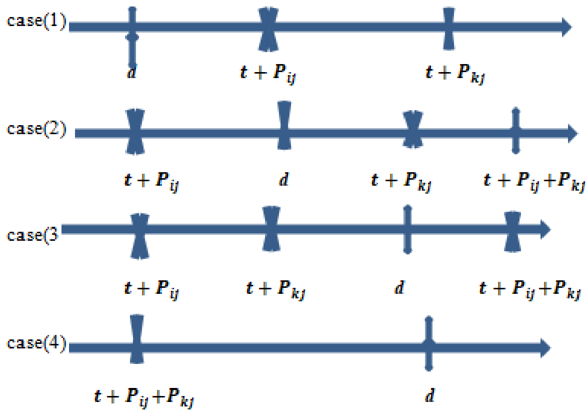

Figure 2. Cases for Theorem (2).

Case (1): if $d \leq t + P_{ij} \leq t + P_{kj}$ (i.e. both of the jobs (i) and (k) are always tardy)
(see Figure 2 case (1))

Proof: since the jobs (i) and (k) are both tardy then $E_{ij}=E_{kj}=0$

$\Delta_{ikj}(t) = F_{ikj}(t) - F'_{kij}(t) =$
$[t + P_{ij})+(t + P_{ij}\text{-}d)+0+(t + P_{ij}+P_{kj})+(t + P_{ij}+P_{kj}\text{-}d)+0]$
$-[t + P_{kj})+(t + P_{kj}\text{-}d)+0+(t + P_{ij}+P_{kj})+(t + P_{ij}+P_{kj}\text{-}d)+0]$
$=[4t + 2P_{ij}+4P_{kj} -2d]\text{-}[4t + 4P_{ij}+2P_{kj} - 2d]$
$=2 (P_{ij}+P_{kj}) \leq 0$, Since $P_{ij} \leq P_{kj}$, then $i \rightarrow k$

Case (2): If $t + P_{ij} \leq d \leq t + P_{kj}$,(the job (i) is tardy if not scheduled first and the job (k) is tardy always)
(See Figure 2 case (2))

Proof: As the job (k) is tardy always, then $E_{kj}=0$, and the job(i) is tardy if not scheduled first, then

$\Delta_{ikj}(t) = F_{ikj}(t) - F'_{kij}(t) =$
$[t + P_{ij})+(d - t - P_{ij}) +(t + P_{ij}+P_{kj})+(t + P_{ij}+P_{kj}\text{-}d)+0]$
$-[t + P_{kj})+(t + P_{kj}\text{-}d)+0+(t + P_{ij}+P_{kj})+(t + P_{ij}+P_{kj}\text{-}d)+0]$
$=[2t + 2P_{ij}+2P_{kj} -2d]\text{-}[4t + 2P_{ij}+4P_{kj} -2d]$
$=2 (P_{ij}+P_{kj}) \leq 0$, Since $t + P_{kj}, \geq d$ then $i \rightarrow k$

Case (3): If $t + P_{ij} \leq d \leq t + P_{ij} + P_{kj}$ (both jobs (i) and (k) are tardy if not scheduled firstly)(see Figure 2 case(3))

Proof:

$\Delta_{ikj}(t) = F_{ikj}(t) - F'_{kij}(t) =$
$= [t + P_{ij})+(d - t - P_{ij}) +(t + P_{ij}+P_{kj})+(t + P_{ij}+P_{kj}\text{-}d)+0]$
$- [t + P_{Kj})+(d - t + P_{Kj}) +(t + P_{ij}+P_{kj})+(t + P_{ij}+P_{kj}\text{-}d)+0]$
$=[2t + 2P_{ij}+2P_{kj}]\text{-}[4t + 2P_{ij}+4P_{kj}]= 0$,then $i \rightarrow k$

Case (4): If $t + P_{ij}+P_{kj} \leq d$, (both jobs (i) and (k) are tardy) (see Figure 2 case (4))

Proof: $\Delta_{ikj}(t) = F_{ikj}(\mathrm{t}) - F'_{kij}(\mathrm{t})$
$=[t + P_{ij})+(d - t - P_{ij}) +(t + P_{ij}+P_{kj})+(0+)+(d - t - P_{ij}-P_{kj}]$ - $[t + P_{ij})+(d - t - P_{ij})+(t + P_{ij}+P_{kj})+(0+)+(d - t - P_{ij}-P_{kj})$ $=[2d]$ — $[2d] = 0$, then $i \rightarrow k$

CONCLUSIONS

In this research, eleven cases of dominance rules were derived for the multiple objectives of the problem (P), which helps us to reduce the time required to find the optimal solution in the BAB method. As for our work in the future, we improve the solutions by approximate methods such as (genetic algorithm, bat algorithm, and wolf's algorithm), which in turn it makes us get the best outputs (e.g., results) and it reduces the time factor and with the least possible losses.

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