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Construction of Complete $(k;r)$-Arcs from Orbits in $PG(3,11)$

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ABSTRACT

The aim of this research is to partition $PG(3,11)$ into orbits using the subgroups of $PGL(4,11)$ which are determined by the nontrivial positive divisors of the order of $PG(3,11)$. These orbits were also studied from the perspective of arcs by finding complete and incomplete arcs.

KEYWORDS: Arc, Complete arc, Companion matrix, Projective space.

الخلاصة

الهدف من هذا البحث هو تجزئة $PG(3,11)$ إلى مدارات باستخدام الزمر الجزئية من $PGL(4,11)$ والتي يتم تحديدها بواسطة القواسم الموجبة غير التافهة لعدد عناصر $PG(3,11)$. ايضا تمت دراسة هذه المدارات من منظور الأقواس من خلال ايجاد اقواس كاملة واقواس غير المكتملة.

INTRODUCTION

In $PG(3,q)$ the projective space of dimension three and order q has $\theta(3,11) = q^3 + q^2 + q + 1$ points, and by the duality has $q^3 + q^2 + q + 1$ planes, $(q^2 + 1)(q^2 + q + 1)$ lines and every plane contains $q^2 + q + 1$ lines, every lines contains $q + 1$ points. Any point of the space has the quadrable form $[x_1, x_2, x_3, x_4]$. Also, there exist five points such that no four of them are on the plane, for example, the points $[1,0,0,0], [0,10,0], [0,0,1,0], [1,1,1,1]$ which are called the standard frame of $PG(3,11)$. The points of $PG(3,q)$ have a unique forms which are $[1,0,0,0], [x,1,0,0], [x,y,1,0], [x,y,z,1]$ for all x,y,z in F_q. A plane π in $PG(3,q)$ is the set of all points $[x_1,x_2,x_3,x_4]$ satisfying a linear equation $u_1x_1 + u_2x_2 + u_3x_3 + u_4x_4 = 0$.This plane denote by $\pi[u_1,u_2,u_3,u_4]$ where u_1,u_2,u_3,u_4 are elements in $F_q\backslash\{0\}$. The idea of construction arcs of different degrees in projective space has been studied by many researchers in different ways. Some researchers studied the subject through the standard frame points [1]. Others studied the subject by the group action on the projective line [2] and plane [3] [4] as well as studied in three dimensional projective space over the field of order 41, see [5]. The purpose of this research, which is appear main results section, is to study the idea of arcs of different degrees in the projective space on the field of order 11 through action of subgroups in $PGL(4,11)$ on $PG(3,11)$, and study the algebraic properties of these arcs. It is worth noting that, the source [6] contains many results of previous studies on the three dimensional projective spaces.

BASIC DFINTIONS

Definition (1) [7]**:** A $(k;r)$-arc in $PG(n \geq 3,q)$ is a set of k points such that no $r+1$ points are collinear, at most r points of which lie in any plane. Here r is called the degree of coplanar $(k;r)$-arc.

Definition (2) [8]**:** A $(k;r)$-arc is called A complete arc if it is not contained in $(k+1;r)$-arc. The maximum size of arc of degree r is denoted by $m_r(n,q)$ and the smallest size of a complete arc of degree r denoted by $t_r(n,q)$.

Definition (3) [6]**:** Let K be an arc of degree r, an i-secant of K in $PG(n,q)$ is a hyper plane π such that $|K \cap \pi| = i$. The number of i-secants of K denoted by τ_i.

Let Q be a point not on the $(k;r)$-arc, the number of i-secants of K passing through Q denoted by $\sigma_i(Q)$.The number $\sigma_r(Q)$ of r-secants is called the index of Q with respect to K.The set of all points of index i will be denoted by C_i and the cardinality of C_i denoted by c_i. The sequence $(t_0, \cdots, t_r)$ will be represented the secant distribution and the sequences $(c_0, \cdots, c_d)$ refer to the index distribution.

***Definition (4)*:** [6] The group of projectivity of $PG(n.q)$ is called the projective general linear group, $PGL(n+1,q)$.
The elements of $PGL(n+1,q)$ are non-singular matrices of dimension $n+1$, and its cardinality is

$$\frac{(q^{n+1}-1)(q^{n+1}-q)\ldots(q^{n+1}-q)}{(q-1)}$$

***Definition (5)*:** [9] Let $f(x)=x^{n+1}-a_nx^n-\cdots-a_1x-a_0$ be a primitive polynomial over F_q of degree $n+1$. A companion matrix for f is a $(n+1)\times(n+1)$ matrix

$$C_f=\begin{pmatrix} 0 & & I_N \\ 0 & & \\ \vdots & & \vdots \\ a_0 & \cdots & a_n \end{pmatrix}$$

The points of $PG(n,q)$ are found by the formula: $P(i)=P_0C_f^i$, where $P_0=[1,0,\ldots,0]$ and i from 0 to $\theta(n,q)-1$. The companion matrix C_f formed a cyclic group of $PGL(n+1,q)$ that is called the *Singer group*. By the Lagrange theorem, any natural number m divided the order of the Singer group generated by C_f will give a cyclic subgroup of order m.

Algorithm of Construction Complete Arcs:
The following lemma is needed to start the algorithm.

***Lemma (6)*:** (i) There exist 14 non trivial cyclic subgroups of $PGL(4,11)$ of order t divided $\theta(3,11)$=1464.
(ii) There exist 14 equivalence classes up to projective space $PG(3,11)$ of order t in y such that $t\cdot i=\theta\ (3,11)$.
***Proof*:**
(i) Let y be the set of non-trivial factors of $\theta(3,11)$. The companion matrix C_f has order $\theta(3,11)$ also which is give cyclic subgroup, $\langle C_f\rangle$ of $PGL(4,11)$ such that $PG(3,11)$ invariant with respect to it, all elements of y divided the order of C_f and give cyclic subgroups of $\langle C_f\rangle$ denoted by S^i, $i\in y$. $PGL(4,11)$ has other cyclic subgroups of an order divided $\theta(3,11)$ will be a copy isomorphic to S^i for $i\in y$.
(ii) Since $i\in y$ by (i), so the action of the subgroups S^i on projective space $PG(3,11)$ will divided the space points into i equivalence classes (orbits) of order $t\in y$, that is $t=\frac{\theta(3,11)}{n}$. The equivalence classes i will be projectively equivalent by C_f.
The algorithm summarized below has been used to construct the complete arcs by the action of the subgroups on the projective space $PG(3,11)$**.**
1. Finding the points of $PG(3,11)$ by formula P_i=(1,0,0,0)$C(f)^i$, $i=0,1,2,\ldots,1463$, where

$$C(f)=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a^8 & a & 1 & a^2 \end{pmatrix}$$

$f(x)=x^4-a^2x^3-x^2-ax-a^8$ be primitive polynomial over F_{11}, and $a=2$ be primitive element of $F_{11}=\{0,a,a^2,a^3,\ldots,a^9|a^{10}=1\}$.
By duality, the planes constructed using the formula π_i=$\pi_0\ C(f)^i, i=0,1,2,\ldots,1463$.
Here π_0 is the plane that passing through point whose fourth coordinate is zero as given below:
π_0={1,2,3,13,19,28,58,59,62,81,95,105,108,111,115,120,121,127,130,136,141,144,145,149,163,177,201,221,239,256,263,274,276,287,304,308,315,316,346,350,353,372,385,386,413,431,434,442,452,453,458,460,467,504,506,509,526,528,533,540,542,551,563,571,595,625,629,649,656,676,699,720,728,741,749,751,757,766,769,788,789,802,820,830,832,836,845,873,880,890,896,903,905,907,931,957,968,995,1000,1005,1011,1020,1037,1058,1066,1069,1088,1092,1102,1108,1109,1117,1165,1167,1172,1227,1239,1225,1293,1298,1311,1316,1339,1342,1361,1375,1387,1392,1422,1426,1427,1439}.
2. Depending on the nontrivial divisors of 1464, which are {2,3,4,6,8,12,24,61,122,183,244,366,488,732} generated subgroups S^i of $PGL(4,11)$, Lemma 6 (i).
3. Finding the orbits of each S^i. Let $O_i[i,t]$ be representative of these orbits where $i\cdot t=\theta(3,11)$
4. Check if $O_i[i,t]$ is arc. If yes, then finding the degree of $O_i[i,t]$ and check it, complete or not.
5. If $O_i[i,t]$ is not complete arc, then the following steps are used to make it complete.
Step i: Determined the set of index zero points of the arc, $O_i[i,t]=B$.

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Step ii: Adding points of index zero in C_0 to make it complete.
Step iii: If Step 2 succeeds we will stop.
Step iv: If Step 3 fails, then continue to add other points until the step succeeds. Let $C_0^{O_i}$ be the set of adding points to $O_i[i,t]$.
A complete $(k;r)$-arc will be $A = O_i[i,t] \cup C_0^B$.
The number 1464 has 14 non trivial positive divisors which are 2,3,4,6,8,12,24, 61,122,183,244,366,488,732, so we have 14 orbits. The above algorithm is executed by GAP-(Groups-Algorithms-Programing) a system for computational discrete algebra [10]. Below the program steps:
PG: symbol refers to the set of point's space.
Lines: symbol refers to the set of line's space.
Planes: symbol refers to the set of plane's space.

```
Read (points);; Read(lines);; Read(planes);;
1- Find the orbits
nf ≔ t; ; nm ≔ i; ;    where t · i = θ (3,11).
xG1:=[];; xGG1:=[];; test:=[];; xpG1:=[];;
for j in [1..q^2+q+1] do
xxG:=[];; xxpG:=[];;
for i in [0..nm-1] do
s:=(b^nf)^i;
r:=(PG[j]*s) ;
p:=Position(PG,y);
Add(xxG,p); Add(test,p);   Add(xxpG,y);od;
Add(xpG1,xxpG);Add(xG1,xxG);
Add(xGG1,Set(xxG));  od;
IINx:= Function compute the τ_i -distribution of an orbit and gave the degree of an arc.
sec:=Function compute the  c_i values.
2. Check the completeness of an arc.
nc:=Maximum(Set(IINx)) ;;
cod:=xG1[1];;
CC0:=Difference([1..q^2+q+1],cod);;
lc:=[];; cp:=[];;
for i in [1..qn] do
l:=plane[i];
I:=Size (Intersection(cod,l));
if I=nc then, append (cp,l); Add(lc,l);
fi;od; Size (C0); C0:=Difference(CC0,Set(cp));
```

MAIN RESULTS

***Theorem (7)*:** The 14 equivalence classes $O_i[i,t]$ up to projective space $PG(3,11)$ are divided into complete and incomplete arcs as follows:

(i) Thirteen complete arcs of degrees 72,48,36,24,21,12,9,13,12,6,4,3,2.

(ii) One incomplete arc of degree 5.

***Proof*:**

(i) Complete arcs.

1. The action of the cyclic subgroup $\langle S^2\rangle$ on $PG(3,11)$ get two orbits $O_2[2,732]$ of size 732 points. This orbit will be arc of degree 72 since it's τ_i-distribution of this orbit is $(\tau_{61}, \tau_{72}) = (732,732)$ and c_i values are $c_{61} = 732$, $c_i = 0$ for $i \neq 61$.
2. The action of the cyclic subgroup $\langle S^3\rangle$ on $PG(3,11)$ get three orbits $O_3[3,488]$ of size 488 points. This orbit will be the arc of a degree 48 since it's τ_i-distribution of this orbit is $(\tau_{37}, \tau_{48}) = (488,976)$ and c_i values are c_{85}=976, $c_i = 0$ for $i \neq 85$.
3. The action of the cyclic subgroup $\langle S^4\rangle$ on $PG(3,11)$ get four orbits $O_4[4,366]$ of size 366 points. This orbit will be the arc of degree 36 since it's τ_i-distribution of this orbit is (τ_{25}, τ_{36})= $(366,1098)$ and c_i values are $c_{97=}1098$, $c_i = 0$ for $i \neq 97$.
4. The action of the cyclic subgroup $\langle S^6\rangle$ on $PG(3,11)$ get six orbits $O_6[6,244]$ of size 244 points. This orbit will be arc of degree 24 since it's τ_i-distribution of this orbit is $(\tau_{13}, \tau_{24}) = (244,1220)$ and c_i values are $c_{109} = 1220$, $c_i = 0$ for $i \neq 109$.
5. The action of the cyclic subgroup $\langle S^8\rangle$ on $PG(3,11)$ get eight orbits $O_8[8,183]$ of size 183 points. This orbit will be arc of degree 21 since it's τ_i-distribution of this orbit is $(\tau_9, \tau_{15}, \tau_{16}, \tau_{18}, \tau_{21}) = (183,366,183,366,366)$ and c_i values are c_{27}=366, $c_{30} = 183$, $c_{34} = 366$, $c_{36} = 366$, $c_i = 0$ for $i \neq 27,30,34,36$.
6. The action of the cyclic subgroup $\langle S^{12}\rangle$ on $PG(3,11)$ get 12 orbits $O_{12}[12,122]$ of size 122 points. This orbit will be arc of degree 12 since it's τ_i-distribution of this orbit is $(\tau_1, \tau_{12}) = (122,1342)$ and c_i values are $c_{121} = 1342$, $c_i = 0$ for $i \neq 121$.
7. The action of the cyclic subgroup $\langle S^{24}\rangle$ on $PG(3,11)$ get 24 orbits $O_{24}[24,61]$ of size 61 points. This orbit will be arc of degree 9 since it's τ_i-distribution of this orbit is $(\tau_0, \tau_1, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7, \tau_8, \tau_9) = (61, 61, 122, 122, 244, 366, 244, 122, 122)$ and c_i values are $c_6 = 183$, $c_7 = 122$, $c_{10} = 366$, $c_{12} = 366$, $c_{14} = 366$, $c_i = 0$ for $i \neq 6,,10,12,14$.

8. The action of the cyclic subgroup $\langle S^{61}\rangle$ on $PG(3,11)$ get 61 orbits $O_{61}[61,24]$ of size 24 points. This orbit will be arc of degree 13 since it's τ_i-distribution of this orbit is $(\tau_2,\tau_{13}) = (1440,24)$ and c_i values are $c_2 = 1440$, $c_i = 0$ for $i \neq 2$.
9. The action of the cyclic subgroup $\langle S^{122}\rangle$ on $PG(3,11)$ get 122 orbits $O_{122}[122,12]$ of size 12 points. This orbit will be arc of degree 12 since it's τ_i-distribution of this orbit is $(\tau_1,\tau_{12}) = (1452,12)$ and c_i values are $c_1 = 1452$, $c_i = 0$ for $i \neq 1$
10. The action of the cyclic subgroup $\langle S^{244}\rangle$ on $PG(3,11)$ get 244 orbits $O_{244}[244,6]$ of size 6 points. This orbit will be arc of degree 6 since it's τ_i- distribution of this orbit is $(\tau_0,\tau_1,\tau_6) = (726,726,12)$ and c_i values are $c_1 = 1452$, $c_{12} = 6$, $c_i = 0$ for $i \neq 1,12$.
11. The action of the cyclic subgroup $\langle S^{366}\rangle$ on $PG(3,11)$ get 366 orbits $O_{366}[366,4]$ of size 4 points. This orbit will be arc of degree 4 since it's τ_i-distribution of this orbit is $(\tau_0,\tau_1,\tau_4) = (968,484,12,)$ and c_i values are $c_1 = 1452$, $c_{12} = 8$ for $i \neq 1,12$.
12. The action of the cyclic subgroup $\langle S^{488}\rangle$ on $PG(3,11)$ get 488 orbits $O_{488}[488,3]$ of size 3 points. This orbit will be arc of degree 3 since it's τ_i- distribution of this orbit is $(\tau_0,\tau_1,\tau_3) = (1089,363,12)$ and c_i values are $c_1 = 1452$, $c_{12} = 9$, $c_i = 0$ for $i \neq 1,12$.
13. The action of the cyclic subgroup $\langle S^{732}\rangle$ on $PG(3,11)$ get 732 orbits $O_{732}[732,2]$ of size 2 points. This orbit will be arc of degree 2 since it's τ_i-distribution of this orbit is $(\tau_0,\tau_1,\tau_2) = (1210,242,12)$ and c_i values are $c_1 = 1452$, $c_{12} = 10$, $c_i = 0$ for $i \neq 1,12$. The point's orbits are 1 and 733 which are lie on the line $\ell = \{1,123,245,367,489,611,733,855,977, 1099,1221,1343\}$. But this line is lie on 12 planes of the orders 105, 227, 349, 471, 593, 715, 837, 959, 1081, 1203, 1325, 1447, and these planes are cover the space. This explains why this orbit is complete arc. For details, see Table 1.

(ii) An incomplete arc.

1. The cyclic subgroup $\langle S^{183}\rangle$ act on $PG(3,11)$ and gave 183 orbits $O_{183}[183,8]$ of size 8 points. This orbit is incomplete arc of degree 5 since τ- distribution is $(\tau_0,\tau_1,\tau_2,\tau_4,\tau_5) = (640,640,160,16,8)$ and $c_0 = 640$, $c_1 = 640$, $c_2 = 160$, $c_4 = 16$. The orbit $O_{183}[183,8]$ will be complete arc of degree 5 when adding 15 points to it. See Table 2.

***Examples (8)*:**

1.$(488;48)$-arc is complete of degree 48.

***Solution*:** Since τ_i-distribution of this orbit is $(\tau_{37},\tau_{48}) = (488,976)$, so the degree of this arc is 48. Since the complete size of this orbit is $c_{85} = 976$, $c_i = 0$ for $i \neq 85$ and $O_3[3,488]$. So, the arc is a complete of size 488.

2. $(6;6)$-arc is a complete of degree 6.

***Solution*:** Since τ_i-distribution of this orbit is (τ_0,τ_1,τ_6)=$(726,726,12)$, so the degree of this arc is 6. Since the complete size of this orbit is $c_1 = 1452$, $c_{12} = 6$, $c_i = 0$ for $i \neq 1,12$ and $O_{244}[244,6]$. So the arc is a complete of size 6.

Note: The results of this article are possible to transform into results in linear codes and graph, see [11] [12] [13].

***Corollary (9)*:**

$m_{12}(3,11) \geq 122$ and $t_{12}(3,11) \leq 12$.

***Proof*:** The orbit O_{12} [12,122] is a complete arc of sizes 122 and degree 12 and O_{122} [122,12] is a complete arc of sizes 12 and degree 12.

CONCLUSIONS

The action of the cyclic subgroups of $PGL(4,11)$, $\langle S^2\rangle$, $\langle S^3\rangle$, $\langle S^4\rangle$, $\langle S^6\rangle$, $\langle S^8\rangle$, $\langle S^{12}\rangle$, $\langle S^{24}\rangle$, $\langle S^{61}\rangle$, $\langle S^{122}\rangle$, $\langle S^{183}\rangle$, $\langle S^{244}\rangle$, $\langle S^{366}\rangle$, $\langle S^{488}\rangle$, $\langle S^{732}\rangle$ on the space $PG(3,11)$ gave 14 orbits. These orbits divided into 13 complete arcs, $(732;72)$-arc, $(488;48)$-arc, $(366;36)$-arc, $(244;24)$-arc, $(183;21)$-arc, $(122;12)$-arc, $(61;9)$-arc, $(24;13)$-arc $(12;12)$-arc (just the line ℓ), $(6;6)$-arc (subset of the line ℓ), $(4;4)$-arc(subset of the line ℓ), $(3;3)$-arc(subset of the line ℓ), $(2;2)$-arc(subset of the line ℓ), and one incomplete arc, $(8;5)$-arc. Let # denote the number of added points to the orbit to be complete.

Table 1. Details about the complete arcs.

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j	**Orbit**	**τ_i-distribution**	**c_i-distribution**
1	O_2[2,372]	(τ_{61}, τ_{72})	(c_{61})
		(732,732)	(732)
2	O_3 [3,488]	(τ_{37}, τ_{48})	(c_{85})
		(488,976)	(976)
3	O_4 [4,366]	(τ_{25}, τ_{36})	(c_{97})
		(366,1098)	(1098)
4	O_6[6,244]	(τ_{13}, τ_{24})	(c_{109})
		(244,1220)	(1220)
5	O_8 [8,183]	$(\tau_9, \tau_{15}, \tau_{16}, \tau_{18}, \tau_{21})$	$(c_{27}, c_{30}, c_{34}, c_{36})$
		(183,366,183,366,366)	(366,183,366,366)
6	O_{12} [12,122]	(τ_1, τ_{12})	(c_{121})
		(122,1342)	(1342)
7	O_{24}[24,61]	$(\tau_0, \tau_1, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7, \tau_8, \tau_9)$	$(c_6, c_7, c_{10}, c_{12}, c_{14})$
		(61,61,122,122,244,366,244,122,122)	(183, 122,366,366,366)
8	O_{61} [61,24]	(τ_2, τ_{13})	(c_2)
		(1440,24)	(1440)
9	O_{122} [122,12]	(τ_1, τ_{12})	(c_1)
		(1452,12)	(1452)
10	O_{244} [244,6]	(τ_0, τ_1, τ_6)	(c_1, c_{12})
		(726,726,12)	(1452,6)
11	O_{366} [366,4]	(τ_0, τ_1, τ_4)	(c_1, c_{12})
		(968,484,12)	(1452,8)
12	O_{488} [488,3]	(τ_0, τ_1, τ_3)	(c_1, c_{12})
		(1089,363,12)	(1452,9)
13	O_{732} [732,2]	(τ_0, τ_1, τ_2)	(c_1, c_{12})
		(1210,242,12)	(1452,10)

Table 2. Details about the incomplete arc.

j	**Orbit**	**τ_i-distribution**	**c_i-distribution**	#
1	O_{183}[183,8]	$(\tau_0, \tau_1, \tau_2, \tau_4, \tau_5)$	(c_0, c_1, c_2, c_4)	15
		(640,640,160,16,8)	(640,640,160,16)	

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