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Hyperfactord of Shi arrangement $\mathrm{Sh}(\mathrm{A}_2)$ and $\mathrm{Sh}(\mathrm{A}_3)$

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ABSTRACT

In this paper, we introduce the region and the faces poset of shi arrangement that it was firstly introduced by J.-Y. Shi. This is an affine arrangement, each of whose hyperplane is parallel to some hyperplane of Coxeter arrangement (Braid arrangement), the degrees and the exponents of this arrangement were found and we prove the shi arrangement is ahyperfactored arrangement when n=3 and not hyperfactored arrangement when n=4 arrangement.

KEYWORDS Hyperplane arrangement, Braid arrangement, Shi arrangement.

الخلاصة

تم حساب الاساس (bases) وناقلات الاس exponent vector)) للترتيبة $Sh(A_3)$ والترتيبة $Sh(A_2)$ ثم الحصول على التجزئة (partition) لهذه الترتيبات ومنها برهنا ان $Sh(A_3)$ ليست قابلة للحل الفوقي بينما $Sh(A_2)$ غير قابلة للحل الفوقي.

INTRODUCTION

Throughout this work, V is a finite-dimensional vector space over a field K (C or R) and S is symmetric algebra $[S \cong K[x_1, x_2, \dots, x_n]$.

Arrangement of a hyperplane A is a set of finite affine hyperplane in V vector space. The dimension of A is denoted by dim (A) is defined to be dim(V), and the rank of A is the rank of any maximal element in A,$(rk(x) = codim(x) = n - dim(A))$. When the $rk(A) = dim(A)$ we say this arrangement is essential, If $\cap_{H\in A} H \neq \emptyset$ then A is central, and if the intersection of the finite family of A is empty then A is called centerless. Let (L_A) is the intersection poset of A containes all the nonempty intersection of hyperplanes partially ordered by reverse inclusion [1]. Let $\{e_1, e_2, \dots, e_n\} \subseteq V$, be the dual basis of $\{x_1, x_2, \dots, x_n\}$. We define $D_i = D_{e_i}$, $1 \le i \le n$, is the derivation $\frac{\partial}{\partial x_i}$, $D_i(f) = \frac{\partial f}{\partial x_i}, f \in S$, Notice that $\{D_1, D_2, \dots, D_n\}$ is a basis for $Der_k(S)$ over S. Therefore any θ is a derivation of S over K is

$\theta = f_1 D_1 + f_2 D_2 + \cdots + f_n D_n$, $f_1, f_2, \dots, f_n \in S$,

Thus $Der_k(S)$ is free S-module of rank n .

Now A non-zero $\theta \in Der_k(S)$ is (homogeneous of polynomial degree) p if $\theta = \sum_{j=1}^{n} f_i D_i$ and $f_i \in S_p$ for $1 \le i \le n$, we say $pdeg\theta = p$, and $tdeg\theta = pdeg\,\theta - 1 =$. Let A be an arrangement with defining polynomial, then a module of A-derivation $D_s(A)$ of $Der_k(S)$ is define by $D_s(A) = \{\theta \in Der_k(S) | \theta(Q) \in QS\}$ A is free arrangement if $D(A)$ is a free module over S. The Shi arrangement of affine hyperplanes is the arrangement in R^n of the form, $x_i - x_j = 0,1$ for $1 \le i < j \le n$ [2].

Definition 1.1 [3]**:** A region of a hyperplane arrangement A, is a connected component of the complement, $R^n - \cup_{H\in A} H$.

Definition 1.2 [4]**:**
The faces of A are the nonempty intersections of the form $F = \cap_{H\in A} H^{\sigma_H}$. where $\sigma_H \in \{+, -, 0\}$ and $H^0 = H$.

Faces of any hyperplane arrangement A can be described by specifying for every $H \in A$, which side of H contains the face. That is, for any $H \in A$ we define H^+ and H^- as two closed half spaces determined by H (the choice of which one is H^- is arbitrary), and let $H^0 = H$.

Example 1.3 [5]**:**
The A_3 Braid arrangement [6] $Q(A) = (x - y)(x - z)(y - z)$. Notice that the sign sequences for the thirteen faces, with the maximum element ($x = y = z$) as shown in Figure 1.

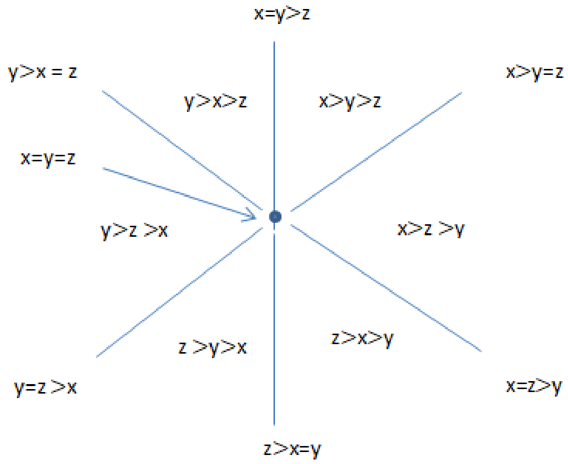

Figure 1. The faces of A_3 Braid arrangement.

2. The Region of Shi arrangement $Sh(A_2), Sh(A_3)$ [7, 8]:

The Shi arrangement dissects R^n into$(n+1)^{n-1}$ region, as was first proved by Shi [2]. There for, when n =3 this arrangement has (16) regions. Shi began by defining rank n sign types as triangular arrays $(X = (X_{ij}))\ 1 \leq i < j \leq n$ with entries form $\{+,-,0\}$ The admissible sign types correspond to the region of his arrangement. He defined them as the sign types which satisfy the following condition: for all $1 \leq i < t < j \leq n$, the triple as belongs to the set $D_A$of admissible sign types of rank 3of $Sh(A_2)$. and D_A is the set

$$\{ {}_{+}{}^{+}{}_{+},\ {}_{+}{}^{+}{}_{0},\ {}_{0}{}^{+}{}_{+},\ {}_{+}{}^{+}{}_{-},\ {}_{-}{}^{+}{}_{+},\ {}_{0}{}^{+}{}_{0},\ {}_{0}{}^{0}{}_{0},\ {}_{+}{}^{0}{}_{-},\ {}_{-}{}^{0}{}_{+},\ {}_{0}{}^{0}{}_{-},\ {}_{-}{}^{0}{}_{0},\ {}_{-}{}^{-}{}_{-},\ {}_{+}{}^{-}{}_{-},\ {}_{-}{}^{-}{}_{+},\ {}_{-}{}^{-}{}_{0},\ {}_{0}{}^{-}{}_{-} \}$$

If we order the symbols $\{0,+,-\}$ as $- < 0 < +$, then D_A can be seen as the rank 3 sign types where either $x_{12} \leq x_{13} \leq x_{23}$ or $x_{23} \leq x_{13} \leq x_{12}$, together with $x_{13} = +, x_{12} = x_{23}$= 0. In Figure 2 each region has been labeled with its type of sign.

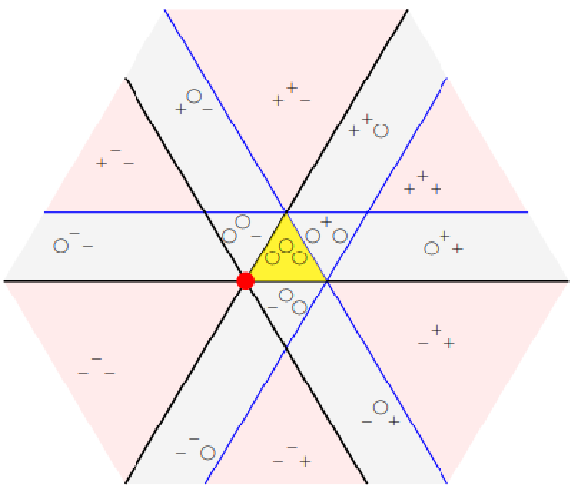

Figure 2. The region of Shi arrangement Sh(A₂).

Example 2.1:

In $Sh(A_2)$ arrangement, each region defined by coordinate inequalities $x_1 > x_2 > x_3$.

Table 1. The regions of($\mathcal{A}_2$)

Region	Sign vector x_{13}\`   $x_{12}$\`x_{23}\`	Corresponding Block-Ordered Partition
$x_2` > x_1` > x_3`$	+ + +	(({2}, {1}, {3}))
$x_1` = x_2` > x_3`$	+ 0 +	(({1, 2}, {3}))
$x_1` > x_2` > x_3`$	+ - +	(({1}, {2}, {3}))
$x_1` = x_3` > x_2`$	0 - +	(({1, 3}, {2}))
$x_1` > x_3` > x_2`$	- - +	(({1}, {3}, {2}))
$x_2` = x_3` > x_1`$	- - 0	(({2, 3}, {1}))
$x_3` > x_1` > x_2`$	- - -	(({3}, {1}, {2}))
$x_3` > x_1` = x_2`$	- 0 -	(({3}, {1, 2}))
$x_3` > x_2` > x_1`$	- + -	(({3}, {2}, {1}))
$x_2` > x_1` = x_3`$	0 + -	(({2}, {1, 3}))
$x_2` > x_3` > x_1`$	+ + -	(({2}, {3}, {1}))
$x_1` > x_2` = x_3`$	+ + 0	(({1}, {2, 3}))
$x_1` = x_2` = x_3`$	0 0 0	(({1, 2, 3}))
$x_3` = x_1` > x_2`$	+ 0 0	({3, 1}, {2})
$x_3` > x_2` = x_1`$	0 0 -	({3}, {2, 1})
$x_1` > x_3` = x_2`$	0 - 0	({1}, {3, 2})

3. The faces of Shi arrangement Sh(A₂), Sh(A₃) [9]:

In order to compute the number of faces in the Shi arrangement we used the formula

$$f_k = \binom{n}{k} \sum_{i=0}^{n-k} \binom{n-k}{i} (-1)^i (n-i+1)^{n-1}$$

where f_k is the number of k-dimensional faces of the Shi arrangement in R^n for $0 \leq k \leq n$ [4]. Thus, the $Sh(A_2)$ when the dimension is (2) this arrangement has (21) faces and if the dimension is (1) the Shi arrangement has (6) faces. Then the set of all faces of $Sh(A_2)$ is (43) with the (16) region. Now for the $Sh(A_3)$ when the dimension is (3) the Shi arrangement $Sh(A_3)$ has (244) faces, and when the dimension is (2) this arrangement has (144), and if the dimension is (1) it has (24) faces, with (125) region.

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Definition 3.1 [6]**:**
Let A is a free arrangement for any homogenous basis $\theta_1, \theta_2, \dots, \theta_n$ of $D(A)$Then the degree of A is $degA = \{tdeg\ \theta_1, tdeg\theta_2, \dots, tdeg\theta_n\}$.

Definition 3.2 [9]**:**
Let A be free arrangement with $\deg A = \{a_1, a_2, \dots, a_n\}$, we define the exponent of A by $expA = \{b_1, b_2, \dots, b_n\}$, where $b_i = a_i + 1, 1 \le i \le n$, i.e., $expA = \{pdeg\ \theta_1, pdeg\theta_2, \dots, pdeg\theta_n\}$, where $\{\theta_1, \theta_2, \dots, \theta_n\}$is a basis of $D(A)$.

Definition 3.3 [9]**:**
$b = (b_1, b_2, \dots, b_n)$ exponent vector or $(b-vector)$ where $b_i = |P_i|$ for $1 \le i \le n$ if $|P_i| = 1$ then P_i is a singleton.

Definition 3.4 [10]**:**
Let P be the partition of a hyperplane arrangement A, then P is nice for if:

1. It is independent.
2. The induced partition P_x admit a block which is a singleton$\forall\ X \in L_A \backslash \{V\}$.

Theorem 3.5 [8]**:**
The hyperplane arrangement is said a hyperfactored if it is a nice partition.

Theorem 3.6:
Shi arrangement $Sh(A_2)$ is a hyperfactored arrangement.
Proof:
By using program (1) we have $D_1(f) = \frac{\partial f}{\partial x_1}$, $D_2(f) = \frac{\partial f}{\partial x_2}$, $\frac{D_3(f)=\partial f}{\partial x_3}$ of $Sh(A_2)$ $D_1(f) = \frac{\partial f}{\partial x_1}$
(x2-x3)*(x3-x2+1)*(x2-2*x1+x3+1)*(x2-2*x1+x3-2*x1*x2-2 *x1*x3+2*x2*x3+2*x1^2).

$$\frac{D_2(f) = \partial f}{\partial x_2}$$

-(x1-x3)*(x1-2*x2+x3)*(x3-x1+1)*(x3-x1+2*x1*x2-2*x1*x3+2*x2*x3-2*x2^2+1) .

$$\frac{D_3(f) = \partial f}{\partial x_3}$$

-(x1-x2)*(x2-x1+1)*(x1+x2-2*x3-1)*(x1+x2-2*x3-2*x1*x2+2*x1*x3+2*x2*x3- 2*x3^2).
so, the degree of $Sh(A_2)$ is $\{-1, 2, 2\}$. and hence the exponent vector of $Sh(A_2)$ is $\{0, 3, 3\}$.
The partition of this arrangement is $P = \{P_1, P_2, P_3\}$ where $\{P_1 = \{\phi\}, P_2 = \{H_1, H_3, H_5\}, P_3 = \{H_2, H_4, H_6\}$.
we find the $A_{x_i}, \forall x_i \in rank\ 2$.
A_{x_1} = {H1, H3, H5}, A_{x_2} = {H1, H4, H6}, A_{x_3} = {H2, H3},
$A_{x_4} = \{H_2, H_5\}$, $A_{x_5} = \{H_2, H_4, H_6\},$, $A_{x_6} = \{H_3, H_6\}$
Now, we compute the induced partition of $Sh(A_2)$, as follows:
$P_1 \cap A_{x_i} = \{\phi\}, \forall i\ = 1, \dots, 6$
$P_2 \cap A_{x_i} = \{x_1 = x_2 + 1 = x_3 + 1\}$, $\forall i = 1, \dots, 6$.
$P_3 \cap A_{x_i} = \{x_1 - 1 = x_2 = x_3 + 1\},$, $\forall\ i = 1, \dots, 6$
Notice that the induced partition of $Sh(A_2)$has singleton block $\forall X \in L_A \backslash \{V\}$, then this arrangement has a nice partition and hence by Th. (3.5) $Sh(A_2)$ is a hyperfactored arrangement.

Theorem 3.7:
Shi arrangement $Sh(A_3)$ is not hyperfactored.
proof:
By using program (2) we found that $\frac{D_1(f)=\partial f}{\partial x_1}$, $D_2(f) = \frac{\partial f}{\partial x_2}$, $\frac{D_3(f)=\partial f}{\partial x_3}$, $D_4(f) = \frac{\partial f}{\partial x_4}$ of $Sh(A_3)$:

$$D_1(f) = \frac{\partial f}{\partial x_1}$$

(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) + (x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) + (x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1).

$$D_2(f) = \frac{\partial f}{\partial x_2}$$

(x1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–

)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1) + (x1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) + (x1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1).

$$D_3(f) = \frac{\partial f}{\partial x_3}$$

(x1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1) + (x1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) + (x1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1).

$$D_4(f) = \frac{\partial f}{\partial x_4}$$

(x1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1) + (x1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) + (x1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x1 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1) - (–1 - x–)*(x1 - x–)*(x2 - x–)*(x2 - x–)*(x3 - x–)*(x2 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x3 - x– + 1)*(x4 - x– + 1)*(x4 - x– + 1).

Therefore, the degree of $Sh(A_3)$ is $\{-1, 3, 3, 3\}$. and the exponent vector of $Sh(A_3)$ is $\{0, 4, 4, 4\}$The partition of this arrangement is $P = \{P_1, P_2, P_3, P_4\}$ where:

$P_1 = \{\phi\}$, $P_2 = \{H_1, H_2, H_3, H_4\}$, $P_3 = \{H_5, H_6, H_7, H_8\}$, $P_4 = \{H_9, H_{10}, H_{11}, H_{12}\}$.

The Ax_i, $\forall x_i \in rank\ 2$ has been found in the following Table 2.

Table 2. The $\mathcal{A}x_i$ of $\mathcal{S}h(A_3)$

$A_{x_1} = \{H_1, H_3, H_7\}$	$A_{x_{17}} = \{H_2, H_{11}\}$
$A_{x_2} = \{H_1, H_4, H_8\}$	$A_{x_{18}} = \{H_2, H_{12}\}$
$A_{x_3} = \{H_1, H_5, H_9\}$	$A_{x_{19}} = \{H_3, H_8\}$
$A_{x_4} = \{H_1, H_6, H_9\}$	$A_{x_{20}} = \{H_3, H_{10}\}$
$A_{x_5} = \{H_2, H_6, H_9\}$	$A_{x_{21}} = \{H_4, H_5\}$
$A_{x_6} = \{H_3, H_5, H_{11}\}$	$A_{x_{22}} = \{H_4, H_9\}$
$A_{x_7} = \{H_3, H_6, H_{12}\}$	$A_{x_{23}} = \{H_4, H_{10}\}$
$A_{x_8} = \{H_4, H_6, H_{11}\}$	$A_{x_{24}} = \{H_5, H_8\}$
$A_{x_9} = \{H_7, H_9, H_{11}\}$	$A_{x_{25}} = \{H_5, H_{10}\}$
$A_{x_{10}} = \{H_2, H_7, H_{12}\}$	$A_{x_{26}} = \{H_5, H_{12}\}$
$A_{x_{11}} = \{H_2, H_4, H_7\}$	$A_{x_{27}} = \{H_6, H_7\}$
$A_{x_{12}} = \{H_8, H_{10}, H_{11}\}$	$A_{x_{28}} = \{H_6, H_8\}$
$A_{x_{13}} = \{H_1, H_3, H_5, H_7, H_9, H_{11}\}$	$A_{x_{29}} = \{H_8, H_9\}$
$A_{x_{14}} = \{H_2, H_3\}$	$A_{x_{30}} = \{H_9, H_{12}\}$
$A_{x_{15}} = \{H_2, H_5\}$	$A_{x_{31}} = \{H_5, H_7\}$
$A_{x_{16}} = \{H_2, H_{10}\}$	$A_{x_{32}} = \{H_1, H_{12}\}$

Now, we compute the induced partition of $h(A_3)$, as follows:

$P_1 \cap A_{x_i} = \{\phi\}, \forall i = 1, \dots, 12$

$P_2 \cap A_{x_i} = \{H_1, H_2, H_3, H_4\}, \forall i = 1, \dots, 12$

$P_3 \cap A_{x_i} = \{H_5, H_6, H_7, H_8\}, \forall i = 1, \dots, 12$

$P_4 \cap A_{x_i} = \{H_9, H_{10}, H_{11}, H_{12}\}, \forall i = 1, \dots, 12$

Notice that the induced partition of $Sh(A_3)$ has no singleton block $\forall X \in L_A \backslash \{V\}$, then this arrangement has no nice partition therefore $Sh(A_3)$ is not a hyperfactored arrangement.

Program 1.

```
syms x1 x2 x3
hp1=x1-x2
hp2=x1-x2-1
hp3=x1-x3
hp4=x1-x3-1
hp5=x2-x3
hp6=x2-x3-1
L=hp1*hp2*hp3*hp4*hp5*hp6
I1=diff(L,x1)
I2=diff(L,x2)
I3=diff(L,x3)
I1=simplify(I1)
I2=simplify(I2)
I3=simplify(I3)
```

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Program 2.

```
syms x1 x2 x3 x4
hp1=x1-x2
hp2=x1-x2-1
hp3=x1-x3
hp4=x1-x3-1
hp5=x1-x3
hp6=x1-x4-1
hp7=x2-x3
hp8=x2-x3-1
hp9=x2-x4
hp10=x2-x4-1
hp11=x3-x4
hp12=x3-x4-1
L=hp1*hp2*hp3*hp4*hp5*hp6*hp7*hp8*hp9*hp
10*hp11*hp12
I1=diff(L,x1)
I2=diff(L,x2)
I3=diff(L,x3)
I4=diff(L,x4)
I1=simplify(I1)
I2=simplify(I2)
I3=simplify(I3)
I4=simplify(I4)
```

CONCLUSIONS

We found the degrees and the exponents of Sh(A_{n-1}) arrangement and we compute the induced partition of $Sh(A_2)$ and $Sh(A_3)$ we prove the shi arrangement is ahyperfactored arrangement when n=3 and not hyperfactored arrangement when n=4 arrangement.

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