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New Types of Separation Axioms via Wp-Open Sets

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ABSTRACT

In this work, new kinds of separation axioms using w_p open sets, some results, properties, examples, and the relationship between these concepts have been given to support our work.

KEYWORDS: w-open, pre-open, regular, and normal spaces

الخلاصة

في هذا العمل، نوع جديد من بديهيات الفصل باستخدام المجموعات المفتوحة، وقد تم اعطاء بعض النتائج، الخصائص، الأمثلة والعلاقة بين هذه المفاهيم لدعم عملنا.

INTRODUCTION

In our work the definitions of w_p-T_i ,i=0, 1, 2, 3, 4 are defined by using w_p- open set. We also offer some notions by utilizing w_p- pen sets and studying some of their facts. Throughout this paper (x, τ) or X is always a topological space. The intersection of w_p closed sets containing A we call it w_p closer of A. the largest w_p open set contained in A we call it w_p interior of A. There are more researches about w-open set [1-5].

ω_P-OPEN SETS AND SOME RESULTS

Definition 1 [6, 7]**:** A set A taken from space X called preopen set if A ⊆ int(cl(A)).

The researchers in our introduction define w-open set as follows:

Definition 2: The set A taken from space X we call it w- open if we find open set U containing any point x in A such that U-A=countable.

Definition 3 [8]**:** A set A taken from space X, we call it w_p open if for every point x belongs to A we have a preopen set U containing x, in which U-A=countable.
So a subset F of space X is w_p closed if X-F is w_p open.

Remark 1: Every open set is w_p-open set but the converse maybe not true.

Example 1: In the indiscrete space (R, τ_{ind}) the set of whole rational numbers is w_p-open but not open

Lemma 1: A subset U of space X is ω_p-open if and only if every point in U is an ω_p-interior point.
Proof:
Suppose U is ω_p-open,then it is ω_p-neighborhood to each of its points. So every point is an ω_p-interior point.
Conversely, since U = $\cup_{x\in U}$ {x} and every point has ω_p-open set V_x such that $x \in V_x \subseteq U$ then U=$\cup_{x\in U}\{V_x\}$ and the union of ω_p-open sets is ω_p-open so U ω_p-open set.

Definition 4: *The space X* will be named:
1- $\omega_p T_0$ −space if for different elements x,,y at X, we find $\omega_p$open set W in X containing x but not y or vice versa.
2- $\omega_p T_1$-space if for different elements x, y in X, there are ω_p open sets W_1, W_2 in X such that $x \in W_1, y \notin W_1$ and $y \in W_2, x \notin W_2$.
3- $\omega_p T_2$ −space if for all different elements x, y in X, there are $\omega_p$open sets have no mutual point W_1, W_2 in X such that $x \in W_1$ and $y \in W_2$.

Example 2: Let X= {1, 2, 3} and τ= {$\emptyset$, X, {1}, {3}, {1, 3}}, so (X,τ) is $\omega_p T_0$-space and, $\omega_p T_1$-$\omega_p T_2$-space, but neither T_1 *nor* T_2-space.

Remark 2:
1-If (X,τ) is T_i-space, that it is $\omega_p T_i$-space where i=0, 1, 2.
2- If (X,τ) is $\omega_p T_2$-space, then it is $\omega_p T_1$ and $\omega_p T_0$-spaces.

Example 3:
1- The indiscrete space (Z,τ_{ind}) is $\omega_p T_0$-space, $\omega_p T_1$ and $\omega_p T_2$-space but not T_0 , $T_1$and T_2-space.
2- The co-finite topological space (R,τ_{cof}) is $\omega_p T_0$-space and $\omega_p T_1$-space, but not $\omega_p T_2$-space.

Proposition 1: Suppose Y be an open subspace of space X, if W is pre-open in X, then W$\cap Y$ is pre-open in Y.

Proof: Since W is pre-open in X, then $W \subseteq \overline{W}^{\circ}$ so
$W \cap Y \subseteq \overline{W}^{\circ} \cap Y = (\overline{W}^{\circ} \cap Y)^{\circ Y} \subseteq (\overline{W} \cap Y)^{\circ Y} \subseteq (\overline{W \cap Y})^{\circ Y}$
$(since\ Y\ is\ open)$
$\subseteq (\overline{W \cap Y})^{\circ Y} \cap Y^{\circ Y} = (\overline{W \cap Y} \cap Y)^{\circ Y}$
, therefore $W \cap Y \subseteq (\overline{W \cap Y}^{Y})^{\circ Y}$.

Proposition 2: If $W$is $\omega_p - open\ in$ a topological space(X,τ_x) and(Y,τ_Y) is a partial open set of X, $then\ W \cap Y\ is\ \omega_p - open\ set\ in\ Y$,.
Proof: Set $x \in W \cap Y, so\ x \in W$ with $x \in Y$, $hence\ there\ is\ pre-open\ G$ in X, with $x \in G$ and G-W be $countable$, but (G-W)⋂Y⊆(G-W), thus $(G$-$W)\cap Y$ is countable, and (G-W)⋂Y=(G⋂Y)-(W⋂Y) will become countable, where G⋂Y is preopen set in Y (by proposition1), we get $W \cap Y$ is $\omega_p$open set in Y.

Corollary1: If M is ω_p-closed in the $space\ (X,\mu)\ with\ (Y,\mu_Y)$ is subspace from X, then $M \cap Y$ is $\omega_p$closed set in Y.

Definition 6: A function f from space X into space Y we call it pre**open function when *image* of all preopen *set* at X is preopen set at Y.

Lemma1: If $f: (X,\tau_X) \longrightarrow (Y,\tau_Y)$ is bijective pre-**open function, $then\ the$ image of all $\omega_p$open set at X is $\omega_p$open set at Y.
Proof: If H is an $\omega_p$open $set\ at\ X$, and $y \in f(H)$, $so\ there\ is\ x \in X$ so that $f(x)$=y (because $f is$ bijective), since it is $y \in f(H)$, then x=$f^{-1}(y) \in f^{-1}(f(H))$=$H$ (f is one to one), hence $x \in H$ which is $\omega_p$open, thus we have preopen set $W\ at\ X$ where $x \in W$ and W-H will be countable, so $f(W$-$H)$ be $countable\ subset$ in Y, however $f(W$-$H)$=$f(W)$-$f(H)$, since $f(W)$ is preopen $at\ Y$ (because of $f\ is$ pre**open function), with $x \in W$, so $f(x)$=$y \in f(W)$, therefore $f(H\ )\ is\ \omega_p$open set in Y.

Proposition 3: A property of spaces which is $\omega_p T_i$-space, i= 0, 1, 2 is open hereditary property.
Proof: Take Y is a subspace of $\omega_p T_0 space\ X$ and x, y as distinct points in Y, hence x, y are distinct points in X which is $\omega_p T_0$-space, so there is $\omega_p$open subset W in X such that $x \in W, y \notin W$. We have $W \cap Y$ is ω_p-open subset of $Y(since\ f\ is\ bijective),(\ by\ lemma\ 1)$ with $x \in W \cap Y, y \notin W \cap Y$ (because $x \in W$ and $x \in Y$ but $y \notin W$), therefore Y is $\omega_p T_0$-space, which means $\omega_p T_0$-space is hereditary property.

Proposition 4: Set f is $function\ of\ X$ into Y be homumorphism then whether X $\omega_p T_i$-space then Y is $\omega_p T_i$, where i = 0, 1, 2.
Proof: To prove Y is $\omega_p T_0$-space, whenever X is ωT_0-space and y_1, y_2 are distinct points in Y, then there are distinct points x_1, x_2 in X with $f(x_1)$=y_1, $f(x_2)$=y_2 (since f is a bijective function), so there exists ω_p-open subset W of X with $x_1 \in W$ and $x_2 \notin W$ (because X is ωT_0-space) there is open set G containing x with G-W=countable. So f(G– W)= f(G) – f(W) is countable. f(G) ⊆ $\underline{f(G)}$ then f(G) = $(f(G))^{\circ} \subseteq \underline{f(g)}^{\circ}$ then $f(G) is$ preopen set $so\ f(W)\ is\ \omega_p$open set $f(x_1)$=$y_1 \in f(W)$ and $f(x_2)$=$y_2 \notin f(W)$,
where $f(W)\ will\ be\ \omega_p$open partial set of Y, therefore Y is $\omega_p T_0 space$.
$By\ same\ method$, we can prove the other part.

Proposition 5: A topological space (X,τ_X) is $\omega_p T_1$-space if and only if any singleton subset $\{x\}$ of X is ω_p-closed subset of X.

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Proof: Suppose any singleton subset $\{x\}$ of X *is* ω_p-closed subset of X for any $x \in X$, and let x, y be distinct points in X, so $\{x\}^c, \{y\}^c$ are ω_p-open sets containing y, x respectively, so X is $\omega_p T_1$-space.

Conversely, let X be an $\omega_p T_1$-space, to prove $\{x\}$ is ω_p-closed subset of X, that means to prove $\{x\}^c$ is $\omega_p$open partial set from X. Let $y \in \{x\}^c$, so $y \neq x$ and there is $\omega_p$open set W at X such that $y \in W, x \notin W$, so $y \in W \subseteq \{x\}^c$, thus $\{x\}^c$ is ω_p-open set, therefore $\{x\}$ is ω_p-closed, but x is arbitrary point in X, that means every singleton subset of X is ω-closed.

Proposition 6: A topological space (X, τ_X) is ωT_0space iff $cl_{\omega_p}(\{x\}) \neq cl_{\omega_p}(\{y\})$ for each distinct points x *and* y in X.

Proof: Suppose $cl_{\omega_p}(\{x\}) \neq cl_{\omega_p}(\{y\})$ with different elements x *and* y in X, so there is at least one element exists in one of them and not in the other, say $a \in cl_{\omega_p}(\{x\})$, $a \notin cl_{\omega_p}(\{y\})$, and suppose $x \notin cl_{\omega_p}(\{y\})$, because if $x \in cl_{\omega_p}(\{y\})$ then $cl_{\omega_p}(\{x\}) \subseteq cl_{\omega_p}\left(cl^{\mu}_{\widehat{\omega}}(\{y\})\right) = cl_{\omega_p}(\{y\})$, then $a \in cl_{\omega_p}(\{x\}) \subseteq cl_{\omega_p}(\{y\})$ and that is a contradiction, therefore $x \in X$-$cl_{\omega_p}(\{y\})$, now X-$cl_{\omega_p}(\{y\})$ is ω_p-open set containing x but not y, that implies X is $\omega_p T_0$-space. Conversely, if X is $\omega_p T_0$-space and x,y are distinct points in X, so there is ω_p-open set U of X with $x \in U$ and $y \notin U$, then X-U is ω_p-closed set contains y but not x, from definition of $cl_{\omega_p}(\{y\})$ we get, $cl_{\omega_p}(\{y\}) \subseteq X$-$U$, which means $x \notin cl_{\omega_p}(\{y\})$ but $x \in cl_{\omega_p}(\{x\})$, so that $cl_{\omega_p}(\{x\}) \neq cl_{\omega_p}(\{y\})$.

Definition 7: *Any space* X *called* :

1- $\omega_p$regular space *for* any *point* $x \in X$ and with M closed at X and $x \notin M$, there is no mutual points of ω_p-open sets W_1, W_2 in X at which $x \in W_1$ and $M \subseteq W_2$.

2- ${\omega_p}^*$-regular space if for any *point* $x \in X$ with all $\omega_p$closed *subset* M of X with $x \notin M$, so we have sets $W_1, W_2 \in \tau$, with $W_1 \cap W_2 = \emptyset$ in which $x \in W_1$ and $M \subseteq W_2$.

3- ${\omega_p}^{**}$-regular space if for any *point* $x \in X$ with all $\omega_p$closed subset M *of* X with $x \notin M$, so we have ω_p*open* sets W_1, W_2 *in* X, *with* $W_1 \cap W_2 = \emptyset$ in which $x \in W_1$ and $M \subseteq W_2$.

Remark 3:

1) Regular space is $\omega_p$regular but the couverse is not true.
2) Every ω_p^*- regular space is ω_p-regular.
3) Every ω_p^*- regular space is ω_p^{**}-regular.

Example 4:

1- X={1,2,3}, τ= {∅ ,X,{1}} , we have τ^c= {X,∅,{2,3}}.So X is not regular but it is ω_p-regular.

2- Let X= {1, 2, 3}, τ = indiscrete so (X, τ_{ind}) is regular, ${\omega_p}^{**}$-regular, and ω_p-regular space, but not ${\omega_p}^*$-regular.

Example 5: The co-finite supra topological space$\left(Z, \tau_{cof}\right)$ is ω_p-regular and ${\omega_p}^{**}$-regular space while it is neither ${\omega_p}^*$-regular, nor a regular.

Theorem 1: A space (X, τ) be ${\omega_p}^*$regular space *iff* for all point x in X with all $\omega_p$neighborhood K *to* x, having neighborhood W in X of x with $cl\ (W) \subseteq K$.

Proof: If X be ${\omega_p}^*$-regular space, let $x \in X$ and K be a ω_p-neighborhood to x, so there exists $\omega_p$open set E at X and $x \in E \subseteq K$, hence E^c is ω_p-closed set at X and $x \notin E^c$, but X is ${\omega_p}^*$regular, thus we *have two* different sets open W, B at X with $x \in W, E^c \subseteq B$, then W is neighborhood to x and $W \subseteq B^c$, where B^c is closed set in X, therefore $cl\ (W) \subseteq cl\ (B^c) = B^c$, which means $cl^{\mu}\ (W) \subseteq B^c$…. (1), and since $E^c \subseteq B$ then $B^c \subseteq E^{c^c} = E \subseteq K$…..(2). From (1) & (2) we have, $cl\ (W) \subseteq B^c \subseteq E \subseteq K \Rightarrow cl\ (W) \subseteq K$;

Conversely, Set $x \in X$ and M is $\omega_p$closed set at X with $x \notin M$, so $x \in M^c$ which is ω_p-open set in X, then M^c is $\omega_p$neighborhood to x, so we have neighborhood W at X to x *such that* $cl\ (W) \subseteq M^c$(from,,hypothesis), since W is the neighborhood to x, then there $will\ be\ open$ set W_1 in X with $x \in W_1 \subseteq W$, from $cl\,(W) \subseteq M^c$ we get $M \subseteq (cl\,(W))^c$, thus $(cl(W))^c$ is open subset of X and since $W \cap W^c$=∅, then $W_1 \cap (cl\,(W))^c$=∅ (because $W_1 \subseteq W$ and because $W \subseteq cl\,(W)$, so $(cl^{\mu}(W))^c \subseteq W^c$), $so\ for\ all$ point x in X with all $\omega_p$closed set M in $X\ where\ x \notin M$ there is $disjoint\ open$ sets $W_1, (cl\,(W))^c$ such that $x \in W_1$ and $M \subseteq (cl\,(W))^c$, that implies X is $\omega_p{}^*$regular.

Proposition 7:
Every open subspace of w_p regular space is w_p regular.
Proof:
If Y be subspace of an ω_p regular space X, take M as a closed set at Y and q as any point at Y such that q∉M, so there is closed set M' in X such $that\ M$ equal $M' \cap Y$, it obvios $q \notin M$, because if opposite, we get q∈M′∩Y that equal M which is a contradiction, so $q \notin M$, but X is ω_p regular, so that there are $two\ \omega_p\ open\ sets\ W, B$ in X with $q \in W$, M′⊆B, $with\ W \cap B$=∅, thus $W \cap Y$,B∩Y are ω_p open in Y, $in\ which\ q \in W \cap Y$ and M′∩Y=M⊆B∩Y and $(W \cap Y) \cap (B \cap Y)$= $(W \cap B) \cap Y$ =∅∩Y that equal ∅, so Y is ω_p regular space,

Lemma 2:
Let f: X → Y be a homeomorphism function if A is preopen in X , then f(A) is preopen in Y.
Proof:
Set $A\ is$ preopen in X then A⊆ int (cl(A)) so by take f for two sides then $f(A)$ ⊆ f(int (cl (A)) …..(1) but $is\ continuous$, then$f$ (cl (A)) ⊆ $clf(A)$ by take interior for two sides we get $int(f\ (cl\ (A))$ ⊆ $int\ (cl$ (A)) ……(2) so by (1) and (2) we have f(A) ⊆ $int\ (cl\ (A))$ that is f(A) is preopen in Y.

Lemma 3:
Let f from X into Y be homeomorphism then if G is preopen and G ⊆Y, then f^{-1}(G) is also preopen in X.
Proof:
Since G is pre-open at Y so G ⊆ $\underline{G}^{\circ}$ then by take f^{-1} for both sides we have f^{-1}(G) ⊆ $f^{-1}(\underline{G}^{\circ})$=$\underline{f^{-1}\ (G)}^{\circ}$so f^{-1}(G)⊆ $\underline{f^{-1}\ (G)}^{\circ}$ so f^{-1}(G) is preopen in X.

Lemma 4:
Let f from X into Y be $homeomorphism\ function$ if A be ω_p open and A ⊆ X ,then f(A) $\omega_p$open and f(A) ⊆ Y.

Proof:
Set A be $\omega_p$open at X to prove f(A)is $\omega_p$open at Y , let y ∈ f(A) then , x = f^{-1}(y) ∈ f^{-1}(f(A)) = A but A is ω_p open set then there exists preopen set G in X containing x such that G – A = countable so f(G-A) = countable since f is bijective but f(G-A) = f(G) – f(A) but f(G) is also preopen set by above lemma , then f(G) – f(A) = countable , then f(A) is also ω_p open set .

Lemma 5:
Let f be a homeomorphism function then the inverse image of $\omega_p$open set is also $\omega_p$open.

Proposition 8:
1- The property of space X being $\omega_p$regular is *topological property.*
2- The $property$ of space X we call it $\omega_p{}^*$regular is *topological property.*
3- A $property$ of $space\ X$ we call it $\omega_p{}^{**}$regular is *topological property.*

Proof:
1- Suppose (X, τ_X) is $\omega_p$regular, with f is $homeomorphism\ function$ from a topological space Xinto a topological space$Y$, let$M$ be closed in$Y$ and q is arbitrary point in$Y$ in which $q \notin M$, so there is point$p \in X$ with$f(p)$=q, we get$f^{-1}\ (M)$ is closed at X ($because\ f\ is\ continuous\ function$) and $f^{-1}(q) = p \notin f^{-1}(M)$, but X is an $\omega_p$regular space, so there are two $\omega_p$open sets W, B in X where $p \in W$, $f^{-1}(M) \subseteq B$, with$W \cap B$=∅, so $f(p) = q \in f(W)$ with$f(f^{-1}(M)) = M \subseteq f(B)$ where $f(W), f(B)$ are $\omega_p$open sets in$Y$ ($by\ \ lemma\ **$),
likewise $f(W) \cap f(B)$=$f(W \cap B)$=f(∅)=∅,
hence$Y$ is $\omega_p$regular, therefore the property space X being $\omega_p$regular is topological property.

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2- If f : X → Y is a homeomorphism function and X be ω_p^* regular space to prove Y is also ω_p^* regular, let $q \in$ M be ω_p closed set in Y and $q \notin$ M then p=f^{-1}(M) is $\omega_p$closed in X by (lemma 4) and Xisω_p^* regular space, then we have two open sets H, K with p ∈ H and f^{-1}(M) ⊆ K and H ⋂ K = ∅ , q =f (p) ∈ f(H) and M = ff^{-1}(M) ⊆ f (K) also f (H) and f (K) will be also open sets $since f\ is$ homeomrphism and f (H) ∩ f (K) = f (H ∩ K) = f(∅) = ∅ then Y is ω_p^* regular space.

3- Prove by the same context.

Theorem 2:
A topological space (X, τ_X) is $\omega_p$regular iff for every $x \in X\ and\ for\ all$ open set U in X contained x, there is $\omega_p$open V in X with $x \in V \subseteq cl_{\omega_p}(V) \subseteq U$.

Proof:
Suppose X is $\omega_p$regular space, set $x \in X$and U is an open in $X\ such\ that\ x \in U$, soU^c be closed in $X\ that\ not\ containing\ x$, but X is $\omega_p$regular space, so there are two $\omega_p$open sets V, W with $x \in V, U^c \subseteq W$ and $V \cap W = \emptyset$, so $V \subseteq W^c$, thus $cl_{\omega_p}(V) \subseteq cl_{\omega_p}(W^c) = W^c$ … (1), and since $U^c \subseteq W$, then $W^c \subseteq U$… (2), from (1) and (2) we get, $x \in V \subseteq cl_{\omega_p}(V) \subseteq W^c \subseteq U$, which means $x \in V \subseteq cl_{\omega_p}(V) \subseteq U$.

Conversely, let M be closed in$X$ and x is arbitrary point in X with $x \notin M$, so M^c is open in $X\ containing\ x$, then from a hypothesis $there$ is $\omega_p$open set V in X in which $x \in V \subseteq cl_{\omega_p}(V) \subseteq M^c$, since $cl_{\omega_p}(V) \subseteq M^c$, hence $M^{c^c} = M \subseteq (cl_{\omega_p}(V))^c$ (since $cl_{\omega_p}(V)$ is $\omega_p$closed set, so $(cl^{\mu}_{\hat{\omega}}(V))^c$ is $\omega_p$open set), and since $V \subseteq cl^{\mu}_{\hat{\omega}}(V)$, hence $V \cap (cl_{\omega_p}(V))^c = \emptyset$. Therefore, there are two $\omega_p$open sets $V, (cl^{\mu}_{\hat{\omega}}(V))^c$ in X such that $x \in V, M \subseteq cl_{\omega_p}(V))^c$, and $V \cap\ (cl_{\omega_p}(V))^c = \emptyset$, then X is $\omega_p$regular space.

Definition 8:
A topological space (X, τ) we call it:-
1- $\omega_p T_3$space if it be $\omega_p$regular T_1-space.
2- $\omega_p{}^* T_3$-space if it is $\omega_p{}^*$regular T_1-space.
3- $\omega_p{}^{**} T_3$-space if it is $\omega_p{}^{**}$regular space and supra T_1-space.

Example 6:
The discrete topological space(R, τ_D) is $\omega_p, \omega_p{}^*$ and $\omega_p{}^{**} T_3$-space.

Remark 4:
1- Every $\omega_p T_3$-space is ω_p regular.
2- $\omega_p$regular space need not be T_2-space (R, τ_{ind}).
3- $\omega_p T_1$-space need not be ω_p regular(R, τ_{cof}).

Example 7:
(R, τ_{ind}) is $\omega_p$regular and $\omega_p T_2$-space but neither ω_p^*regular nor $\omega_p T_3$-space.

Definition 9 [4]**:**
If X is space we called X to be Excluded space, if τ_{Ex}={U: U ⊆ X , $x_\circ \notin$ U , for some $x_\circ \in$ X } ∪ {X}. Excluded space is neither T_1 nor regular space.

Definition 10 [4]**:**
Set X be space we called X is Included space if τ_{In}={U: U ⊆ X , $x_\circ \in$ U , for some $x_\circ \in$ X} ∪ {∅}. Included space is T_1 but not regular so it is not T_3.

Example 8:
To show that (R,τ_{In}) is $\omega_p T_3$ since it is T_1 space now let {2} ⊆ X (since 1 ∉ {2} i.e. R-{2} $\subseteq_{open}$ R so {2} = (R-{2}$)^C \subseteq_{closed}$ R) and 3 ∈ R with 3 ∉ {2} , since {1,3} $\subseteq_{open}$ R so it is $\omega_p$open , T.p. {2} is $\omega_p$open , 2 ∈ {2} , ∃ {2} ⊆ R containing 2 , $\underline{\{2\}}$ = {2} $\underline{\{2\}}^\circ = \emptyset \Longrightarrow \{2\} \not\subseteq \underline{\{2\}}^\circ$ i.e. $\omega_p$open.
The following scheme is helpful.

Proposition 9:
For all $\omega_p T_3$space be $\omega_p T_2$-space.

Proof:
Suppose(X, τ_x) is a $\omega_p T_3$space, let x, y any different points in X, $we\ haveX$ is T_1space $(from\ definition\ of\ \omega_p T_3$space), so $\{x\}$ is closed and $y \notin \{x\}$, since X is $\omega_p$regular so there are $\omega_p$open sets W, B with $\{x\} \subseteq W$, $y \in B$ also $W \cap B$ equal $\emptyset$, since $x \in \{x\} \subseteq w$, therefore X is $\omega_p T_2$-space.

Proposition 10:

1- Every $\omega_p T_3$-space is $\omega_p T_1$-space.

2-Every $\omega_p{}^* T_3$-space is $\omega_p T_2$-space.

3- Every $\omega_p{}^{**} T_3$-space is $\omega_p T_2$-space.

Proof:

1- As in proposition (9).

2- Set x, y∈ X and x , y are not equal, since X is $\omega_p^* T_3$space ⟹ X is T_1 ⟹ [x] is closed in X , x ∈ [x] , [x] is closed , y ∉ [x] so [x] is ω_p closed but X is ω_p^*reguler so then exists two open sets W, B such that [x] ⊆ W, y ∈ B but every open is $\omega_p$open so we have W,B is two $\omega_p$open sets and x ∈ [x] ⊆ W, therefor X is $\omega_p T_2$-space.

3- Similarly to the above proposition.

Definition 11:

A space X we call it $\omega_p$normal if for every closed two sets F_1 , F_2 in X then there exist two different $\omega_p$open sets H, K containing F_1 , F_2 respectively.

Definition 12:

A space X we call it ω_p^*normal if for any two $\omega_p$closed sets F_1, F_2, we have two different open sets H, K containing F_1, F_2 respectively.

Definition 13:

A space X we called it ω_p^{**}normal if for any two $\omega_p$closed sets F_1, F_2in X, we have two different $\omega_p$open set H, K containing F_1 and F_2 respectively.

Remarks 5:

1- Every normal space is $\omega_p$normal but the converse is not true.

2- Every ω_p^*normal is $\omega_p$normal.

3- Every ω_p^*normal is ω_p^{**}normal.

4- Every ω_p^{**}normal space is $\omega_p$normal but the converse not true.

Proposition 11:

A space X is $\omega_p$normal if to each closed set F with an open set U containing F there exist an $\omega_p$open set V, containing F so that F ⊆ V ⊆ $\omega_p cl$(V) ⊆ U.

Theorem 3:

A space X is $\omega_p$normal $iff\ for\ all$ closed set F with any $set\ G\ containing\ F$, there is an $\omega_p$open set V with F ⊂ V ⊂ $\underline{V}$ ⊂ G.

Proof:

Set F is any closed set and G is an open set with F⊂ G, so $G^c$is closed set and F $\cap G^c = \emptyset$. But X is an $\omega_p$normal, then we have $\omega_p$open sets U, V with $G^c \subset$ U , F ⊂ V and U ∩ V = ∅ so thatV⊂ U^c so $\omega_p cl$ (V) C $\omega_p cl$ (U^c) = U^c…..(1) , since U^c is $\omega_p$closed set , but $G^c \subset$ U , then U^c ⊂ G …...(2) by (1) and (2) we get $\omega_p cl$ (V) ⊂ G therefore there exists $\omega_p$open set V such that F ⊂ V and $\omega_p cl$ (V) ⊂ G .

Conversely. Let L and M be $closed\ subsets$ of X with L ∩ M = ∅ so that L ⊂ M^c , so by hypothesis there is an $\omega_p$open set V with L ⊂ V and $\omega_p cl$ (V) ⊂ M^c so M ⊂ ($\omega_p cl$(V))^c also V ∩ ($\omega_p cl$(V))^c = ∅ then V with ($\omega_p cl$(V))^c are two different $\omega_p$open sets, with L ⊂ V, M ⊂ ($\omega_p cl$ (V))^c, therefore, X is $\omega_p$normal space

CONCLUSIONS

In this paper, we introduce new types of separation axioms via Wp-Open sets. In addition to this, we get many results and the most important of which are:

1. If $W$is $\omega_p - open\ in$ a topological space(X, τ_x) and(Y, τ_Y) is a partial set of X, $then\ W \cap Y\ is\ \omega_p - open\ set\ in\ Y$,.
2. A property of spaces which is $\omega_p T_i$-space, i= 0, 1, 2 is hereditary property.
3. A space (X, τ) be $\omega_p{}^*$regular space iff for all point x in$X$ with all $\omega_p$neighborhood $K\ to\ x$, having neighborhood W in X of x with $cl\ (W) \subseteq K$.
4. A topological space (X, τ_X) is $\omega_p$regular iff for every$x \in X and\ for\ all$ open set U in$X$ contained$x$, there is $\omega_p$open$V$ in X with $x \in V \subseteq cl_{\omega_p}(V) \subseteq U$.
5. A space X is $\omega_p$normal $iff\ for\ all$ closed set F with any $set\ G\ containing\ F$, there is an $\omega_p$open set V with F ⊂ V ⊂ $\underline{V}$ ⊂ G.

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REFERENCES

[1] G. S. Ashaea and Y. Y. Yousif. "Weak and Strong Forms of □-Perfect Mappings." Iraqi Journal of Science (2020), 45-55. https://doi.org/10.24996/ijs.2020.SI.1.7

[2] G. S. Ashaea, and Y. Y. Yousif. "Some types of mappings in bitopological spaces." Baghdad Science Journal 18(1), (2021),149-155. https://doi.org/10.21123/bsj.2021.18.1.0149

[3] Humadi, N. K., & Ali, H. J. (2020). Certain types of functions by using supra ˆω-open sets. *Italian Journal of Pure and Applied Mathematics*, *44*, 589-601.

[4] N. Rajesh, on weakly w-closed sets in topological spaces, Math. Maced 3, (2005), 15-24.

[5] A. Samer, and W. Zareer. "Omega open sets in generalized topological spaces." J. Nonlinear Sci. Appl 9(5), (2016), 3010-3017. https://doi.org/10.22436/jnsa.009.05.93

[6] S. Jafari, "on certain types of notions via preopen sets", Tamkang Journal of Mathematics, 37(4), 2006, 391-398. https://doi.org/10.5556/j.tkjm.37.2006.152

[7] H. Maki, K. C. Rao, and A. Nagoor Gani. "On generalizing semi-open sets and preopen sets." Pure and Applied Mathematika Sciences, 49(1/2), (1999): 17-30.

[8] Hussain, K. A., Noman, L. M., & Ali, H. J. On supra Cωp-Lindelöf spaces.

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