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On Intuitionistic Fuzzy Quasi Metric Space

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ABSTRACT

In this article the fixed point of intuitionistic fuzzy quasi metric space instabilities on two metrics $d_x(f, g)$ and $D_x(f, g)$ on complicity space X have been studied and presented in details depended on (0,1]-fuzzy contractive map. The illustrative example is proposed to explain the present results using quick sort algorithm.

KEYWORDS: Polycyclic Aromatic Hydrocarbons; leaf Plants.

الخلاصة

في هذا البحث تمت دراسة النقطة الصامدة في شبه الفضاءات المترية الضبابية الحدسية على فضائين متريين d_x (f, g) و $D_x(f, g)$ باستخدام خوارزميات التعقيد للفضاء X وعرضها بالتفصيل اعتماداً على (0,1) للدوال الانقباضية الضبابية. تم اقتراح المثال التوضيحي لشرح النتائج الحالية باستخدام خوارزمية الفرز السريع .

INTRODUCTION

Firstly, the concept of fuzzy sets is introduced by Zadeh in 1963, [10]. More researchers concept on fuzzy metric space, can be found in Kramos and Michalek [6]. Grabiec in [4] provided conclusive evidence for basic space which is called fuzzy metric spaces and contraction theory. Additionally, George and Veramani in [3] extended the concept of fuzzy space by using the t-norm. Some authors have investigated fuzzy fixed-point theorems such as Sojaei [9], George and Veramani in [3] also Grabiec [4], Kramosil and Michalek [6] and Sharma [8]. In [5], The Banach fixed point has been studied by applied a context of words on fuzzy metric space. In [7], the fixed point studied on intuitionistic fuzzy metric spaces. In [1] the complexity analysis sport to contractive maps with some important results in fuzzy quasi-metric spaces are provided. In [2,4] the concept of intuitionistic related to fuzzy quasi-metric spaces with application that explain the fixe point theorem have been provided in details

The aim of this paper focused on the fixed point of intuitionistic fuzzy quasi metric space, so we are depended on two metrics for each concept of $M_{d_x}(f, g, t)$ and $N_x(f, g, t)$, for each f, $g \in X$ and $t > 0$.

BASIC DEFINITION

Definition(2.1),[1]:

Consider the following space

$X = \left\{\hat{f} : w \to (0, \infty): \sum_{n=0}^{\infty} 2^{-n} \frac{1}{\hat{f}(n)} < \infty\right\}$

Such that w defined as the set of integers with sign nonnegative and d_x is the quasi metric on X given by

$d_x(\hat{f}, \hat{g}) = \sum_{n=0}^{\infty} 2^{-n} ((\frac{1}{\hat{g}(n)} - \frac{1}{\hat{f}(n)}) \vee 0)$

where $\vee$ that is proved or zero

In order of defined (2.1) we develop the as following definition.

Definition(2.2):

The complexity intuitionistic fuzzy quasi metric defined as follows:

The auxiliary functions for $\hat{f}, \hat{g} \in X$, $t > 0$, and defined a continuous function $\alpha_1, \alpha_2 : X \to X$ as

$\alpha_1(\hat{f}) = \hat{f}$, $\alpha_2(\hat{g}) = \hat{g}$

$D_x(\hat{f}, \hat{g}, t) = \sum_{k=n}^{\infty} 2^{-k} ((\frac{1}{\hat{g}(k)} - \frac{1}{\hat{f}(k)}) \vee 0)$, also

$V_x(\hat{g},\hat{f},t) = \sum_{k=n}^{\infty} 2^{-k} ((\frac{1}{\hat{f}(k)} - \frac{1}{\hat{g}(k)}) \vee 0)$

where t ∈ (n, n+1] such that n ∈ ℕ , and t the time between two algorithm $\hat{f},\hat{g}$ provided $\hat{f}(k) \neq \hat{g}(k) \neq 0$ for k = n → ∞ ,

Remark(2.3):

For each $\hat{f},\hat{g} \in X$ and t > 0 we have

$D_x(\hat{f},\hat{g},t)$

$\leq \sum_{k=0}^{\infty} 2^{-k} ((\frac{1}{\hat{g}(k)} - \frac{1}{\hat{f}(k)}) \vee 0) = d_x(\hat{f},\hat{g})$, also

$V_x((\hat{g},\hat{f},t)$

$\geq \sum_{k=0}^{\infty} 2^{-k} ((\frac{1}{\hat{f}(k)} - \frac{1}{\hat{g}(k)}) \vee 0) = d_x(\hat{f},\hat{g})$, also

In particular, for each $\hat{f},\hat{g} \in X$ and t ∈(0, 1] , we have

$D_x(\hat{f},\hat{g},t) = d_x(\hat{f},\hat{g})$ also $V_x((\hat{g},\hat{f},t) = d_x(\hat{g},\hat{f})$.

MAIN RESULTS

Lemma(3.1):

For each $\hat{f},\hat{g},\hat{h} \in X$ and t , s > 0 it follows

1. $D_x(\hat{f},\hat{g},t+s) \leq D_x(\hat{f},\hat{h},t) + D_x(\hat{h},\hat{g},s)$ also
2. $V_x(\hat{g},\hat{f},t+s) \geq V_x((\hat{g},\hat{h},t) + V_x(\hat{h},\hat{f},t)$.

Proof:

Let t ∈ [n ,n+l] and s ∈ [m , m+l] with *n* ,*m* ∈ w . Then t+s ∈[n+m,n+m+l] or t+s ∈[n+m+l, n+m+2]. Hence

$D_x(\hat{f},\hat{g},t+s) \leq \sum_{k=n+m}^{\infty} 2^{-k} ((\frac{l}{\hat{g}(k)} - \frac{l}{\hat{f}(k)}) \vee 0)$

$\leq \sum_{k=n+m}^{\infty} 2^{-k} ((\frac{1}{\hat{h}(k)} - \frac{1}{\hat{f}(k)}) \vee 0)$

$+ \sum_{k=n+m}^{\infty} 2^{-k} ((\frac{l}{\hat{g}(k)} - \frac{l}{\hat{h}(k)}) \vee 0)$

$\leq \sum_{k=n}^{\infty} 2^{-k} ((\frac{1}{\hat{h}(k)} - \frac{1}{\hat{f}(k)}) \vee 0)$

$= D_x(\hat{h},\hat{f},t) + D_x(\hat{g},\hat{h},s)$. Also

$V_x(\hat{f},\hat{g},t+s) \geq \sum_{k=n+m}^{\infty} 2^{-k} ((\frac{l}{\hat{f}(k)} - \frac{l}{\hat{g}(k)}) \vee 0)$

$\geq \sum_{k=n+m}^{\infty} 2^{-k} ((\frac{1}{\hat{h}(k)} - \frac{1}{\hat{g}(k)}) \vee 0)$

$+ \sum_{k=n+m}^{\infty} 2^{-k} ((\frac{1}{\hat{f}(k)} - \frac{1}{\hat{h}(k)}) \vee 0)$

$\geq \sum_{k=n}^{\infty} 2^{-k} ((\frac{1}{\hat{h}(k)} - \frac{1}{\hat{g}(k)}) \vee 0)$

$+ \sum_{k=m}^{\infty} 2^{-k} ((\frac{1}{\hat{f}(k)} - \frac{1}{\hat{h}(k)}) \vee 0)$

$= V_x((\hat{h},\hat{g},t) + V_x(\hat{f},\hat{h},t)$.

Definition(3.2):

1: M(x, y, t) + N(x, y, t) ≤ 1 for all t ≥ 0;
2: M(x, y, 0) = 0;
3: x = y if and only if M(x, y, t) = M(y, x, t) = 1 for all t > 0;
4: M(x, y, t) ∧ M(y, z, s) ≤ M(x, z, t + s) for all t, s ≥ 0
5: M(x, y,-) : [0, ∞) → [0, 1] is left continuous;
6: N(x, y, 0) = 1;
7: x = y if and only if N(x, y, t) = N(y, x, t) = 0 for all t > 0
8: N(x, y, t) ∨N(y, z, s) ≥ N(x, z, t + s) for all t, s ≥ 0;
9: N(x, y,-) : [0,∞) → [0, 1] is left continuous.

Theorem(3.3):

For each $D_x(\hat{f},\hat{g},t)$ and $V_x(\hat{g},\hat{f},t) \in X \times X \times [0,\infty)$

Let $M_x(\hat{f},\hat{g},t) = \frac{t}{t+D_x(\hat{f},\hat{g},t)}$, $M_x(\hat{f},\hat{g},0) = 0$,

and $N_x(\hat{g},\hat{f},t) = \frac{1-V_x(\hat{g},\hat{f},t)}{t+1-V_x(\hat{g},\hat{f},t)}$, $N_x(\hat{g},\hat{f},0) = 1$

whenever t > 0 . Then (M_x , N_x , ∧ , ∨)

is a intuitionistic fuzzy quasi metric on X,

t the time between two algorithm, $\hat{f},\hat{g}$ where ∧ denotes the continuous t- norm and ∨ denotes the continuous t- conorm given by a∧b = min{a,b} and a∨b = max{a,b}

Furthermore for each $\hat{f},\hat{g} \in X$,

$M_x(\hat{f},\hat{g},t) = M_{d_x}(\hat{f},\hat{g},t)$ Also

$N_x(\hat{g},\hat{f},t) = N_{d_x}(\hat{g},\hat{f},t)$. Whenever t >1 where (M_x , N_x , ∧ , ∨) is satisfy the following

a

From $M_{dx}(\hat{f},\hat{g},t) = \frac{t}{t+d_x(\hat{f},\hat{g})}$, also $N_{dx}(\hat{g},\hat{f},t) = \frac{d_x(\hat{f},\hat{g})}{t+d_x(\hat{f},\hat{g})}$

For all $\hat{f},\hat{g} \in X$ and t > 0 . Thus by remark(2.3) We have that .

$M_x(\hat{f},\hat{g},t) = M_{d_x}(\hat{f},\hat{g},t)$, also $N_x(\hat{g},\hat{f},t) = N_{d_x}(\hat{g},\hat{f},t)$ whenever $t \in (0,1]$ also $M_x(\hat{f},\hat{g},t) > M_{d_x}(\hat{f},\hat{g},t)$ also

$N_x(\hat{g},\hat{f},t) < N_{d_x}(\hat{g},\hat{f},t)$ whenever t > l. Next we show that (M_x , N_x , ∧ , ∨) is a intuitionistic fuzzy quasi metric on X.

1: Since $M_x(\hat{f},\hat{g},t)$ = l and $N_x(\hat{g},\hat{f},t) = 0$

Then clearly

$M_x(\hat{f},\hat{g},t) + N_x(\hat{g},f,t) \leq l$ for all $\hat{f},\hat{g} \in X$ and t > 0

2: Let $\hat{f},\hat{g} \in X$ such that

$M_x(\hat{f},\hat{g},t) = M_x(\hat{g},\hat{f},t) = 1$, also

$D_x(\hat{f},\hat{g},l) = D_x(\hat{g},\hat{f},l) = 0$

By remark (2.3) $d_x(\hat{f},\hat{g}) = d_x(\hat{g},\hat{f}) = 0$

So $\hat{f} = \hat{g}$

Moreover given $\hat{f} \in X$ and $t > 0$ then $M_x(\hat{f},\hat{f},t) = 1$ because $D_x(\hat{f},\hat{f},t) = 0$

3: Let $\hat{f}, \hat{g}, \hat{h} \in X$ and t, s > 0 we want to show that $M_x(\hat{f},\hat{g},t+s) \geq M_x(\hat{f},\hat{h},t) \wedge M_x(\hat{h},\hat{g},s)$

Assume that $M_x(\hat{f},\hat{h},t) \leq M_x(\hat{h},\hat{g},s)$

Then $tD_x(h,\hat{g},s) \leq sD_x(\hat{f},\hat{h},t)$. So by using lemma. (3.1) which show one is used we get $tD_x(\hat{f},\hat{g},t+s) \leq tD_x(\hat{f},\hat{h},t) + tD_x(\hat{h},\hat{g},s) \leq t+s\, D_x(\hat{f},\hat{h},t)$

Therefore $M_x(\hat{f},\hat{g},t+s) = \frac{t+s}{t+s+D_x(\hat{f},\hat{g},t+s)} \geq \frac{t}{t+D_x(\hat{f},\hat{h},t)} = M_x(\hat{f},\hat{h},t)$

4: when $t_m \to t_0$, there is an m_0 such that $D_x(\hat{f},\hat{g},t_m) = D_x(\hat{f},\hat{g},t)$ for all $m \geq m_0$ and thus

$\lim_{m\to\infty} M_x(\hat{f},\hat{g},t_m) = M_x(\hat{f},\hat{g},t)$

5: Let $\hat{f},\hat{g} \in X$ such that

$N_x(\hat{g},\hat{f},t) = N_x(\hat{f},\hat{g},t) = 0$ for all $t > 0$ and hence

$V_x(\hat{g},\hat{f},0) = V_x(\hat{f},\hat{h},0) = 1$

By remark (3.4.10) $d_x(\hat{f},\hat{g}) = d_x(\hat{g},\hat{f}) = 0$

So $\hat{f} = \hat{g}$

Moreover given $\hat{g} \in X$ and $t > 0$ then $N_x(\hat{g},\hat{g},t) = 0$ because $V_x(\hat{g},\hat{g},t) = 1$

6: Let f, $\hat{g},\hat{h} \in X$ and t , s > 0 we want to show that

$N_x(\hat{g},\hat{f},t+s) \leq N_x(\hat{g},\hat{h},t) \vee N_x(\hat{h},\hat{f},s)$

Assume that $N_x(\hat{g},\hat{h},t) \geq N_x(\hat{h},\hat{f},s)$

Then $tV_x(\hat{h},\hat{f},s) \geq sV_x(\hat{g},\hat{h},t)$. So by using lemma(3.1). And the above Inequality we obtain $tV_x(\hat{g},\hat{f},t+s) \geq tV_x(\hat{g},\hat{h},t) + tV_x(\hat{h},\hat{f},s) \geq t+s\, V_x(\hat{g},\hat{h},t)$

Therefore $N_x(\hat{g},\hat{f},t+s) = \frac{D_x(\hat{g},\hat{f},t+s)}{t+s+D_x(\hat{g},\hat{f},t+s)} \leq \frac{D_x(\hat{g},\hat{h},t)}{t+D_x(\hat{g},\hat{h},t)} = N_x(\hat{g},\hat{h},t)$

7: If $t_n \to t_0$, there is an n_0 such that $V_x(\hat{g},\hat{f},t_n) = V_x(\hat{g},\hat{f},t)$

For all $n_0 \geq n$ and thus

$\lim_{n\to\infty} N_x(\hat{g},\hat{f},t_n) = N_x(\hat{g},\hat{f},t)$.

Remark(3.4):

For each $\hat{f},\hat{g} \in X$ and $t \in (0, 1]$ we have $D_x(\hat{f},\hat{g},t_m) = d_x(\hat{f},\hat{g})$.

Therefore $M_x(\hat{f},\hat{g},t) = M_{d_x}(\hat{f},\hat{g},t)$. Also $V_x(\hat{g},\hat{f},t_n) = d_x(\hat{g},\hat{f})$. Therefore $N_x(\hat{g},\hat{f},t) = N_{d_x}(\hat{g},\hat{f},t)$

For all $\hat{f},\hat{g} \in X$ and $t \in (0, 1]$, where $(M_{d_x}, N_{d_x}, \wedge, \vee)$ is the intuitionistic fuzzy quasi metric induced by d_x

Definition(3.5):

Let (X, $\mathfrak{M}$, $\mathfrak{N}$, ⊛ , ⊗) be intuitionistic fuzzy quasi metric space and f : X → X a self map it is said that is a fuzzy contractive map if there exists k ∈ (0, 1) Such that $\mathfrak{M}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)), t)$

$$= \frac{\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t)}{\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t)+k(1-\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t))} \text{ also}$$

$\mathfrak{N}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)), t)$

$$= \frac{k(1-\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t)}{\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t)+k(1-\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t))} .$$

For all $\alpha_1(\varpi), \alpha_2(\varpi) \in X$ and $t > 0$.

Definition(3.6):

Let (X, $\mathfrak{M}$, $\mathfrak{N}$, ⊛ , ⊗) be intuitionistic fuzzy quasi metric space and f: X → X a self map it is said that is f is an (0.1] fuzzy contractive map if there exists k ∈ (0,1) such that

$\mathfrak{M}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)), t) =$

$$\frac{\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t)}{\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t)+k(1-\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t))} \text{ also}$$

$\mathfrak{N}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)), t) =$

$$\frac{k(1-\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t)}{\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t)+k(1-\mathfrak{M}(\alpha_1(\varpi),\alpha_2(\varpi),t))} ,$$

for all $\alpha_1(\varpi), \alpha_2(\varpi) \in X$ and $t > 0$

Theorem(3.7) :

Let (X,d) be a quasi metric space and let (X, , $\mathfrak{N}$, ⊛ , ⊗) be intuitionistic fuzzy quasi metric space on X satisfying ,

$\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t) = \frac{t}{t+d(\alpha_1(\varpi),\alpha_2(\varpi))}$ also

$\mathfrak{N}(\alpha_1(\varpi), \alpha_2(\varpi), t) = \frac{d(\alpha_1(\varpi),\alpha_2(\varpi))}{t+d(\alpha_1(\varpi),\alpha_2(\varpi))}$

1. A sequence $\{\zeta_n\}$ in X is a Cauchy sequence. in the quasi metric space (X, d^s)

Also (X, d^i) if and only if it is Cauchy sequence, in the intuitionistic fuzzy quasi metric space $(X, \mathfrak{M}^i_{d_x}, \mathfrak{N}^s_{d_x}, ⊛, ⊗)$

2. A map X → X is (0,1] intuitionistic fuzzy

contractive if and only if it is contractive map in the quasi metric space (X ,d) .

Proof (1):

Let $\{\zeta_n\}$ be a Cauchy sequence in (X, d^s) also (X,d^i) . flx t ∈ (0,l).

Let $\varepsilon \in (0, l)$ with $t > 1-\varepsilon$ also $t < \varepsilon$.

There exists $n_0 \in \mathbb{N}$ such that ,

$d^s(\alpha(\zeta_n), \alpha(\zeta_m)) < \varepsilon$ also $d^i(\alpha(\zeta_n), \alpha(\zeta_m)) > 1-\varepsilon$ for al1 $n, m \geq n_o$.

So $\mathfrak{M}^i_{d_x}(\alpha(\zeta_n), \alpha(\zeta_m)), t) > \frac{t}{t+\varepsilon} > 1-\varepsilon$ also,

$\mathfrak{N}^s_{d_x}(\alpha(\zeta_n), \alpha(\zeta_m), t) < \frac{\varepsilon}{t+\varepsilon} < \varepsilon$ for all $n, m \geq n_o$

So $\{\zeta_n\}$ is a Cauchy sequence in $(X, \mathfrak{M}^i_{d_x}, \mathfrak{N}^s_{d_x}, \circledast, \otimes)$

Reciprocally if $\{\zeta_n\}_n$ is a Cauchy sequence in the intuitionistic fuzzy quasi metric space $(X, \mathfrak{M}^i_{d_x}, \mathfrak{N}^s_{d_x}, \circledast, \otimes)$ given $\varepsilon \in (0, \frac{1}{2})$ there exists $n_0 \in \mathbb{N}$ such that .

$\mathfrak{M}^i_{d_x}(\alpha(\zeta_n), \alpha(\zeta_m)), t) > 1-\varepsilon$ also $\mathfrak{N}^s_{d_x}(\alpha(\zeta_n), \alpha(\zeta_m), t) < \varepsilon$ for all $n, m \geq n_o$

So $\frac{1}{1+d^s(\alpha(\zeta_n), \alpha(\zeta_m))} > 1-\varepsilon$ also $\frac{d^s(\alpha(\zeta_n), \alpha(\zeta_m))}{1+d^s(\alpha(\zeta_n), \alpha(\zeta_m))} < \varepsilon$ for all $n, m \geq n_o$.

Hence $d^s(\alpha(\zeta_n), \alpha(\zeta_m)) < \frac{\varepsilon}{1-\varepsilon} < 2\varepsilon$ also, $d^i(\alpha(\zeta_n), \alpha(\zeta_m)) < 1 - \frac{\varepsilon}{1-\varepsilon} < 1-2\varepsilon$, for all $n, m \geq n_0$, we conclude that $\{\zeta_n\}$ is a Cauchy sequence in (X, d^i) also (X, d^s).

Proof (2):

Let $f: X \rightarrow X$ and $k \in (0,1)$. Then we have that for all t ∈ (0,1] then,

$$\frac{\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t)}{\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t) + k(1-\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t))} = \frac{t}{t+k\, d(\alpha_1(\varpi), \alpha_2(\varpi))}$$ also,

$$= \frac{k(1-\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t)}{\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t) + k(1-\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t))}$$

$$\frac{k\, d(\alpha_1(\varpi), \alpha_2(\varpi))}{t+k\, d(\alpha_1(\varpi), \alpha_2(\varpi))}.$$

For all $\alpha_1(\varpi), \alpha_2(\varpi) \in X$ and $t > 0$. And $\mathfrak{M}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)), t) = \frac{t}{t+d(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)))}$, also

$\mathfrak{N}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)), t) = \frac{d(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)))}{t+d(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)))}$.

Therefore ,

$d(f(\alpha_1(\varpi)), f(\alpha_2(\varpi))) \leq k\, d(\alpha_1(\varpi), \alpha_2(\varpi))$ if and only if

$$\frac{\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t)}{\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t) + k(1-\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t))} \leq \mathfrak{M}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)), t),$$ also

$d(f(\alpha_1(\varpi)), f(\alpha_2(\varpi))) \geq k d(\alpha_1(\varpi), \alpha_2(\varpi))$ if and only if

$$\frac{k(1-\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t)}{\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t) + k(1-\mathfrak{M}(\alpha_1(\varpi), \alpha_2(\varpi), t))} \geq \mathfrak{N}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi)), t).$$

for all $\alpha_1(\varpi), \alpha_2(\varpi) \in X$. This show that f is an (0,1]- intuitionistic fuzzy contractive map on $(X, , \mathfrak{N}, \circledast, \otimes)$ if and only if it is a contractive map on the quasi-metric space (X, d).

Theorem(3.8):

Let (X, d_x), is bicomplete quasi metric space and $(M_{d_x}, N_{d_x}, \wedge, \vee)$ be intuitionistic fuzzy quasi metric on X such that $M_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t) = \frac{t}{t+d_x(\alpha_1(\varpi), \alpha_2(\varpi))}$, also $N_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t) = \frac{d_x(\alpha_1(\varpi), \alpha_2(\varpi))}{t+d_x(\alpha_1(\varpi), \alpha_2(\varpi))}$, with $t \in (0,1]$.

If $f: X \rightarrow X$ is intuitionistic a fuzzy map which is contractive , then *f* has a unique fixed point.

Proof :

Let $M_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t) = \frac{t}{t+d_x(\alpha_1(\varpi), \alpha_2(\varpi))}$ and,

$N_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t) = \frac{d_x(\alpha_1(\varpi), \alpha_2(\varpi))}{t+d_x(\alpha_1(\varpi), \alpha_2(\varpi))}$, implies that

$$\frac{M_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t)}{M_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t) + k(1-M_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t))} = \frac{t}{t+d(\alpha_1(\varpi), \alpha_2(\varpi))}$$ and,

$$\frac{k(N_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t))}{1-N_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t) + k(N_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t))} = \frac{d(\alpha_1(\varpi), \alpha_2(\varpi))}{t+d(\alpha_1(\varpi), \alpha_2(\varpi))}$$

since $k \in (0,1)$ then $\frac{t}{t+k\, d(\alpha_1(\varpi), \alpha_2(\varpi))} = \frac{t}{t+d_x(\alpha_1(\varpi), \alpha_2(\varpi))}$ and,

$\frac{kd(\alpha_1(\varpi), \alpha_2(\varpi))}{t+kd(\alpha_1(\varpi), \alpha_2(\varpi))} = \frac{d_x(\alpha_1(\varpi), \alpha_2(\varpi))}{t+d_x(\alpha_1(\varpi), \alpha_2(\varpi))}$ then

$M_{d_x}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi), t) = \frac{t}{t+d_x(\alpha_1(\varpi), f(\alpha_2(\varpi))}$

And

$N_{d_x}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi), t) = \frac{d_x(\alpha_1(\varpi), f(\alpha_2(\varpi))}{t+d_x(\alpha_1(\varpi), f(\alpha_2(\varpi))}$

If $\frac{M_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t)}{M_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t) + k(1-M_{d_x}(\alpha_1(\varpi), \alpha_2(\varpi), t))} \leq$

$M_{d_x}(f(\alpha_1(\varpi)), f(\alpha_2(\varpi), t)$

Implies that $d(f(\alpha_1(\omega), f(\alpha_2(\omega)) \leq kd(\alpha_1(\omega), \alpha_2(\omega))$

Implies that $t + d(f(\alpha_1(\omega), f(\alpha_2(\omega)) \leq t + k\, d(\alpha_1(\omega), \alpha_2(\omega))$

Implies that $\frac{t}{t+d(f(\alpha_1(\omega),f(\alpha_2(\omega))} \geq \frac{t}{t+k\,d(\alpha_1(\omega),\alpha_2(\omega))}$

$$\geq \frac{M_{d_x}(\alpha_1(\omega),\alpha_2(\omega),t)}{M_{d_x}(\alpha_1(\omega),\alpha_2(\omega),t)+k(1- M_{d_x}(\alpha_1(\omega),\alpha_2(\omega),t))}$$

Implies that $M_{d_x}(f(\alpha_1(\omega), f(\alpha_2(\omega), t)$

$$\geq \frac{M_{d_x}(\alpha_1(\omega), \alpha_2(\omega))}{M_{d_x}(\alpha_1(\omega), \alpha_2(\omega)) + k\left(1 - M_{d_x}(\alpha_1(\omega), \alpha_2(\omega))\right)}$$

If

$$\frac{k(N_{d_x}(f(\alpha_1(\omega), f(\alpha_2(\omega), t))}{1 - N_{d_x}(\alpha_1(\omega), \alpha_2(\omega), t) + k\left(N_{d_x}(f(\alpha_1(\omega), f(\alpha_2(\omega), t)\right)}$$

$\geq 1 - N_{d_x}(f(\alpha_1(\omega), f(\alpha_2(\omega), t)$

Implies that

$d(f(\alpha_1(\omega), f\left(\alpha_2(\omega)\right) \geq kd\left(\alpha_1(\omega), \alpha_2(\omega)\right)$

Implies that

$t + d(f(\alpha_1(\omega), f\left(\alpha_2(\omega)\right) \geq t + k\, d(\alpha_1(\omega), \alpha_2(\omega))$

Implies that

$$\frac{k\,d(\alpha_1(\omega),\alpha_2(\omega)}{t+d(f(\alpha_1(\omega),f(\alpha_2(\omega))} \leq \frac{k\,d(\alpha(\omega)_1(\omega),\alpha_2}{t+k\,d(\alpha_1(\omega),\alpha_2(\omega))}$$

$$\leq \frac{k(N_{d_x}(\alpha_1(\omega), \alpha_2(\omega)))}{1 - N_{d_x}(\alpha_1(\omega), \alpha_2(\omega)) + k\left(N_{d_x}(\alpha_1(\omega), \alpha_2(\omega))\right)}$$

Implies that $N_{d_x}(f(\alpha_1(\omega), f(\alpha_2(\omega), t)$

$$\leq \frac{k(N_{d_x}(\alpha_1(\omega), \alpha_2(\omega)))}{1 - N_{d_x}(\alpha_1(\omega), \alpha_2(\omega)) + k\left(N_{d_x}(\alpha_1(\omega), \alpha_2(\omega))\right)}$$

Hence, by the Banach fixed point theorem f has a unique fixed point.

Example (3.9):

Let T be the recurrence equation of a Quicksort algorithm given by:

T(l) = 0, and $T(n) = \frac{2(n-l)}{n} + \frac{(n+l)}{n} T(n-l)$, $n \geq 2$.

We associate to T the functional $\Psi : X \to X$ given by: $(\Psi(\hat{f}))_l = T(l)$ and

$(\Psi(\hat{f}))_n = \frac{2(n-l)}{n} + \frac{(n+l)}{n}\hat{f}_{n-1}$ for all $n \geq 2$. Next we show that Ψ is a contraction on the bicomplete quasi-metric space (X, d_x), with contraction constant 1/2. We have:

$$d_x\left(\Psi\hat{f}(n), \Psi\hat{g}(n)\right) = \sum_{n=1}^{\infty} 2^{-n}\left(\left(\frac{l}{\Psi\hat{g}(n)} - \frac{l}{\Psi\hat{f}(n)}\right) \vee 0\right) =$$

$$\sum_{n=2}^{\infty} 2^{-n}\left(\left(\frac{l}{\frac{2(n-l)}{n}+\frac{(n+l)}{n}\hat{g}_{n-l}} - \frac{l}{\frac{2(n-l)}{n}+\frac{(n+l)}{n}\hat{f}_{n-l}}\right)\vee 0\right)$$

also $d_x\left(\Psi\hat{g}(n), \Psi\hat{f}(n)\right) =$

$$\sum_{n=1}^{\infty} 2^{-n}\left(\left(\frac{l}{\Psi\hat{f}(n)} - \frac{l}{\Psi\hat{g}(n)}\right)\vee 0\right)=$$

$$\sum_{n=2}^{\infty} 2^{-n}\left(\left(\frac{l}{\frac{2(n-l)}{n}+\frac{(n+l)}{n}\hat{f}_{n-l}}\right) - \frac{l}{\frac{2(n-l)}{n}+\frac{(n+l)}{n}\hat{g}_{n-l}}\right)\vee 0)$$

$$d_x(\Psi\hat{f}(n), \Psi\hat{g}(n)) \leq \sum_{n=2}^{\infty} 2^{-n}\left(\left(\frac{\frac{(n+l)}{n}(\hat{f}_{n-l}-\hat{g}_{n-l})}{(\frac{2(n-l)}{n}+\frac{(n+l)}{n}\hat{g}_{n-l})(\frac{2(n-l)}{n}+\frac{(n+l)}{n}\hat{f}_{n-l})}\right)\vee 0\right)$$

also

$$d_x\left(\Psi\hat{f}(n), \Psi\hat{g}(n)\right) \leq \sum_{n=2}^{\infty} 2^{-n}\left(\left(\frac{\hat{f}_{n-l}-\hat{g}_{n-l}}{\hat{g}_{n-l}\hat{f}_{n-l}}\right)\vee 0\right),$$

also

$$d_x(\Psi\hat{g}(n), \Psi\hat{f}(n)) \geq \sum_{n=2}^{\infty} 2^{-n}\left(\left(\frac{\hat{g}_{n-l}-\hat{f}_{n-l}}{\hat{f}_{n-l}\hat{g}_{n-l}}\right)\vee 0\right)$$

$$d_x\left(\Psi\hat{f}(n), \Psi\hat{g}(n)\right) \leq \frac{1}{2}\sum_{n=2}^{\infty} 2^{-n}\left(\left(\frac{\hat{f}_n-\hat{g}_n}{\hat{g}_n\hat{f}_n}\right)\vee 0\right)$$

also

$$d_x\left(\Psi\hat{g}(n), \Psi\hat{f}(n)\right) \geq \frac{l}{2}\sum_{n=2}^{\infty} 2^{-n}\left(\left(\frac{\hat{g}_n-\hat{f}_n}{\hat{f}_n\hat{g}_n}\right)\vee 0\right)$$

$$d_x\left(\Psi\hat{f}(n), \Psi\hat{g}(n)\right) \leq \frac{l}{2}\sum_{n=2}^{\infty} 2^{-n}\left(\left(\frac{l}{\hat{g}_{(n)}} - \frac{l}{f_{(n)}}\right)\vee 0\right) = \frac{l}{2d_x\hat{f}(n)\hat{g}(n)},$$ also

$$d_x(\Psi\hat{g}(n), \Psi\hat{f}(n)) \geq \frac{l}{2}\sum_{n=2}^{\infty} 2^{-n}\left(\left(\frac{l}{\hat{f}_{(n)}} - \frac{l}{\hat{g}_{(n)}}\right)\vee 0\right) = \frac{l}{2d_x\hat{g}(n)\hat{f}(n)}.$$

So, by Theorem (3.7) we have that Ψ is an (0,1] fuzzy contraction on the complexity intuitionistic fuzzy quasi metric space (X, $\mathfrak{M}$, $\mathfrak{N}$, ⊛, ⊗) with contraction constant l/2, so by Theorem (3.8) we conclude that Ψ has a unique fixed point which is, obviously, the unique solution to the recurrence equation T associated to the Quicksort algorithm.

CONCLUSIONS

The fixed point theorems studied on intuitionistic fuzzy quasi metric space is efficient for studying the complexity of algorithm technique and solved this problems with two complement fuzzy sets.

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