

# A Reliable Numerical Algorithm for Stabilizing of the 2-Dimensional Logistic Hyperchaotic Trajectory

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## ABSTRACT

The dynamical system is the concept used to describe the behavior of several phenomena in our daily life. It comes in two types; linear and nonlinear. Two essential properties characterize the latter, stability and chaos, which in turn are classified into two categories, continuous and discrete, for the models that exhibiting chaotic behavior, which sometimes needs to be stabilized and synchronized. There are various approaches for such a purpose. In this work, the chaotic behavior of the 2D-logistic map is stabilized without adding any control parameters. This approach is considered efficient for models whose analytic solutions are challenging to find. Moreover, the modulus for the Jacobian matrix eigenvalues is greater than unity. Finally, the feasibility and effectiveness of this stabilizing method are demonstrated through some Numerical analysis.

**KEYWORDS:** 2-Dimension logistic map; Stability analysis; Lyapunov exponent; controlling chaos

## الخلاصة

النظام الديناميكي هو المفهوم المستخدم لوصف سلوك العديد من الظواهر في حياتنا اليومية. يأتي في نوعين؛ خطي وغير خطي. هناك خاصيتان أساسيتان تميزان الأخير، الاستقرار والفوضى، والتي يتم تصنيفها بدورها إلى فئتين، مستمرة ومنفصلة، للنماذج التي تظهر سلوكاً فوضوياً، والتي تحتاج أحياناً إلى الاستقرار والمزامنة. هناك طرق مختلفة لهذا الغرض. في هذا العمل، تم تثبيت السلوك الفوضوي للخريطة اللوجستية ثنائية الأبعاد دون إضافة أي معلمات تحكم. يعتبر هذا النهج فعالاً للنماذج التي يصعب العثور على حلولها التحليلية. علاوة على ذلك، فإن معامل قيم eigenvalues لمصفوفة يعقوبي أكبر من الوحدة. أخيراً، تم توضيح جدوى وفعالية طريقة التثبيت هذه من خلال بعض التحليلات العددية.

## INTRODUCTION

The mathematical model is used in many disciplines to describe the behavior of a system and the effect of its component. There are two types of mathematical models; continuous and discrete. The formal is described by differential equations, where difference equations describe the latter. Since the dynamic of these systems is complex, it attracts scholarly attention. We refer the reader to see [1-3] as examples of the continuous type.

On the other hand, discrete-time dynamical systems or difference equations have been increasingly used to model the biological and ecological systems for which there is a time interval between each measurement [4-5]. This modeling approach is made using iterative maps. Iterative maps are an essential part of nonlinear systems dynamics. They allow us to take the output

of the system's previous state and fit it back to the next iteration. However, in general, it is not easy to solve explicitly a system of difference equations. Stability theory in the field of discrete-time dynamical systems deals with the stability of solutions of difference equations and orbits of dynamical systems under small perturbations of initial conditions. From a dynamical systems point of view, bifurcation theory addresses the changes in the qualitative behavior or topological structure of a family of difference equations solution [6-7]. Chaos theory is a branch of dynamical systems that focuses on studying its chaotic states, which are often governed by deterministic laws. Its solutions demonstrate irregular behavior and are highly sensitive to initial conditions [8]. The discrete-time chaotic system has various applications in almost most applied sciences branches. This is due to its ability to describe the behavior of several situations

in real life. For example, most creatures that reproduce once a year like a fish, plant, etc. These applications included but were not limited to, are in biology and medicine [9], engineering [10-11], economics and finance [12], cryptography and security [14-18], and many others. Although chaotic behavior is essential in some applications, the impossibility of long-term prediction and high sensitivity to the initial conditions, and the random influence are still reasons for avoiding such behavior. To make such systems behave as desired this required introduction of external forces to withstand the perturbations and make the system trajectory skew towards stability. There are many numerical and analytical methods for stabilizing the discrete chaotic systems behavior; we refer the reader to see [19-21]. In this work, the hyperchaotic behavior of the 2D-Logistic map [22] is stabilized to a fixed point without using any controlling variables or any prior analytical knowledge. The classical formwork of 2D-Logistic map in discrete time has the form [22]:

$$\begin{aligned} x_{i+1} &= (1 - \mu)x_i + 4\mu y_i(1 - y_i) \\ y_{i+1} &= (1 - \mu)y_i + 4\mu x_i(1 - x_i) \end{aligned} \tag{1}$$

where  $x_i$  represent the size of the population of a certain species at time  $i$  and  $\mu$  be the growth rate of the population from one generation to another, such that  $0 < \mu < 1$ .

This paper organized as follows: Section 2 presents the analytical study of the 2D-Logistic map. The dynamic behavior of the 2D-Logistic map is discussed in section 3. Section 4 introduced the general strategy to control discrete chaotic systems. Section 5 presents the numerical stabilization method for the 2D-Logistic map. The analytical result is proved numerically led in section 6. Finally, section 7 concluded the paper.

### Analytical analysis of 2d-logistic map

The fixed points of the discrete-time system are determined, and the stability conditions are examined. The possible fixed points are obtained by solving system of equations.

$$\begin{aligned} x_{i+1} &= (1 - \mu)x_i + 4\mu y_i(1 - y_i) = f_1(x_i, y_i) \\ y_{i+1} &= (1 - \mu)y_i + 4\mu x_i(1 - x_i) = f_2(x_i, y_i) \end{aligned} \tag{2}$$

A simple calculation shows that system (2) has three fixed points. The Jacobain matrix of (2) is:

$$J = \begin{bmatrix} 1 - \mu & 4\mu(1 - 2y) \\ 4\mu(1 - 2x) & 1 - \mu \end{bmatrix}$$

Solving this system (2) we get the following equilibrium points  $E_0 = (0,0)$  &  $E_1 = (\frac{3}{4}, \frac{3}{4})$ ,  $E^* = (x^*, y^*)$  The Jacobain matrix at each fixed point is calculated and as follows

At the point,  $E_0 = (0,0)$ , we have:

$$J_{E_0} = \begin{bmatrix} 1 - \mu & 4\mu \\ 4\mu & (1 - \mu) \end{bmatrix}$$

The eigenvalues of  $E_0$  are  $\lambda_1 = (1 + 3\mu)$  and  $\lambda_2 = (1 - 5\mu)$ . Hence, by a simple calculation, the local dynamics of fixed point  $E_0$  is shown. The local dynamics of  $E_0$  illustrated in:

Proposition 1: For the fixed  $E_0$ , the following station holds true:-

$E_0$  is a *source* point if and only if  $|\lambda_1| > 1$  for  $\mu > 0$ , and  $|\lambda_2| > 1$  for  $\mu > 0$ .

$E_0$  is a local asymptotic stable point if and only if  $|\lambda_1| < 1$  for  $\mu < 0$ , &  $|\lambda_2| < 1$  for  $0 < \mu < 0.4$ .

$E_0$  is a saddle point if and only for  $|\lambda_1| < 1$  for  $\mu < 0$  and  $|\lambda_2| > 1$  for  $\mu > 0.4$ .

$E_0$  is a non-hyperbolic point if and only if  $|\lambda_2| = 1$ , for  $\mu = 0.4$ .

At  $E_1 = (\frac{3}{4}, \frac{3}{4})$  we have,  $J_{E_1} = \begin{bmatrix} 1 - \mu & -2\mu \\ -2\mu & 1 - \mu \end{bmatrix}$

The eigenvalues of  $E_1$  are;  $\lambda_1 = (1 - 3\mu)$  and  $\lambda_2 = (1 + \mu)$ . Therefore, the local dynamics of  $E_1$  is illustrated in Proposition 2.

Proposition 2: For the fixed point  $E_1$ , the following station holds.

$E_1$  is a *source* point if and only  $|\lambda_1| > 1$  for  $\mu < 0$ , and  $|\lambda_2| > 1$  for  $\mu > 0$ .

$E_1$  is a local asymptotic stable if and only if  $|\lambda_1| < 1$  for  $\mu < 0$ , &  $|\lambda_2| < 1$  for  $0 < \mu < 0.4$ .

$E_1$  is saddle point if and only if  $|\lambda_1| > 1$  for  $\mu > 0.6$ , and  $|\lambda_2| < 1$  for  $\mu < 0$ .

$E_1$  is a non-hyperbolic point if and only if  $\mu = 0.667$  for  $|\lambda_1| = 1$ .

For the fixed point  $E^* = (x^*, y^*)$ , the Jacobain matrix of 2 is

$$J_{E^*} = \begin{bmatrix} 1 - u & 4u(1 - 2y^*) \\ 4u(1 - 2x^*) & 1 - u \end{bmatrix}$$

The complex pair eigenvalues of  $E^*$  are  $\lambda_1 = (1 - \mu) + 2\mu i$ ,  $\lambda_2 = (1 - \mu) - 2\mu i$  where  $x^* = \frac{5+\sqrt{5}}{8}$ ,  $y^* = \frac{5-\sqrt{5}}{8}$

It is well known that the eigenvalue of J determines the stability of the fixed point. In the light of this information, we can give the following proposition.

Proposition 3: Assume  $E^*$  is a positive fixed point of system (2), then  $E^*$  is:

sink fixed point if  $\mu \in (0,0.4)$  with  $|\lambda_1| < 1$  and  $|\lambda_2| < 1$ .

Source fixed point if  $\mu > 0.4$  with  $|\lambda_1| > 1$  and  $|\lambda_2| > 1$ .  $\lambda_1$  and  $\lambda_2$  are pair of conjugate complex roots  $|\lambda_1| = 1$  and  $|\lambda_2| = 1$ , if and only if  $B^2 - 4C < 0$  and  $C = 1$  for  $\mu = 0.4$ .

## Dynamic analysis of 2d-logistic map

### 1. Trajectory (Phase space)

The trajectory of (1) is series of values, which shows the shifting path of the system outputs Fig.1 describes the trajectory of (1) in the xy-plane for initial value  $x_0 = 0.1, y_0 = 0.1$  with the parameter  $\mu$ . The trajectory of the 2D-Logistic map is shown in Figure 1.

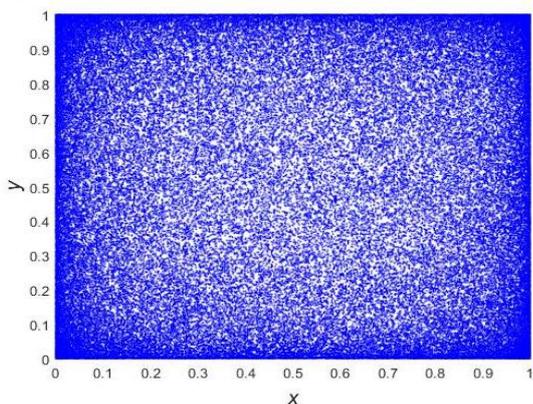


Figure 1. 2D-Logistic map trajectory for  $\mu = 1$ .

### 2. Bifurcation analysis

The bifurcation is the value of the parameter that effecting the system behavior. This behavior is described through a diagram that illustrates the change of the dynamic. The bifurcation diagram of (1) is shown in Figure 2.

For  $0 < \mu < 1$  system (1) has chaotic behavior with the range  $\mu \in [0.85, 0.87]$  and it is hyperchaotic when  $\mu \in [0.89, 0.94] \cup [0.95, 1]$ .

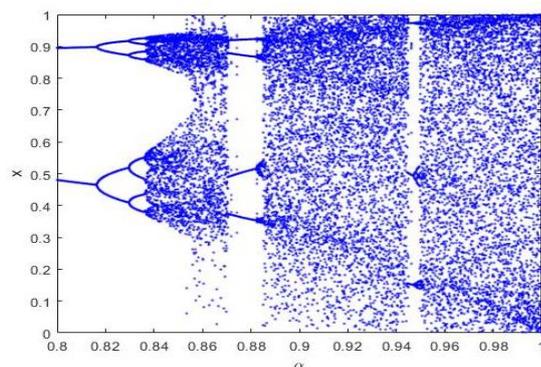


Figure 2. Bifurcation diagram of the 2D-Logistic map.

### 3. Lyapunov exponent (LE)

The extreme sensitivity to the initial condition essentially characterizes the chaoticity of the nonlinear dynamical systems. Suppose two adjacent trajectories of a dynamical system diverge exponentially. In that case, this arbitrary invariant is used to characterize a chaotic system known as the Lyapunov exponent. The chaoticity of (1) is investigated by the LE, as shown in Figure 3. It shows that the proposed system has chaotic behavior in some parameters and hyperchaotic behavior in others. The average of LE is used to define the local instability of a given system.

Note that, for  $0 < \mu < 1$ , system (1) has chaotic behavior with the range  $\mu \in [0.85, 0.87]$  and it is hyperchaotic when  $\mu \in [0.89, 0.94] \cup [0.95, 1]$ .

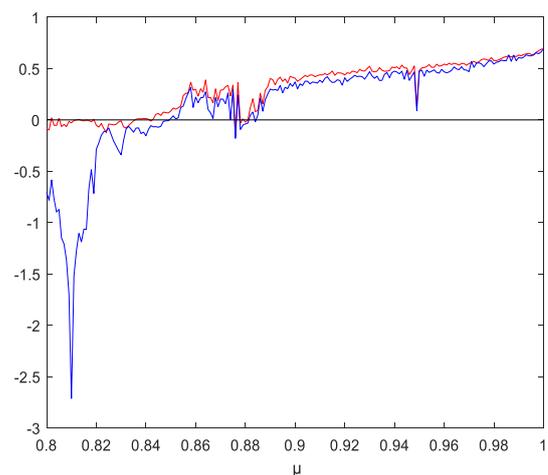


Figure 3: Lyapunov Exponent of the 2D-Logistic map.

### 4. Controlling unstable Discrete Chaos

Consider an n-dimensional dynamic system defined by

$$x_{k+1} = F(x_k) \quad (3)$$

where  $x \in R^n$  is an n-dimensional vector, F is a nonlinear vector-valued function. Let  $x_f$  be the fixed point of the system (1). To stabilize the chaotic orbit of this fixed point, we take variable feedback control, which described by:

$$x_{k+1} = F(x_k) + u(x_k) \quad (4)$$

Substitute in (4) feedback control  $u(x_k) = M(F(x_k) - x_k)$  we get:

$$x_{k+1} = F(x_k) + M(F(x_k) - x_k) \quad (5)$$

Define an infinitesimal deviation of  $x_k$  from  $x_f$  as  $\delta x_k = x_k - x_f$ . Then from 5 after applying of Taylor series about  $x_f$ , we have

$$\begin{aligned}
x_{k+1} &\cong F(x_f) + \frac{\partial F}{\partial x}(x_k - x_f) + \frac{1}{2} \frac{\partial^2 F}{\partial^2 x}(x_k - x_f)^2 + \dots + M \left( \frac{\partial F}{\partial x}(x_k - x_f) - 0 \right) \\
x_{k+1} &\cong x_f + J\delta x_k + MJ \delta x_k \\
x_{k+1} - x_f &\cong J\delta x_k + M(J - I) \delta x_k \\
\delta x_{k+1} &= J\delta x_k + M(J - I) \delta x_k \quad (6)
\end{aligned}$$

where  $J = \left. \frac{\partial F}{\partial x_k} \right|_{x_k=x_f}$  is the Jacobain matrix of the origin system F that evaluated at the fixed point  $x_f$  and I is the nxn identity matrix.

The goal of controlling here is to make

$$[\delta x_k] = 0$$

To achieve this goal, it requires:

$$\delta x_{k+1} = Q\delta x_k \quad (7)$$

where Q is a n×n matrix and takes the form:

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \quad (8)$$

Substituting Eq. (7) and Eq. (8) into Eq. (6)

Choosing special form matrix  $Q=qI$ ,  $q \in (-1,1)$

We get

$$\delta x_{k+1} = J\delta x_k + M(J - I) \delta x_k$$

$$Q\delta x_k = J\delta x_k + M(J - I) \delta x_k$$

$$\text{where } (J + M(J - I) - Q) \delta x_k = 0.$$

$$\text{We observe } J + M(J - I) - Q = 0.$$

$$\text{Note that } M(J - I) = Q - J.$$

$$M = \begin{pmatrix} \frac{(32\mu - 64\mu x_{2f})x_{1f} + q - 15\mu + 32\mu x_{2f} - 1}{(64\mu x_{2f} - 32\mu)x_{1f} + 15\mu - 32\mu x_{2f}} & \frac{(4q + 8x_{2f} - 8qx_{2f} - 4)}{(64\mu x_2 - 32\mu)x_{1f} + 15\mu - 32\mu x_{2f}} \\ \frac{((8 - 8q)x_{1f} + 4q - 4)}{(64\mu x_{2f} - 32\mu)x_{1f} + 15\mu - 32\mu x_{2f}} & \frac{(32\mu - 64\mu x_{2f})x_{1f} + q - 15\mu + 32\mu x_{2f} - 1}{(64\mu x_{2f} - 32\mu)x_{1f} + 15\mu - 32\mu x_{2f}} \end{pmatrix} \quad (12)$$

**Step 4.** Choose the parameter  $(x_{1f}, x_{2f}) = (0.9045, 0.3455)$  in (10) we get,  $(q_1, q_2) = (0.3, 0.5)$

**Step 5.** Compute the matrix M in Eq. (8) at  $q_1 = 0.3$  &  $q_2 = 0.5$

We get respectively,

$$\begin{aligned}
M_1 &= \begin{pmatrix} -0.88235 & 0.14542 \\ -0.38073 & -0.88235 \end{pmatrix}, \text{ and} \\
M_2 &= \begin{pmatrix} -0.83528 & 0.20359 \\ -0.53302 & -0.83528 \end{pmatrix} \quad (13)
\end{aligned}$$

**Step 6.**

From system (2), substitute Eq.(13) into Eq. (10) we obtain:

For M1

$$\begin{aligned}
x_{k+1} &= (1 - \mu)x_k + 4\mu y_k(1 - y_k) \\
&\quad - 0.882[(1 - \mu)x_k \\
&\quad + 4\mu y_k(1 - y_k) - x_k] \\
&\quad + 0.1454[(1 - \mu)y_k \\
&\quad + 4\mu x_k(1 - x_k) - y_k]
\end{aligned}$$

Finally, we get:

$$M = (qI - J)(J - I)^{-1} \quad (9)$$

### 5. Stabilizing of the 2D-logistic map

The 2D-logstic system is stabilized using the following algorithm:

**Input Data:** The chaotic system

$$\begin{aligned}
x_{k+1} &= (1 - \mu)x_k + 4\mu y_k(1 - y_k) \\
y_{k+1} &= (1 - \mu)y_k + 4\mu x_k(1 - x_k) \quad (10) \\
k &= 0, 1, \dots, n
\end{aligned}$$

**Output** The stable system.

**Algorithm Steps:**

**Step 1.** Compute fixed point of the 2D-Logistic system.

We get the fixed point (0.9045, 0.3455) by a fixed point iteration method.

**Step 2.** Compute the Jacobain matrix that correspond the fixed point  $(x_{1f}, x_{2f})$  such that:

$$J = \begin{pmatrix} (1 - \mu) & 4\mu(1 - 2x_{2f}) \\ 4\mu(1 - 2x_{1f}) & (1 - \mu) \end{pmatrix} \quad (11)$$

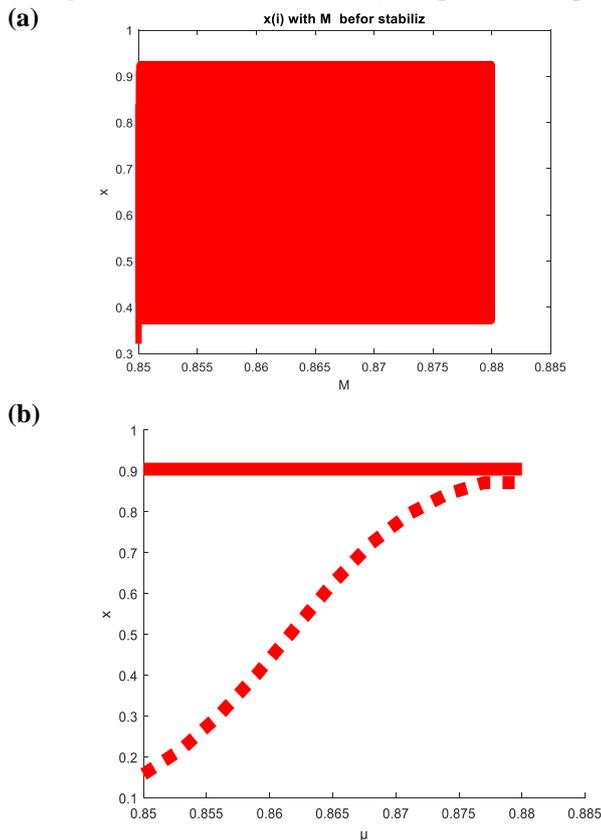
**Step 3.** Compute the matrix M from Eq. (9) after calculation matrix J from (11) and compute the inverse  $(J - I)^{-1}$

$$\begin{aligned}
y_{k+1} &= (1 - \mu)y_k + 4\mu x_k(1 - x_k) \\
&\quad - 0.38073[(1 - \mu)x_k \\
&\quad + 4\mu y_k(1 - y_k) - x_k] \\
&\quad - 0.882[(1 - \mu)y_k \\
&\quad + 4\mu x_k(1 - x_k) - y_k]
\end{aligned}$$

For M2

$$\begin{aligned}
x_{k+1} &= (1 - \mu)x_k + 4\mu y_k(1 - y_k) \\
&\quad - 0.835[(1 - \mu)x_k \\
&\quad + 4\mu y_k(1 - y_k) - x_k] \\
&\quad + 0.2035[(1 - \mu)y_k \\
&\quad + 4\mu x_k(1 - x_k) - y_k] \\
y_{k+1} &= (1 - \mu)y_k + 4\mu x_k(1 - x_k) \\
&\quad - 0.533[(1 - \mu)x_k \\
&\quad + 4\mu y_k(1 - y_k) - x_k] \\
&\quad - 0.352[(1 - \mu)y_k \\
&\quad + 4\mu x_k(1 - x_k) - y_k]
\end{aligned}$$

In this section, the numerical results are shown in Figures 4&5. In Figure 4(a), the 2D-Logistic map is chaotic before adding stabilization for  $x_i$  with different parameters of  $\mu \in [0.85,0.88]$ , while in Figure 4(b), the 2D-logistic map is stable after adding stabilization for  $x_i$  for  $\mu \in [0.85,0.88]$ .



**Figure 4.** (a)  $x_i$  versus  $\mu$  before the stabilization, whenever (b) represent  $x_i$  versus  $\mu$  after the stabilization.

## CONCLUSION

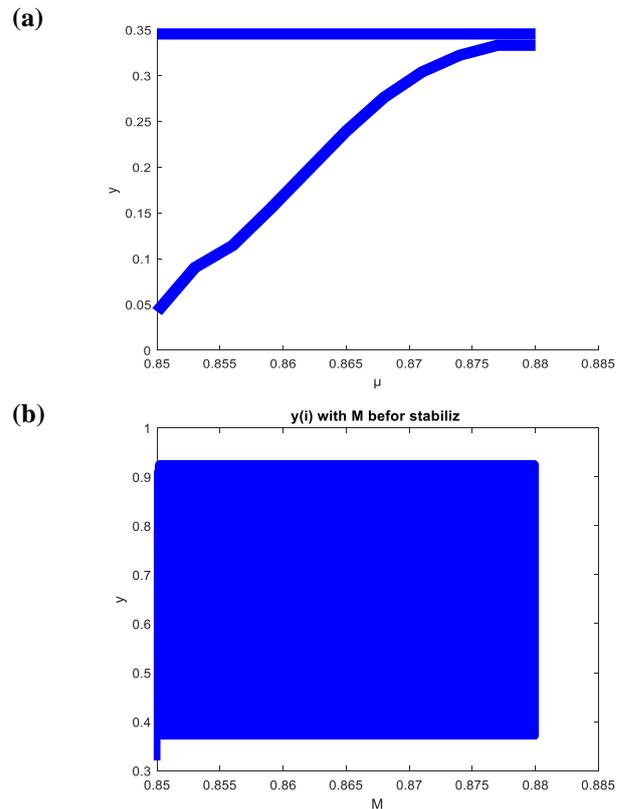
In this paper, the 2D-logistic map is considered. It has three fixed points. The behavior of these points is discussed to show its chaotic behavior for some eigenvalues. A straightforward method for stabilizing the chaotic behavior of this map is used. This method did not require adjustable parameters for controlling the chaotic behavior. The results show the efficiency of this method, which is demonstrated through numerical investigations before and after stabilization for different ranges of  $\mu$  value that strongly affected the stability of the 2D-logistic map.

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In Figure 5 (a), the 2D-logistic map is chaotic before adding stabilization for  $y_i$  with different parameters of  $\mu \in [0.85,0.88]$ , while in Figure 5(b), the 2D-Logistic map is stable after adding stabilization for  $y_i$  with  $\mu \in [0.85,0.88]$ .



**Figure 5.** (a) Represents  $y_i$  versus  $\mu$  before the stabilization, whenever, (b) represent  $y_i$  versus  $\mu$  after the stabilization.

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## How to Cite

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