

Weibull Reliability Estimation of (3+1) Cascade Model

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ABSTRACT

In this paper presents the R reliability mathematical formula of (3+1) Weibull Cascade model. The reliability of the model is expressed by Weibull random variables, which are stress and strength distributions. The reliability model was estimated by six dissimilar methods (ML, Mo, LS, WLS, Rg and Pr) and simulation was performed using MATLAB 2012 program to compare the results of the reliability model estimates using the MSE criterion, the results indicated that the best estimator among the six estimators was ML.

KEYWORDS: Standby redundancy; Weibull distribution; distributed Identically.

الخلاصة

في هذا البحث تم تقديم المعادلة الرياضية لمعولية ويبل لنموذج كاسكاد 3+1. تم التعبير عن معولية النموذج بواسطة متغيرات ويبل العشوائية، وهي توزيعات الأجهاد و المتانة. تم تقدير نموذج المعولية بست طرق مختلفة هي (الأماكن الأعظم، العزوم، المربعات الصغرى، المربعات الصغرى الحرجة، الأنحدار و الرتب المئوية)، وتم إجراء المحاكاة باستخدام برنامج ماتلاب 2012 لمقارنة نتائج تقديرات نماذج المعولية باستخدام مقياس متوسط مربعات الخطأ MSE، أشارت النتائج إلى أن أفضل مقدر من بين ستة مقدرات هو طريقة الأماكن الأعظم ML.

INTRODUCTION

Many researches have been performed on reliability estimation $R = p(X > Y)$ in the field of strength and stress models. The Cascade is a special kind of stress-strength model. Cascade redundancy is a hierarchical standby redundancy in which a standby unit with different stress substitutes for a system. When a system unit fails, it is replaced by a standby unit and the stress changed k Times the previous stress. In a previous study Karam and Khaleel (2019) presented a study of (2+1) Cascade model, which the model consists of two main components and one redundancy standby[9]. In this paper, we assumed that the (3+1) of Cascade with $(U_1, U_2, U_3 \text{ and } U_4)$ Units, in which three units $U_1, U_2, U_3 \text{ and } U_4$ are work and the unit U_4 is a standby unit. Assume that X_1, X_2, X_3, X_4 denote the unit strengths $(U_1, U_2, U_3 \text{ and } U_4)$ respectively and Y_1, Y_2, Y_3, Y_4 indicated the enforcement of stress. Here, if the active unit U_1 is a failure then the standby component U_4 is activated,

where $X_4 = mX_1$ and $Y_4 = kY_1$, if the active unit U_1 is a failure then the standby component U_4 is activated, where $X_4 = mX_2$ and $Y_4 = kY_2$ and if the active unit R_3 is a failure then the standby component R_4 is activated, where $X_4 = mX_3$ and $Y_4 = kY_3$ Where "k" and "m" denote the stress and strength attenuation factors respectively, such that $0 < m < 1$ and $k > 1$ Reddy (2016) [15] presents of $R = p(X > Y)$ by discussing model stress – strength of a cascade, assuming all the parameters are independent and following Weibull stress-strength distribution in one parameter and calculating first four cascade reliability for different stress-strength values. Mutkekar and Munoli (2016) [13], (1+1) exponential distribution cascade model is derived with the common effect of the force and stress reduction factors. Kumar and Vaish (2017) [12], discussed that Gompertz distribution is stress and that strength is power distribution parameters. Karam and Khaleel (2018) [9]

derived a special (2+1) stress-strength reliability cascade model for the distribution of Weibull. Khaleel and Karam (2019) [10] discussed the reliability of the (2+1) cascade inverse distribution Weibull model, reliability can be found when reverse Weibull random variables with unknown parameters scale and known shape parameter are distributed with strength-stress and used six different estimations method to estimate reliability. Karam and Khaleel (2019) [8], expression for model confidence is found when strength and stress distribution are generalized in reversed Rayleigh random variable Rayleigh, derived from mathematical formulas for Reliability to Special (2+1). Khaleel (2021) [11], (3+1) exponential distribution cascade model is derived with the common effect of the force and stress reduction factors.

Suppose, for the four units (three basic and one redundant standby), the random strength-stress variables of the four units $j = 1,2,3,4$ each independently and identically distributed of the parameter scale $\beta_i, i = 1,2,3,4$ and scale $\mu_j, j = 1,2,3,4$.

Weibull Distribution:[5]

$$f(x, \alpha, \theta) = \frac{\alpha}{\theta} x^{\alpha-1} e^{-\frac{x^\alpha}{\theta}}$$

$$, x > 0, \alpha > 0, \theta > 0$$

$$\text{Let } \beta = \frac{1}{\theta} \rightarrow f(x, \alpha, \beta)$$

$$= \alpha \beta x^{\alpha-1} e^{-\beta x^\alpha} \tag{1}$$

$$R(x) = e^{-\frac{x^\alpha}{\theta}} = e^{-\beta x^\alpha} \tag{2}$$

$$h(x) = \frac{\alpha}{\theta} x^{\alpha-1} = \alpha \beta x^{\alpha-1} \tag{3}$$

$$E(x) = \theta \left(\frac{1}{\alpha}\right) \Gamma\left(\frac{1}{\alpha} + 1\right) = \left(\frac{1}{\beta}\right) \left(\frac{1}{\alpha}\right) \Gamma\left(\frac{1}{\alpha} + 1\right) \tag{4}$$

The Cumulative distribution function of $Wei(\alpha, \beta)$ is :

$$F(x) = 1 - e^{-\frac{x^\alpha}{\theta}} = 1 - e^{-\beta x^\alpha} \quad x > 0, \alpha > 0, \beta > 0 \tag{5}$$

The Cumulative distribution function of $Wei(\alpha, \mu)$ is :

$$G(y) = 1 - e^{-\mu y^\alpha}, y > 0, \alpha > 0, \mu > 0 \tag{6}$$

Reliability Model for Weibull Distribution

(R_{Wei})

Let $X_i \sim Wei(\alpha, \beta_i); i = 1,2,3,4$ and $Y_j \sim Wei(\alpha, \mu_j); j = 1,2,3,4$ be strength and stress random variables of

the three components (three components are basic and one is standby) with unknown scale parameters β_i, μ_j and common known shape parameter α , where X_i and Y_j are independently and identically distributed Weibull random variables .

The reliability function for (3+1) cascade model

is:

$$R = P[X_1 \geq Y_1, X_2 \geq Y_2, X_3 \geq Y_3]$$

$$+ P[X_1 < Y_1, X_2 \geq Y_2, X_3 \geq Y_3, X_4 \geq Y_4]$$

$$+ P[X_1 \geq Y_1, X_2 < Y_2, X_3 \geq Y_3, X_4 \geq Y_4]$$

$$+ P[X_1 \geq Y_1, X_2 \geq Y_2, X_3 < Y_3, X_4 \geq Y_4] \tag{7}$$

$$R = R_1 + R_2 + R_3 + R_4$$

$$P_1 = P(X_1 \geq Y_1)$$

$$= \int_{y_1}^{\infty} [1 - F_1(y_1)] f(y_1) dy_1$$

$$= \int_0^{\infty} e^{-\beta_1 y_1^\alpha} \alpha \mu_1 y_1^{\alpha-1} e^{-\mu_1 y_1^\alpha} dy_1$$

$$= \int_0^{\infty} \alpha \mu_1 y_1^{\alpha-1} e^{-(\beta_1 + \mu_1) y_1^\alpha} dy_1$$

$$= \frac{-\mu_1}{\beta_1 + \mu_1} \int_0^{\infty} -(\beta_1 + \mu_1) e^{-(\beta_1 + \mu_1) y_1^\alpha} dy_1$$

$$\times \alpha y_1^{\alpha-1} e^{-(\beta_1 + \mu_1) y_1^\alpha} dy_1$$

$$= \frac{-\mu_1}{\beta_1 + \mu_1} [e^{-(\beta_1 + \mu_1) y_1^\alpha}]_0^{\infty}$$

$$= \frac{-\mu_1}{\beta_1 + \mu_1} (0 - 1)$$

$$= \frac{\mu_1}{\beta_1 + \mu_1} \tag{8}$$

$$P_2 = P(X_2 \geq Y_2)$$

$$= \int_{y_2}^{\infty} [1 - F_2(y_2)] f(y_2) dy_2$$

$$= \int_0^{\infty} e^{-\beta_2 y_2^\alpha} \alpha \mu_2 y_2^{\alpha-1} e^{-\mu_2 y_2^\alpha} dy_2$$

$$= \int_0^{\infty} \alpha \mu_2 y_2^{\alpha-1} e^{-(\beta_2 + \mu_2) y_2^\alpha} dy_2$$

$$= \frac{-\mu_2}{\beta_2 + \mu_2} \int_0^{\infty} -(\beta_2 + \mu_2) e^{-(\beta_2 + \mu_2) y_2^\alpha} dy_2$$

$$\times \alpha y_2^{\alpha-1} e^{-(\beta_2 + \mu_2) y_2^\alpha} dy_2$$

$$= \frac{-\mu_2}{\beta_2 + \mu_2} [e^{-(\beta_2 + \mu_2) y_2^\alpha}]_0^{\infty}$$

$$= \frac{-\mu_2}{\beta_2 + \mu_2} (0 - 1)$$

$$= \frac{\mu_2}{\beta_2 + \mu_2} \tag{9}$$

$$P_3 = P(X_3 \geq Y_3)$$

$$= \int_{y_3}^{\infty} [1 - F_3(y_3)] f(y_3) dy_3$$

$$= \int_0^{\infty} e^{-\beta_3 y_3^\alpha} \alpha \mu_3 y_3^{\alpha-1} e^{-\mu_3 y_3^\alpha} dy_3$$

$$= \int_0^{\infty} \alpha \mu_3 y_3^{\alpha-1} e^{-(\beta_3 + \mu_3) y_3^\alpha} dy_3$$

$$= \frac{-\mu_3}{\beta_3 + \mu_3} \int_0^{\infty} -(\beta_3 + \mu_3) e^{-(\beta_3 + \mu_3) y_3^\alpha} dy_3$$

$$\times \alpha y_3^{\alpha-1} e^{-(\beta_3 + \mu_3) y_3^\alpha} dy_3$$

$$\begin{aligned}
 &= \frac{-\mu_3}{\beta_3 + \mu_3} \left[e^{-(\beta_3 + \mu_3)y_3^\alpha} \right]_0^\infty \\
 &= \frac{-\mu_3}{\beta_3 + \mu_3} (0 - 1) \\
 &= \frac{\mu_3}{\beta_3 + \mu_3} \\
 P_{11} &= P[X_1 < Y_1, X_4 \geq Y_4] \\
 &= \int_{y_1} F_1(y_1) \left[1 - F_1\left(\frac{k}{m}y_1\right) \right] f(y_1) dy_1 \\
 &= \int_0^\infty (1 - e^{-\beta_1 y_1^\alpha}) \left[e^{-\beta_1 \left(\frac{k}{m}y_1\right)^\alpha} \right] \\
 &\times \alpha \mu_1 y_1^{\alpha-1} e^{-\mu_1 y_1^\alpha} dy_1 \\
 &= \int_0^\infty (1 - e^{-\beta_1 y_1^\alpha}) \alpha \mu_1 y_1^{\alpha-1} \\
 &\times e^{-(\beta_1 \left(\frac{k}{m}\right)^\alpha + \mu_1) y_1^\alpha} dy_1 \\
 &= \int_0^\infty \alpha \mu_1 y_1^{\alpha-1} e^{-(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1) y_1^\alpha} \\
 &- \alpha \mu_1 y_1^{\alpha-1} e^{-(\beta_1 (1 + \frac{k^\alpha}{m^\alpha}) + \mu_1) y_1^\alpha} dy_1 \\
 &= \int_0^\infty \frac{-\mu_1}{\left(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1\right)} \left[-\left(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1\right) \right] \\
 &\times \alpha y_1^{\alpha-1} e^{-(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1) y_1^\alpha} \\
 &- \frac{\mu_1}{\left(\beta_1 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_1\right)} \\
 &\times \left(\beta_1 \left(1 + \frac{k^\alpha}{m^\alpha} + \mu_1\right) \alpha y_1^{\alpha-1}\right) \\
 &\times e^{-(\beta_1 (1 + \frac{k^\alpha}{m^\alpha}) + \mu_1) y_1^\alpha} dy_1 \\
 &= \left[\frac{-\mu_1}{\left(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1\right)} e^{-(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1) y_1^\alpha} \right. \\
 &+ \frac{\mu_1}{\left(\beta_1 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_1\right)} \\
 &\times e^{-(\beta_1 (1 + \frac{k^\alpha}{m^\alpha}) + \mu_1) y_1^\alpha} \left. \right]_0^\infty \\
 &= 0 - \left[\frac{-\mu_1}{\left(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1\right)} \right. \\
 &+ \frac{\mu_1}{\left(\beta_1 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_1\right)} \left. \right] \\
 &= \frac{\mu_1}{\left(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1\right)} - \frac{\mu_1}{\left(\beta_1 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_1\right)} \\
 &= \frac{\mu_1 \beta_1}{\left(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1\right) \left(\beta_1 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_1\right)} \\
 P_{22} &= P[X_2 < Y_2, X_4 \geq Y_4] \\
 &= \int_{y_2} F_2(y_2) \left[1 - F_2\left(\frac{k}{m}y_2\right) \right] f(y_2) dy_2 \\
 &= \int_0^\infty (1 - e^{-\beta_2 y_2^\alpha}) \left[e^{-\beta_2 \left(\frac{k}{m}y_2\right)^\alpha} \right] \\
 &\times \alpha \mu_2 y_2^{\alpha-1} e^{-\mu_2 y_2^\alpha} dy_2 \\
 &= \int_0^\infty (1 - e^{-\beta_2 y_2^\alpha}) \alpha \mu_2 y_2^{\alpha-1}
 \end{aligned}$$

(10)

$$\begin{aligned}
 &\times e^{-(\beta_2 \left(\frac{k}{m}\right)^\alpha + \mu_2) y_2^\alpha} dy_2 \\
 &= \int_0^\infty \alpha \mu_2 y_2^{\alpha-1} e^{-(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2) y_2^\alpha} \\
 &- \alpha \mu_2 y_2^{\alpha-1} e^{-(\beta_2 (1 + \frac{k^\alpha}{m^\alpha}) + \mu_2) y_2^\alpha} dy_2 \\
 &= \int_0^\infty \frac{-\mu_2}{\left(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2\right)} \left[-\left(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2\right) \right] \\
 &\times \alpha y_2^{\alpha-1} e^{-(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2) y_2^\alpha} \\
 &- \frac{\mu_2}{\left(\beta_2 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_2\right)} \\
 &\times \left(\beta_2 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_2\right) \alpha y_2^{\alpha-1} \\
 &\times e^{-(\beta_2 (1 + \frac{k^\alpha}{m^\alpha}) + \mu_2) y_2^\alpha} dy_2 \\
 &= \left[\frac{-\mu_2}{\left(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2\right)} e^{-(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2) y_2^\alpha} \right. \\
 &+ \frac{\mu_2}{\left(\beta_2 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_2\right)} \\
 &\times e^{-(\beta_2 (1 + \frac{k^\alpha}{m^\alpha}) + \mu_2) y_2^\alpha} \left. \right]_0^\infty \\
 &= 0 - \left[\frac{-\mu_2}{\left(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2\right)} \right. \\
 &+ \frac{\mu_2}{\left(\beta_2 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_2\right)} \left. \right] \\
 &= \frac{\mu_2}{\left(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2\right)} - \frac{\mu_2}{\left(\beta_2 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_2\right)} \\
 &= \frac{\mu_2 \beta_2}{\left(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2\right) \left(\beta_2 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_2\right)} \tag{12}
 \end{aligned}$$

(11)

$$\begin{aligned}
 P_{33} &= P[X_3 < Y_3, X_4 \geq Y_4] \\
 &= \int_{y_3} F_3(y_3) \left[1 - F_3\left(\frac{k}{m}y_3\right) \right] f(y_3) dy_3 \\
 &= \int_0^\infty (1 - e^{-\beta_3 y_3^\alpha}) \left[e^{-\beta_3 \left(\frac{k}{m}y_3\right)^\alpha} \right] \\
 &\times \alpha \mu_3 y_3^{\alpha-1} e^{-\mu_3 y_3^\alpha} dy_3 \\
 &= \int_0^\infty (1 - e^{-\beta_3 y_3^\alpha}) \alpha \mu_3 y_3^{\alpha-1} \\
 &\times e^{-(\beta_3 \left(\frac{k}{m}\right)^\alpha + \mu_3) y_3^\alpha} dy_3 \\
 &= \int_0^\infty \alpha \mu_3 y_3^{\alpha-1} e^{-(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3) y_3^\alpha} \\
 &- \alpha \mu_3 y_3^{\alpha-1} e^{-(\beta_3 (1 + \frac{k^\alpha}{m^\alpha}) + \mu_3) y_3^\alpha} dy_3 \\
 &= \int_0^\infty \frac{-\mu_3}{\left(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3\right)} \left[-\left(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3\right) \right] \\
 &\times \alpha y_3^{\alpha-1} e^{-(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3) y_3^\alpha} \\
 &- \frac{\mu_3}{\left(\beta_3 \left(1 + \frac{k^\alpha}{m^\alpha}\right) + \mu_3\right)}
 \end{aligned}$$

$$\begin{aligned}
 & \times \left(\beta_3 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_3 \right) \alpha y_3^{\alpha-1} \\
 & \times e^{-(\beta_3(1+\frac{k^\alpha}{m^\alpha})+\mu_3)y_3^\alpha} dy_3 \\
 & = \left[\frac{-\mu_3}{\left(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3 \right)} e^{-(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3)y_3^\alpha} \right. \\
 & \left. + \frac{\mu_3}{\left(\beta_3 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_3 \right)} \right. \\
 & \times e^{-(\beta_3(1+\frac{k^\alpha}{m^\alpha})+\mu_3)y_3^\alpha} \Big]^\infty_0 \\
 & = 0 - \left[\frac{-\mu_3}{\left(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3 \right)} \right. \\
 & \left. + \frac{\mu_3}{\left(\beta_3 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_3 \right)} \right] \\
 & = \frac{\mu_3}{\left(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3 \right)} \\
 & - \frac{\mu_3}{\left(\beta_3 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_3 \right)} \\
 & = \frac{\mu_3 \beta_3}{\left(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3 \right) \left(\beta_3 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_3 \right)} \tag{13}
 \end{aligned}$$

$$\begin{aligned}
 R_1 &= P[X_1 \geq Y_1, X_2 \geq Y_2, X_3 \geq Y_3] \\
 &= P[X_1 \geq Y_1]P[X_2 \geq Y_2]P[X_3 \geq Y_3]
 \end{aligned}$$

$$\begin{aligned}
 R_1 &= P_1 P_2 P_3 \\
 \rightarrow R_1 &= \left[\frac{\mu_1}{\beta_1 + \mu_1} \right] \left[\frac{\mu_2}{\beta_2 + \mu_2} \right] \left[\frac{\mu_3}{\beta_3 + \mu_3} \right] \tag{14}
 \end{aligned}$$

$$\begin{aligned}
 R_2 &= P[X_1 < Y_1, X_2 \geq Y_2, \\
 & X_3 \geq Y_3, mX_1 \geq kY_1] \\
 &= P[X_1 < Y_1, mX_1 \geq kY_1] \\
 &\times P[X_2 \geq Y_2]P[X_3 \geq Y_3] \\
 R_2 &= P_{11} P_2 P_3
 \end{aligned}$$

$$\begin{aligned}
 &= \left[\frac{\mu_1 \beta_1}{\left(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1 \right) \left(\beta_1 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_1 \right)} \right] \\
 &\times \left[\frac{\mu_2}{\beta_2 + \mu_2} \right] \left[\frac{\mu_3}{\beta_3 + \mu_3} \right] \tag{15}
 \end{aligned}$$

$$\begin{aligned}
 R_3 &= P[X_1 \geq Y_1, X_2 < Y_2, \\
 & X_3 \geq Y_3, X_4 \geq Y_4] \\
 &= P[X_1 \geq Y_1, X_2 < Y_2, \\
 & X_3 \geq Y_3, mX_2 \geq kY_2] \\
 &= P[X_1 \geq Y_1] \\
 &\times P[X_2 < Y_2, mX_2 \geq kY_2]P[X_3 \geq Y_3] \\
 R_3 &= P_1 P_{22} P_3
 \end{aligned}$$

$$\begin{aligned}
 &= \left[\frac{\mu_1}{\beta_1 + \mu_1} \right] \\
 &\times \left[\frac{\mu_2 \beta_2}{\left(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2 \right) \left(\beta_2 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_2 \right)} \right] \\
 &\times \left[\frac{\mu_3}{\beta_3 + \mu_3} \right] \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 R_4 &= P[X_1 \geq Y_1, X_2 \geq Y_2, \\
 & X_3 < Y_3, X_4 \geq Y_4]
 \end{aligned}$$

$$\begin{aligned}
 &= P[X_1 \geq Y_1, X_2 \geq Y_2, \\
 & X_3 < Y_3, mX_3 \geq kY_3] \\
 &= P[X_1 \geq Y_1]P[X_2 \geq Y_2] \\
 &\times P[X_3 < Y_3, mX_3 \geq kY_3] \\
 R_4 &= P_1 P_2 P_{33} \\
 &= \left[\frac{\mu_1}{\beta_1 + \mu_1} \right] \left[\frac{\mu_2}{\beta_2 + \mu_2} \right] \\
 &\times \left[\frac{\mu_3 \beta_3}{\left(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3 \right) \left(\beta_3 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_3 \right)} \right] \tag{17}
 \end{aligned}$$

$$\begin{aligned}
 R &= R_1 + R_2 + R_3 + R_4 \\
 &= \left[\frac{\mu_1}{\beta_1 + \mu_1} \right] \left[\frac{\mu_2}{\beta_2 + \mu_2} \right] \left[\frac{\mu_3}{\beta_3 + \mu_3} \right] \\
 &+ \left[\frac{\mu_1 \beta_1}{\left(\beta_1 \frac{k^\alpha}{m^\alpha} + \mu_1 \right) \left(\beta_1 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_1 \right)} \right] \\
 &\times \left[\frac{\mu_2}{\beta_2 + \mu_2} \right] \left[\frac{\mu_3}{\beta_3 + \mu_3} \right] \\
 &+ \left[\frac{\mu_1}{\beta_1 + \mu_1} \right] \\
 &\times \left[\frac{\mu_2 \beta_2}{\left(\beta_2 \frac{k^\alpha}{m^\alpha} + \mu_2 \right) \left(\beta_2 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_2 \right)} \right] \\
 &\times \left[\frac{\mu_3}{\beta_3 + \mu_3} \right] \\
 &+ \left[\frac{\mu_1}{\beta_1 + \mu_1} \right] \left[\frac{\mu_2}{\beta_2 + \mu_2} \right] \\
 &\times \left[\frac{\mu_3 \beta_3}{\left(\beta_3 \frac{k^\alpha}{m^\alpha} + \mu_3 \right) \left(\beta_3 \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \mu_3 \right)} \right] \tag{18}
 \end{aligned}$$

Parameters Estimation of Weibull distribution

Maximum Likelihood Estimation Method (ML)

Suppose that a random sample $x_1, x_2, x_3, \dots, x_n$ have $W(\alpha, \beta)$ distribution with sample size n , where β is unknown scale parameter and α is known shape parameter, then the likelihood function "L", the joint probability function with the general form, can be written as follows :[2]

$$\begin{aligned}
 L(x_1, x_2, \dots, x_n, \alpha, \beta) &= f(x_1, \alpha, \beta) f(x_2, \alpha, \beta) \dots f(x_n, \alpha, \beta) \\
 &= \prod_{i=1}^n f(x_i, \alpha, \beta)
 \end{aligned}$$

Then likelihood function using equation (1) will be as:

$$\begin{aligned}
 L(x_1, x_2, \dots, x_n, \alpha, \beta) &= \prod_{i=1}^n [\alpha \beta x_i^{\alpha-1} e^{-\beta x_i^\alpha}] \\
 L(x_1, x_2, \dots, x_n, \alpha, \beta) & \tag{19}
 \end{aligned}$$

$$= \alpha^n \beta^n \left(\prod_{i=1}^n x_i \right)^{\alpha-1} e^{-\beta \sum_{i=1}^n x_i^\alpha}$$

Then natural logarithm function for equation (19) can be written as;

$$\ln L = \ln \left[\alpha^n \beta^n \left(\prod_{i=1}^n x_i \right)^{\alpha-1} e^{-\beta \sum_{i=1}^n x_i^\alpha} \right]$$

$$\ln L = n \ln \alpha + n \ln \beta + (\alpha - 1) \sum_{i=1}^n \ln x_i - \beta \sum_{i=1}^n x_i^\alpha \quad (20)$$

To minimize, natural logarithm in equation (20), must compute the great endings by taking partial derivative with respect to unknown scale parameter β , then will get as:

$$\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} - \sum_{i=1}^n x_i^\alpha \quad (21)$$

Equating partial derivative to zero, thus the right-hand side of (21) will be:

$$\rightarrow \frac{n}{\hat{\beta}} - \sum_{i=1}^n x_i^\alpha = 0 \quad (22)$$

The maximum likelihood estimator for β is given by:

$$\rightarrow \hat{\beta}_{(ML)} = \frac{n}{\sum_{i=1}^n x_i^\alpha} \quad (23)$$

In the same way above, let $y_1, y_2, y_3, \dots, y_m$ a random sample have $W(\alpha, \mu)$ distribution with the sample size m , then the maximum likelihood estimator of unknown scale parameter μ ; says $\hat{\mu}_{(ML)}$; is:

$$\hat{\mu}_{(ML)} = \frac{m}{\sum_{j=1}^m y_j^\alpha} \quad (24)$$

Now, suppose that

$$x_1 \sim W(\alpha, \beta_1), \quad x_2 \sim W(\alpha, \beta_2)$$

$$, \quad x_3 \sim W(\alpha, \beta_3) \text{ and } x_4 \sim W(\alpha, \beta_4)$$

are strengths r.v.'s with the samples sizes n_1, n_2, n_3 and n_4 respectively, where $(\beta_1, \beta_2, \beta_3, \beta_4)$ are the unknown scale parameters and suppose that $y_1 \sim W(\alpha, \mu_1), y_2 \sim W(\alpha, \mu_2), y_3 \sim W(\alpha, \mu_3)$ and $y_4(\alpha, \mu_4)$ are the stresses r.v.'s with samples sizes m_1, m_2, m_3 and m_4 respectively, where $(\mu_1, \mu_2, \mu_3, \mu_4)$ are unknown scale parameters. By using the same way, the maximum likelihood estimators $(\beta_1, \beta_2, \beta_3, \beta_4)$ and $(\mu_1, \mu_2, \mu_3, \mu_4)$ are:

$$\hat{\beta}_{\delta(ML)} = \frac{n_\delta}{\sum_{i_\delta=1}^{n_\delta} x_{i_\delta}^\alpha}, \quad \delta = 1, 2, 3, 4 \quad (25)$$

and

$$\hat{\mu}_{\delta(ML)} = \frac{m_\delta}{\sum_{j_\delta=1}^{m_\delta} y_{j_\delta}^\alpha}, \quad \delta = 1, 2, 3, 4 \quad (26)$$

Substituting (25) and (26) in (18), the maximum likelihood estimator for reliability R ; $\hat{R}_{(ML)}$; invariability will be as:

$$\begin{aligned} \hat{R}_{W(ML)} &= \hat{R}_{1(ML)} + \hat{R}_{2(ML)} \\ &+ \hat{R}_{3(ML)} + \hat{R}_{4(ML)} \\ &= \left[\frac{\hat{\mu}_{1(ML)}}{\hat{\beta}_{1(ML)} + \hat{\mu}_{1(ML)}} \right] \\ &\times \left[\frac{\hat{\mu}_{2(ML)}}{\hat{\beta}_{2(ML)} + \hat{\mu}_{2(ML)}} \right] \left[\frac{\hat{\mu}_{3(ML)}}{\hat{\beta}_{3(ML)} + \hat{\mu}_{3(ML)}} \right] \\ &+ \left[\frac{\hat{\mu}_{1(ML)} \hat{\beta}_{1(ML)}}{\left(\hat{\beta}_{1(ML)} \frac{k^\alpha}{m^\alpha} + \hat{\mu}_{1(ML)} \right) \left(\hat{\beta}_{1(ML)} \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \hat{\mu}_{1(ML)} \right)} \right] \\ &\times \left[\frac{\hat{\mu}_{2(ML)}}{\hat{\beta}_{2(ML)} + \hat{\mu}_{2(ML)}} \right] \left[\frac{\hat{\mu}_{3(ML)}}{\hat{\beta}_{3(ML)} + \hat{\mu}_{3(ML)}} \right] \\ &+ \left[\frac{\hat{\mu}_{1(ML)}}{\hat{\beta}_{1(ML)} + \hat{\mu}_{1(ML)}} \right] \\ &\times \left[\frac{\hat{\mu}_{2(ML)} \hat{\beta}_{2(ML)}}{\left(\hat{\beta}_{2(ML)} \frac{k^\alpha}{m^\alpha} + \hat{\mu}_{2(ML)} \right) \left(\hat{\beta}_{2(ML)} \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \hat{\mu}_{2(ML)} \right)} \right] \\ &\times \left[\frac{\hat{\mu}_{3(ML)}}{\hat{\beta}_{3(ML)} + \hat{\mu}_{3(ML)}} \right] \\ &+ \left[\frac{\hat{\mu}_{1(ML)}}{\hat{\beta}_{1(ML)} + \hat{\mu}_{1(ML)}} \right] \left[\frac{\hat{\mu}_{2(ML)}}{\hat{\beta}_{2(ML)} + \hat{\mu}_{2(ML)}} \right] \\ &\times \left[\frac{\hat{\mu}_{3(ML)} \hat{\beta}_{3(ML)}}{\left(\hat{\beta}_{3(ML)} \frac{k^\alpha}{m^\alpha} + \hat{\mu}_{3(ML)} \right) \left(\hat{\beta}_{3(ML)} \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \hat{\mu}_{3(ML)} \right)} \right] \quad (27) \end{aligned}$$

3.2 Moments Estimation Method (Mo):

To derive method of the moments estimator parameters of WD, assume that $x_i, i = 1, 2, 3, \dots, n$ random sample have $W(\alpha, \beta)$ distribution with the sample size n , first step the mean population of $W(\alpha, \beta)$, obtain by equation (4):[4]

$$E(x) = \frac{1}{\beta^\alpha} \Gamma \left(1 + \frac{1}{\alpha} \right) \quad (28)$$

The second step equating mean sample with corresponding the mean population, then will get as:

$$\frac{\sum_{i=1}^n x_i}{n} = \frac{1}{\beta^\alpha} \Gamma \left(1 + \frac{1}{\alpha} \right) \quad (29)$$

Then the moment estimator of β says $\hat{\beta}_{(Mo)}$ is:

$$\hat{\beta}_{(Mo)} = \left[\frac{\Gamma \left(1 + \frac{1}{\alpha} \right)}{\bar{x}} \right]^\alpha \quad (30)$$

In the same manner, the moments estimator of unknown scale parameter μ ; says $\hat{\mu}_{(MO)}$; is:

$$\hat{\mu}_{(MO)} = \left[\frac{\Gamma\left(1 + \frac{1}{\alpha}\right)}{\bar{y}} \right]^\alpha \tag{31}$$

Now, by using the same technique, the moments estimators of the unknown scale parameters $(\beta_1, \beta_2, \beta_3)$ and (μ_1, μ_2, μ_3) are:

$$\hat{\beta}_{\delta(MO)} = \left[\frac{\Gamma\left(1 + \frac{1}{\alpha}\right)}{\bar{X}_\delta} \right]^\alpha, \delta = 1,2,3,4 \tag{32}$$

and

$$\hat{\mu}_{\delta(MO)} = \left[\frac{\Gamma\left(1 + \frac{1}{\alpha}\right)}{\bar{y}_\delta} \right]^\alpha, \delta = 1,2,3,4 \tag{33}$$

Substitution (32) and (33) in (18), the moments estimator for reliability R_W ; says $\hat{R}_{W(MO)}$; approximately will be as:

$$\begin{aligned} \hat{R}_{W(MO)} &= \hat{R}_{1(MO)} + \hat{R}_{2(MO)} \\ &+ \hat{R}_{3(MO)} + \hat{R}_{4(MO)} \\ &= \left[\frac{\hat{\mu}_{1(MO)}}{\hat{\beta}_{1(MO)} + \hat{\mu}_{1(MO)}} \right] \left[\frac{\hat{\mu}_{2(MO)}}{\hat{\beta}_{2(MO)} + \hat{\mu}_{2(MO)}} \right] \\ &\times \left[\frac{\hat{\mu}_{3(MO)}}{\hat{\beta}_{3(MO)} + \hat{\mu}_{3(MO)}} \right] \\ &+ \left[\frac{\hat{\mu}_{1(MO)}\hat{\beta}_{1(MO)}}{\left(\hat{\beta}_{1(MO)}\frac{k^\alpha}{m^\alpha} + \hat{\mu}_{1(MO)}\right)\left(\hat{\beta}_{1(MO)}\left(1 + \frac{k^\alpha}{m^\alpha}\right) + \hat{\mu}_{1(MO)}\right)} \right] \\ &\times \left[\frac{\hat{\mu}_{2(MO)}}{\hat{\beta}_{2(MO)} + \hat{\mu}_{2(MO)}} \right] \left[\frac{\hat{\mu}_{3(MO)}}{\hat{\beta}_{3(MO)} + \hat{\mu}_{3(MO)}} \right] \\ &+ \left[\frac{\hat{\mu}_{1(MO)}}{\hat{\beta}_{1(MO)} + \hat{\mu}_{1(MO)}} \right] \\ &\times \left[\frac{\hat{\mu}_{2(MO)}\hat{\beta}_{2(MO)}}{\left(\hat{\beta}_{2(MO)}\frac{k^\alpha}{m^\alpha} + \hat{\mu}_{2(MO)}\right)\left(\hat{\beta}_{2(MO)}\left(1 + \frac{k^\alpha}{m^\alpha}\right) + \hat{\mu}_{2(MO)}\right)} \right] \\ &\times \left[\frac{\hat{\mu}_{3(MO)}}{\hat{\beta}_{3(MO)} + \hat{\mu}_{3(MO)}} \right] \\ &+ \left[\frac{\hat{\mu}_{1(MO)}}{\hat{\beta}_{1(MO)} + \hat{\mu}_{1(MO)}} \right] \left[\frac{\hat{\mu}_{2(MO)}}{\hat{\beta}_{2(MO)} + \hat{\mu}_{2(MO)}} \right] \\ &\times \left[\frac{\hat{\mu}_{3(MO)}\hat{\beta}_{3(MO)}}{\left(\hat{\beta}_{3(MO)}\frac{k^\alpha}{m^\alpha} + \hat{\mu}_{3(MO)}\right)\left(\hat{\beta}_{3(MO)}\left(1 + \frac{k^\alpha}{m^\alpha}\right) + \hat{\mu}_{3(MO)}\right)} \right] \end{aligned} \tag{34}$$

3.3 Least Squares Estimation Method (LS):

In this method, used the minimize equation to reduce the non-parametric (\hat{F}) and parametric (F) distribution functions.[1]

The minimizing following equation: [6]

$$S = \sum_{i=1}^n \left(\hat{F}(x_i) - F(x_i) \right)^2 \tag{35}$$

Suppose that $X_1, X_2, X_3, \dots, X_n$ be a random sample have $W(\alpha, \beta)$ distribution with the

sample size n . The procedure attempts to minimize the following function with respect to α and β will get as:

$$S(\alpha, \beta) = \sum_{i=1}^n \left(\hat{F}(x_i) - (1 - e^{-\beta x_i^\alpha}) \right)^2 \tag{36}$$

To obtain the formula of $F(x_i)$; use the equation (5)

$$F(x_i) = 1 - e^{-\beta x_i^\alpha} - \ln(1 - F(x_i)) = \beta x_i^\alpha \tag{37}$$

On the other hand, since $\hat{F}(x_i)$ is unknown, it better to use $\hat{F}(x_{(i)})$ as follows $\hat{F}(x_{(i)}) = P_i$ and P_i is the plotting position where:

$$P_i = \frac{i}{n+1}; i = 1,2, \dots, n \tag{38}$$

Here $x_{(i)}$ is the i : the order statistics of the random sample of the size n from WD.

Hence for the WD, to obtain the LS estimates $\hat{\beta}$ of the parameter β can be define following the function from equation (36):

$$S(\alpha, \beta) = \sum_{i=1}^n (q_i - \beta x_{(i)}^\alpha)^2 \tag{39}$$

$$\begin{aligned} \text{Where } q_i &= -\ln(1 - \hat{F}(x_{(i)})) \\ &= -\ln(1 - p_i) \end{aligned}$$

By taking the derivative equation (39) with respect to the parameter β and equating result to zero:

$$\begin{aligned} \frac{\partial S(\alpha, \beta)}{\partial \beta} &= \sum_{i=1}^n 2(q_i - \beta x_{(i)}^\alpha)(-x_{(i)}^\alpha) - \sum_{i=1}^n q_i x_{(i)}^\alpha \\ &+ \beta \sum_{i=1}^n x_{(i)}^{2\alpha} = 0 \end{aligned} \tag{40}$$

Then the least squares estimator of β ; says $\hat{\beta}_{(LS)}$, will get as:

$$\hat{\beta}_{(LS)} = \frac{\sum_{i=1}^n q_i x_{(i)}^\alpha}{\sum_{i=1}^n x_{(i)}^{2\alpha}} \tag{41}$$

In the same way, the least squares estimator of unknown parameter μ ; says $\hat{\mu}_{(LS)}$; is:

$$\hat{\mu}_{(LS)} = \frac{\sum_{j=1}^m q_j y_{(j)}^\alpha}{\sum_{j=1}^m y_{(j)}^{2\alpha}} \tag{42}$$

Where $\hat{G}(y_{(j)}) = \frac{j}{m+1}; j = 1,2, \dots, m$ and

$$q_j = -\ln(1 - \hat{G}(y_{(j)})) = -\ln(1 - P_j)$$

Now, by using the same way, the last squares estimator of the unknown scale parameters $(\beta_1, \beta_2, \beta_3)$ and (μ_1, μ_2, μ_3) are:

$$\hat{\beta}_{\delta(LS)} = \frac{\sum_{i_\delta=1}^{n_\delta} q_{i_\delta} x_{\delta(i_\delta)}^\alpha}{\sum_{i_\delta=1}^{n_\delta} x_{\delta(i_\delta)}^{2\alpha}}, \delta = 1,2,3,4 \tag{43}$$

and

$$\hat{\mu}_{\delta(LS)} = \frac{\sum_{j_{\delta}=1}^{m_{\delta}} q_{j_{\delta}} y_{\delta(j_{\delta})}^{\alpha}}{\sum_{j_{\delta}=1}^{m_{\delta}} y_{\delta(j_{\delta})}^{2\alpha}}, \delta = 1,2,3,4 \quad (44)$$

Substitution (43) and (44) in (18), the last squares estimator for reliability R_W says $\hat{R}_{W(LS)}$; approximately will be as:

$$\begin{aligned} \hat{R}_{(LS)} &= \hat{R}_{1(LS)} + \hat{R}_{2(LS)} \\ &+ \hat{R}_{3(LS)} + \hat{R}_{4(LS)} \\ &= \left[\frac{\hat{\mu}_{1(LS)}}{\hat{\beta}_{1(LS)} + \hat{\mu}_{1(LS)}} \right] \left[\frac{\hat{\mu}_{2(LS)}}{\hat{\beta}_{2(LS)} + \hat{\mu}_{2(LS)}} \right] \\ &\times \left[\frac{\hat{\mu}_{3(LS)}}{\hat{\beta}_{3(LS)} + \hat{\mu}_{3(LS)}} \right] \\ &+ \left[\frac{\hat{\mu}_{1(LS)} \hat{\beta}_{1(LS)}}{\left(\hat{\beta}_{1(LS)} \frac{k^{\alpha}}{m^{\alpha}} + \hat{\mu}_{1(LS)} \right) \left(\hat{\beta}_{1(LS)} \left(1 + \frac{k^{\alpha}}{m^{\alpha}} \right) + \hat{\mu}_{1(LS)} \right)} \right] \\ &\times \left[\frac{\hat{\mu}_{2(LS)}}{\hat{\beta}_{2(LS)} + \hat{\mu}_{2(LS)}} \right] \left[\frac{\hat{\mu}_{3(LS)}}{\hat{\beta}_{3(LS)} + \hat{\mu}_{3(LS)}} \right] \\ &+ \left[\frac{\hat{\mu}_{1(LS)}}{\hat{\beta}_{1(LS)} + \hat{\mu}_{1(LS)}} \right] \\ &\times \left[\frac{\hat{\mu}_{2(LS)} \hat{\beta}_{2(LS)}}{\left(\hat{\beta}_{2(LS)} \frac{k^{\alpha}}{m^{\alpha}} + \hat{\mu}_{2(LS)} \right) \left(\hat{\beta}_{2(LS)} \left(1 + \frac{k^{\alpha}}{m^{\alpha}} \right) + \hat{\mu}_{2(LS)} \right)} \right] \\ &\times \left[\frac{\hat{\mu}_{3(LS)}}{\hat{\beta}_{3(LS)} + \hat{\mu}_{3(LS)}} \right] \\ &+ \left[\frac{\hat{\mu}_{1(LS)}}{\hat{\beta}_{1(LS)} + \hat{\mu}_{1(LS)}} \right] \left[\frac{\hat{\mu}_{2(LS)}}{\hat{\beta}_{2(LS)} + \hat{\mu}_{2(LS)}} \right] \\ &\times \left[\frac{\hat{\mu}_{3(LS)} \hat{\beta}_{3(LS)}}{\left(\hat{\beta}_{3(LS)} \frac{k^{\alpha}}{m^{\alpha}} + \hat{\mu}_{3(LS)} \right) \left(\hat{\beta}_{3(LS)} \left(1 + \frac{k^{\alpha}}{m^{\alpha}} \right) + \hat{\mu}_{3(LS)} \right)} \right] \quad (45) \end{aligned}$$

Weighted Least Squares Estimation Method (WLS)

This method reflects the behavior of random errors in the model and it can be used with the functions that are either linear or nonlinear in parameters. It works by incorporating extra nonnegative weights or constants associated with all data point into the fitting criterion. The size of weight shows the precision of the information contained in associated observation. The method of weighted last squares can be used in minimizing the following equation [3]:

$$Q = \sum_{i=1}^n W_i \left(\hat{F}(x_i) - F(x_i) \right)^2 \quad (46)$$

Where:

$$\begin{aligned} W_i &= \frac{1}{\text{Var}[F(X_{(i)})]} \\ &= \frac{(n+1)^2(n+2)}{i(n-i+1)}, i = 1,2, \dots, n \end{aligned} \quad (47)$$

Let a random sample $(x_1, x_2, x_3, \dots, x_n)$ size n take from have $W(\alpha, \beta)$ distribution. The procedure attempts to minimize the following function with respect to α and β will get as:

$$Q(\alpha, \beta) = \sum_{i=1}^n W_i \left(\hat{F}(x_i) - (1 - e^{-\beta x_i^{\alpha}}) \right)^2 \quad (48)$$

As steps in equations (37) and (39) will get as:

$$Q(\alpha, \beta) = \sum_{i=1}^n W_i (q_i - \beta x_{(i)}^{\alpha})^2 \quad (49)$$

By taking partial derivative to the equation (49) with respect to β , and equating result to the zero we obtain:

$$\begin{aligned} \frac{\partial Q(\alpha, \beta)}{\partial \beta} &= \sum_{i=1}^n 2W_i (q_i - \beta x_{(i)}^{\alpha}) (-x_{(i)}^{\alpha}) \\ &\rightarrow - \sum_{i=1}^n W_i q_i x_{(i)}^{\alpha} + \hat{\beta} \sum_{i=1}^n W_i x_{(i)}^{2\alpha} = 0 \end{aligned} \quad (50)$$

The weighted least square estimator of β ; says $\hat{\beta}_{(WLS)}$:

$$\hat{\beta}_{(WLS)} = \frac{\sum_{i=1}^n W_i q_i x_{(i)}^{\alpha}}{\sum_{i=1}^n W_i x_{(i)}^{2\alpha}} \quad (51)$$

In the same technique, the weighted least squares estimator of unknown scale parameter μ ; says $\hat{\mu}_{(WLS)}$; is:

$$\hat{\mu}_{(WLS)} = \frac{\sum_{j=1}^m W_j q_j y_{(j)}^{\alpha}}{\sum_{j=1}^m W_j y_{(j)}^{2\alpha}}$$

$$\begin{aligned} \text{Where } W_j &= \frac{1}{\text{Var}[G(y_{(j)})]} \\ &= \frac{(m+1)^2(m+2)}{j(m-j+1)}, j = 1,2, \dots, m \end{aligned} \quad (52)$$

Now, by using the same way, the weighted least squares estimators of the unknown scale parameters $(\beta_1, \beta_2, \beta_3)$ and (μ_1, μ_2, μ_3) are:

$$\hat{\beta}_{\delta(WLS)} = \frac{\sum_{i_{\delta}=1}^{n_{\delta}} W_{i_{\delta}} q_{i_{\delta}} x_{\delta(i_{\delta})}^{\alpha}}{\sum_{i_{\delta}=1}^{n_{\delta}} W_{i_{\delta}} x_{\delta(i_{\delta})}^{2\alpha}}, \delta = 1,2,3,4 \quad (53)$$

and

$$\hat{\mu}_{\delta(WLS)} = \frac{\sum_{j_{\delta}=1}^{m_{\delta}} W_{j_{\delta}} q_{j_{\delta}} y_{\delta(j_{\delta})}^{\alpha}}{\sum_{j_{\delta}=1}^{m_{\delta}} W_{j_{\delta}} y_{\delta(j_{\delta})}^{2\alpha}}, \delta = 1,2,3,4 \quad (54)$$

Substitution (53) and (54) in (18), the weighted least squares estimator for reliability R_W ; says $\hat{R}_{W(WLS)}$; approximately will be as:

$$\begin{aligned} \hat{R}_{W(WLS)} &= \hat{R}_{1(WLS)} + \hat{R}_{2(WLS)} + \hat{R}_{3(WLS)} + \hat{R}_{4(WLS)} \\ &= \left[\frac{\hat{\mu}_{1(WLS)}}{\hat{\beta}_{1(WLS)} + \hat{\mu}_{1(WLS)}} \right] \\ &\times \left[\frac{\hat{\mu}_{2(WLS)}}{\hat{\beta}_{2(WLS)} + \hat{\mu}_{2(WLS)}} \right] \left[\frac{\hat{\mu}_{3(WLS)}}{\hat{\beta}_{3(WLS)} + \hat{\mu}_{3(WLS)}} \right] \end{aligned}$$

$$\begin{aligned}
 & + \left[\frac{\hat{\mu}_1(WLS)\hat{\beta}_1(WLS)}{\left(\hat{\beta}_1(WLS)\frac{k^\alpha}{m^\alpha} + \hat{\mu}_1(WLS)\right)\left(\hat{\beta}_1(WLS)\left(1 + \frac{k^\alpha}{m^\alpha}\right) + \hat{\mu}_1(WLS)\right)} \right] \\
 & \times \left[\frac{\hat{\mu}_2(WLS)}{\hat{\beta}_2(WLS) + \hat{\mu}_2(WLS)} \right] \left[\frac{\hat{\mu}_3(WLS)}{\hat{\beta}_3(WLS) + \hat{\mu}_3(WLS)} \right] \\
 & + \left[\frac{\hat{\mu}_1(WLS)}{\hat{\beta}_1(WLS) + \hat{\mu}_1(WLS)} \right] \\
 & \times \left[\frac{\hat{\mu}_2(WLS)\hat{\beta}_2(WLS)}{\left(\hat{\beta}_2(WLS)\frac{k^\alpha}{m^\alpha} + \hat{\mu}_2(WLS)\right)\left(\hat{\beta}_2(WLS)\left(1 + \frac{k^\alpha}{m^\alpha}\right) + \hat{\mu}_2(WLS)\right)} \right] \\
 & \times \left[\frac{\hat{\mu}_3(WLS)}{\hat{\beta}_3(WLS) + \hat{\mu}_3(WLS)} \right] \\
 & + \left[\frac{\hat{\mu}_1(WLS)}{\hat{\beta}_1(WLS) + \hat{\mu}_1(WLS)} \right] \left[\frac{\hat{\mu}_2(WLS)}{\hat{\beta}_2(WLS) + \hat{\mu}_2(WLS)} \right] \\
 & \times \left[\frac{\hat{\mu}_3(WLS)\hat{\beta}_3(WLS)}{\left(\hat{\beta}_3(WLS)\frac{k^\alpha}{m^\alpha} + \hat{\mu}_3(WLS)\right)\left(\hat{\beta}_3(WLS)\left(1 + \frac{k^\alpha}{m^\alpha}\right) + \hat{\mu}_3(WLS)\right)} \right] \quad (55)
 \end{aligned}$$

Regression Estimation Method (Rg):

Regression is conceptually the simple method for examining functional relations among variables. The relations is expressed in form of an equation or the model connecting the response variable "Y" and one "X" or more expository variables. The simple true relations can be approximated by the standard regression equation:

$$z_i = a + b\mu_i + e_i \quad (56)$$

Where (z_i) is the dependent variable, (u_i) is the independent variable and (e_i) is error random variable independent.

Assume that x_1, x_2, \dots, x_n random samples have $W(\alpha, \beta)$ with the sample size n .

Taking the natural logarithm to CDF [14], obtain by equation (5):

$$\begin{aligned}
 F(x_i) &= 1 - e^{-\beta x_i^\alpha} \\
 (1 - F(x_i))^{-1} &= e^{\beta x_i^\alpha} \\
 Ln[(1 - F(x_i))^{-1}] &= \beta x_i^\alpha
 \end{aligned}$$

Estimating $F(x_{(i)})$ by P_i in equation (38)

$$Ln[(1 - P_i)^{-1}] = \beta x_{(i)}^\alpha \quad (57)$$

Comparing the equation (57) with equation (56), we get:

$$z_i = Ln[(1 - P_i)^{-1}] \quad , a = 0, b = \beta, u_i = x_{(i)}^\alpha \quad (58)$$

Where; $i = 1, 2, \dots, n$

Where b can be estimated by the minimizing summation of the squared error with respect to b , then we get :

$$\hat{b} = \frac{n \sum_{i=1}^n z_i u_i - \sum_{i=1}^n z_i \sum_{i=1}^n u_i}{n \sum_{i=1}^n (u_i)^2 - (\sum_{i=1}^n u_i)^2} \quad (59)$$

By substitution (58) in (59) , the estimator for β ; says $\hat{\beta}_{(Rg)}$; is :

$$\begin{aligned}
 \hat{\beta}_{(Rg)} &= (n \sum_{i=1}^n x_{(i)}^\alpha Ln[(1 - P_i)^{-1}] \\
 & - \sum_{i=1}^n x_{(i)}^\alpha \sum_{i=1}^n Ln[(1 - P_i)^{-1}]) \\
 & \div (n \sum_{i=1}^n [x_{(i)}^\alpha]^2 - \left[\sum_{i=1}^n x_{(i)}^\alpha \right]^2) \quad (60)
 \end{aligned}$$

In the same way, the regression estimator of unknown scale parameter μ ; says $\hat{\mu}_{(Rg)}$; is:

$$\begin{aligned}
 \hat{\mu}_{(Rg)} &= (m \sum_{j=1}^m y_{(j)}^\alpha Ln[(1 - P_j)^{-1}] \\
 & - \sum_{j=1}^m y_{(j)}^\alpha \sum_{j=1}^m Ln[(1 - P_j)^{-1}]) \\
 & \div (m \sum_{j=1}^m [y_{(j)}^\alpha]^2 - \left[\sum_{j=1}^m y_{(j)}^\alpha \right]^2) \quad (61)
 \end{aligned}$$

As in equation (58) where:

$$\begin{aligned}
 z_j &= Ln[(1 - P_j)^{-1}] \\
 , a &= 0, b = \mu, u_j = y_{(j)}^\alpha \\
 ; j &= 1, 2, \dots, m
 \end{aligned}$$

Now, by using the same way above, the regression estimators of the unknown scale parameters $(\beta_1, \beta_2, \beta_3)$ and (μ_1, μ_2, μ_3) are:

$$\begin{aligned}
 \hat{\beta}_{\delta(Rg)} &= (n_\delta \sum_{i_\delta=1}^{n_\delta} x_{\delta(i_\delta)}^\alpha Ln[(1 - P_{i_\delta})^{-1}] \\
 & - \sum_{i_\delta=1}^{n_\delta} x_{\delta(i_\delta)}^\alpha \sum_{i_\delta=1}^{n_\delta} Ln[(1 - P_{i_\delta})^{-1}]) \\
 & \div (n_\delta \sum_{i_\delta=1}^{n_\delta} [x_{\delta(i_\delta)}^\alpha]^2 - \left[\sum_{i_\delta=1}^{n_\delta} x_{\delta(i_\delta)}^\alpha \right]^2) \\
 , \delta &= 1, 2, 3, 4 \quad (62)
 \end{aligned}$$

and

$$\begin{aligned}
 \hat{\mu}_{\delta(Rg)} &= (m_\delta \sum_{j_\delta=1}^{m_\delta} y_{\delta(j_\delta)}^\alpha Ln[(1 - P_{j_\delta})^{-1}] \\
 & - \sum_{j_\delta=1}^{m_\delta} y_{\delta(j_\delta)}^\alpha \sum_{j_\delta=1}^{m_\delta} Ln[(1 - P_{j_\delta})^{-1}]) \\
 & \div (m_\delta \sum_{j_\delta=1}^{m_\delta} [y_{\delta(j_\delta)}^\alpha]^2 - \left[\sum_{j_\delta=1}^{m_\delta} y_{\delta(j_\delta)}^\alpha \right]^2) \\
 , \delta &= 1, 2, 3, 4 \quad (63)
 \end{aligned}$$

Substitution (62) and (63) in (18), the regression estimator for reliability R_W ; says $\hat{R}_{W(Rg)}$; approximately will be as:

$$\begin{aligned}
 \hat{R}_{W(Rg)} &= \hat{R}_{1(Rg)} + \hat{R}_{2(Rg)} \\
 &+ \hat{R}_{3(Rg)} + \hat{R}_{4(Rg)} \\
 &= \left[\frac{\hat{\mu}_{1(Rg)}}{\hat{\beta}_{1(Rg)} + \hat{\mu}_{1(Rg)}} \right] \left[\frac{\hat{\mu}_{2(Rg)}}{\hat{\beta}_{2(Rg)} + \hat{\mu}_{2(Rg)}} \right] \\
 &\times \left[\frac{\hat{\mu}_{3(Rg)}}{\hat{\beta}_{3(Rg)} + \hat{\mu}_{3(Rg)}} \right] \\
 &+ \left[\frac{\hat{\mu}_{1(Rg)} \hat{\beta}_{1(Rg)}}{\left(\frac{\hat{\beta}_{1(Rg)} k^\alpha}{m^\alpha} + \hat{\mu}_{1(Rg)} \right) \left(\hat{\beta}_{1(Rg)} \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \hat{\mu}_{1(Rg)} \right)} \right] \\
 &\times \left[\frac{\hat{\mu}_{2(Rg)}}{\hat{\beta}_{2(Rg)} + \hat{\mu}_{2(Rg)}} \right] \left[\frac{\hat{\mu}_{3(Rg)}}{\hat{\beta}_{3(Rg)} + \hat{\mu}_{3(Rg)}} \right] \\
 &+ \left[\frac{\hat{\mu}_{1(Rg)}}{\hat{\beta}_{1(Rg)} + \hat{\mu}_{1(Rg)}} \right] \\
 &\times \left[\frac{\hat{\mu}_{2(Rg)} \hat{\beta}_{2(Rg)}}{\left(\frac{\hat{\beta}_{2(Rg)} k^\alpha}{m^\alpha} + \hat{\mu}_{2(Rg)} \right) \left(\hat{\beta}_{2(Rg)} \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \hat{\mu}_{2(Rg)} \right)} \right] \\
 &\times \left[\frac{\hat{\mu}_{3(Rg)}}{\hat{\beta}_{3(Rg)} + \hat{\mu}_{3(Rg)}} \right] \\
 &+ \left[\frac{\hat{\mu}_{1(Rg)}}{\hat{\beta}_{1(Rg)} + \hat{\mu}_{1(Rg)}} \right] \left[\frac{\hat{\mu}_{2(Rg)}}{\hat{\beta}_{2(Rg)} + \hat{\mu}_{2(Rg)}} \right] \\
 &\times \left[\frac{\hat{\mu}_{3(Rg)} \hat{\beta}_{3(Rg)}}{\left(\frac{\hat{\beta}_{3(Rg)} k^\alpha}{m^\alpha} + \hat{\mu}_{3(Rg)} \right) \left(\hat{\beta}_{3(Rg)} \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \hat{\mu}_{3(Rg)} \right)} \right] \quad (64)
 \end{aligned}$$

Percentile Estimation Method (Pr)

The method was originally discovered by Kao (1958 – 1959). In case of Weibull distribution, let a random sample $X_i ; i = 1, 2, 3, \dots, n$ with size n have $W(\alpha, \beta)$, it is possible to use this method to obtain the estimator unknown scale parameter β , which is obtain from the CDF, defined in equation (5) [7]:

$$\begin{aligned}
 F(x_i) &= 1 - e^{-\beta x_i^\alpha} \\
 \ln(1 - F(x_i)) &= -\beta x_i^\alpha \\
 x_i &= \left(\frac{-\ln(1 - F(x_i))}{\beta} \right)^{\frac{1}{\alpha}} \quad (65)
 \end{aligned}$$

If $P_i ; i = 1, 2, \dots, n$ put the plotting position instead of $F(x_i ; \alpha, \beta)$. Can be obtained by 99 minimizing

$$\sum_{i=1}^n \left[x_{(i)} - \left(\frac{-\ln(1 - p_i)}{\beta} \right)^{\frac{1}{\alpha}} \right]^2 \quad (66)$$

By taking partial derivative to the (66) with respect to β , and equating the result to zero we obtain:

$$\begin{aligned}
 &\sum_{i=1}^n 2 \left[(x_{(i)}) - (\hat{\beta})^{-\frac{1}{\alpha}} (-\ln(1 - p_i))^{\frac{1}{\alpha}} \right] \\
 &\times \left(\frac{1}{\alpha} (\hat{\beta})^{-\left(\frac{1}{\alpha} + 1\right)} \right) (-\ln(1 - p_i))^{\frac{1}{\alpha}} = 0
 \end{aligned}$$

The percentile estimator of β ; says $\hat{\beta}$ becomes:

$$\hat{\beta}_{(pr)} = \left[\frac{\sum_{i=1}^n (-\ln(1 - p_i))^{\frac{2}{\alpha}}}{\sum_{i=1}^n (x_{(i)}) (-\ln(1 - p_i))^{\frac{1}{\alpha}}} \right]^\alpha \quad (67)$$

In the name way above, the percentile estimator of the unknown parameter $\hat{\beta}$; says $\hat{\mu}$ is:

$$\hat{\mu}_{(pr)} = \left[\frac{\sum_{j=1}^m (-\ln(1 - p_j))^{\frac{2}{\alpha}}}{\sum_{j=1}^m (y_{(j)}) (-\ln(1 - p_j))^{\frac{1}{\alpha}}} \right]^\alpha \quad (68)$$

Now, by using the same manner, the percentile estimators of the unknown scale parameter $(\beta_1, \beta_2, \beta_3)$ and $(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$ are:

$$\hat{\beta}_{\delta(pr)} = \left[\frac{\sum_{i_{\delta=1}}^n (-\ln(1 - p_{i_{\delta}}))^{\frac{2}{\alpha}}}{\sum_{i_{\delta=1}}^n (x_{(i_{\delta})}) (-\ln(1 - p_{i_{\delta}}))^{\frac{1}{\alpha}}} \right]^\alpha, \delta = 1, 2, 3, 4 \quad (69)$$

and

$$\hat{\mu}_{\delta(pr)} = \left[\frac{\sum_{j_{\delta=1}}^m (-\ln(1 - p_{j_{\delta}}))^{\frac{2}{\alpha}}}{\sum_{j_{\delta=1}}^m (y_{(j_{\delta})}) (-\ln(1 - p_{j_{\delta}}))^{\frac{1}{\alpha}}} \right]^\alpha, \delta = 1, 2, 3, 4 \quad (70)$$

Substitution (69) and (70) in (18), the Percentile estimator for reliability R_W ; says $\hat{R}_{W(Pr)}$; approximately will be as:

$$\begin{aligned}
 \hat{R}_{(Pr)} &= \hat{R}_{1(Pr)} + \hat{R}_{2(Pr)} \\
 &+ \hat{R}_{3(Pr)} + \hat{R}_{4(Pr)} \\
 &= \left[\frac{\hat{\mu}_{1(Pr)}}{\hat{\beta}_{1(Pr)} + \hat{\mu}_{1(Pr)}} \right] \left[\frac{\hat{\mu}_{2(Pr)}}{\hat{\beta}_{2(Pr)} + \hat{\mu}_{2(Pr)}} \right] \\
 &\times \left[\frac{\hat{\mu}_{3(Pr)}}{\hat{\beta}_{3(Pr)} + \hat{\mu}_{3(Pr)}} \right] \\
 &+ \left[\frac{\hat{\mu}_{1(Pr)} \hat{\beta}_{1(Pr)}}{\left(\frac{\hat{\beta}_{1(Pr)} k^\alpha}{m^\alpha} + \hat{\mu}_{1(Pr)} \right) \left(\hat{\beta}_{1(Pr)} \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \hat{\mu}_{1(Pr)} \right)} \right] \\
 &\times \left[\frac{\hat{\mu}_{2(Pr)}}{\hat{\beta}_{2(Pr)} + \hat{\mu}_{2(Pr)}} \right] \left[\frac{\hat{\mu}_{3(Pr)}}{\hat{\beta}_{3(Pr)} + \hat{\mu}_{3(Pr)}} \right] \\
 &+ \left[\frac{\hat{\mu}_{1(Pr)}}{\hat{\beta}_{1(Pr)} + \hat{\mu}_{1(Pr)}} \right] \\
 &\times \left[\frac{\hat{\mu}_{2(Pr)} \hat{\beta}_{2(Pr)}}{\left(\frac{\hat{\beta}_{2(Pr)} k^\alpha}{m^\alpha} + \hat{\mu}_{2(Pr)} \right) \left(\hat{\beta}_{2(Pr)} \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \hat{\mu}_{2(Pr)} \right)} \right] \\
 &\times \left[\frac{\hat{\mu}_{3(Pr)}}{\hat{\beta}_{3(Pr)} + \hat{\mu}_{3(Pr)}} \right] \\
 &+ \left[\frac{\hat{\mu}_{1(Pr)}}{\hat{\beta}_{1(Pr)} + \hat{\mu}_{1(Pr)}} \right] \left[\frac{\hat{\mu}_{2(Pr)}}{\hat{\beta}_{2(Pr)} + \hat{\mu}_{2(Pr)}} \right] \\
 &\times \left[\frac{\hat{\mu}_{3(Pr)} \hat{\beta}_{3(Pr)}}{\left(\frac{\hat{\beta}_{3(Pr)} k^\alpha}{m^\alpha} + \hat{\mu}_{3(Pr)} \right) \left(\hat{\beta}_{3(Pr)} \left(1 + \frac{k^\alpha}{m^\alpha} \right) + \hat{\mu}_{3(Pr)} \right)} \right] \quad (71)
 \end{aligned}$$

THE EXPERIMENTAL STUDY

We simulate the outputs of all three estimating methods by using MSE. Study of simulation is replicated several times (500) so that the samples

of three sizes (small, moderate and large) are independently collected.

Algorithm of Simulation

The simulation algorithms are written for estimating R using MATLAB program, according to the following steps:

1. The random sample

$$(x_{11}, x_{12}, \dots, x_{1r_1}), (x_{21}, x_{22}, \dots, x_{2r_2}), (x_{31}, x_{32}, \dots, x_{3r_3})$$

and

$$(y_{11}, y_{12}, \dots, y_{1v_1}), (y_{21}, y_{22}, \dots, y_{2v_2}), (y_{31}, y_{32}, \dots, y_{3v_3})$$

of size

$$(r_1, r_2, r_3, v_1, v_2, v_3)$$

$$= (15, 15, 15, 15, 15, 15), (45, 45, 45, 45, 45, 45)$$

and (95, 95, 95, 95, 95, 95) are generated from Weibull distribution

2. Selected the values of parameters for 6 experiments ($\beta_1, \beta_2, \beta_3, \mu_1, \mu_2, \mu_3$) in the Table 1:

3. Parameters $\beta_1, \beta_2, \beta_3, \mu_1, \mu_2, \mu_3$ were estimated (ML, Mo, LS, WLS, Rg and Pr) in equations: (25), (26), (32), (33), (43), (44), (53), (54), (62), (63), (69) and (70), respectively.

4. R was estimated in equations: (27), (34), (45), (55), (64) and (71).

5. Calculate the mean by $Mean = \frac{\sum_{i=1}^L \hat{R}_i}{L}$

6. The last stage is to use the "Mean square Error" to assess the results of the seven estimation methods:

$$MSE(\hat{R}) = \frac{1}{L} \sum_{i=1}^L (\hat{R}_i - R)^2$$

Table 1. Values of parameters and Reliability, such that $\alpha = 0.1$.

Experiment	k	m	β_1	β_2	β_3	μ_1	μ_2	μ_3	R
1	1.9	0.1	1.8	1.8	1.8	1.2	1.2	1.2	0.1169
2	1.9	0.1	2	2	2	1.2	1.2	1.2	0.0970
3	1.9	0.1	1.8	1.8	1.8	2	2	2	0.2548
4	1.1	0.9	1.8	1.8	1.8	1.2	1.2	1.2	0.1346
5	1.1	0.9	2	2	2	2	2	2	0.2479
6	1.5	0.5	1.1	1.2	1.3	1.4	1.5	1.6	0.3171

SIMULATION RESULTS:

After applying the previous steps of R for sample size

$$(r_1, r_2, r_3, v_1, v_2, v_3): (15, 15, 15, 15, 15, 15),$$

$$(95, 95, 95, 95, 95, 95) \text{ and } (45, 45, 45, 45, 45, 45)$$

Table 2. Values MSE and Mean for 6 experiments.

Exp.	Simple size	Criterion	ML	MO	LS	WLS	Rg	Pr	Best
1	(15,15,15,15,15,15)	MSE	0.0019	0.0035	0.0023	0.0027	0.0033	0.0036	ML
		Mean	0.1195	0.1208	0.1195	0.1200	0.1201	0.1211	
	(45,45,45,45,45,45)	MSE	0.0006	0.0017	0.0007	0.0012	0.0011	0.0019	ML
		Mean	0.1165	0.1170	0.1168	0.1169	0.1171	0.1170	
	(95,95,95,95,95,95)	MSE	0.0003	0.0013	0.0003	0.0008	0.0006	0.0016	ML LS
		Mean	0.1175	0.1206	0.1181	0.1195	0.1188	0.1209	
2	(15,15,15,15,15,15)	MSE	0.0015	0.0027	0.0018	0.0021	0.0026	0.0028	ML
		Mean	0.0999	0.1015	0.1001	0.1006	0.1009	0.1019	
	(45,45,45,45,45,45)	MSE	0.0004	0.0013	0.0005	0.0009	0.0009	0.0015	ML
		Mean	0.0969	0.0977	0.0971	0.0974	0.0975	0.0978	
	(95,95,95,95,95,95)	MSE	0.0002	0.0010	0.0003	0.0006	0.0004	0.0012	ML
		Mean	0.0976	0.1007	0.0982	0.0995	0.0989	0.1010	
3	(15,15,15,15,15,15)	MSE	0.0050	0.0085	0.0059	0.0069	0.0081	0.0089	ML
		Mean	0.2521	0.2497	0.2512	0.2505	0.2491	0.2497	
	(45,45,45,45,45,45)	MSE	0.0015	0.0045	0.0020	0.0031	0.0030	0.0050	ML
		Mean	0.2520	0.2487	0.2518	0.2504	0.2508	0.2480	
	(95,95,95,95,95,95)	MSE	0.0007	0.0034	0.0009	0.0021	0.0015	0.0040	ML
		Mean	0.2547	0.2561	0.2554	0.2561	0.2557	0.2559	
4	(15,15,15,15,15,15)	MSE	0.0024	0.0042	0.0028	0.0033	0.0040	0.0044	ML
		Mean	0.1368	0.1378	0.1368	0.1371	0.1371	0.1381	
	(45,45,45,45,45,45)	MSE	0.0007	0.0021	0.0009	0.0014	0.0014	0.0023	ML
		Mean	0.1340	0.1341	0.1342	0.1342	0.1344	0.1341	
	(95,95,95,95,95,95)	MSE	0.0003	0.0016	0.0004	0.0010	0.0007	0.0019	ML
		Mean	0.1352	0.1382	0.1358	0.1372	0.1365	0.1385	
5	(15,15,15,15,15,15)	MSE	0.0049	0.0084	0.0058	0.0068	0.0080	0.0088	ML
		Mean	0.2457	0.2437	0.2449	0.2443	0.2430	0.2437	
	(45,45,45,45,45,45)	MSE	0.0015	0.0044	0.0019	0.0031	0.0029	0.0049	ML
		Mean	0.2453	0.2424	0.2451	0.2439	0.2443	0.2417	
	(95,95,95,95,95,95)	MSE	0.0007	0.0034	0.0009	0.0021	0.0015	0.0039	ML
		Mean	0.2480	0.2495	0.2487	0.2494	0.2490	0.2494	
6	(15,15,15,15,15,15)	MSE	0.0062	0.0105	0.0073	0.0085	0.0100	0.0110	ML
		Mean	0.3116	0.3073	0.3101	0.3089	0.3067	0.3071	
	(45,45,45,45,45,45)	MSE	0.0019	0.0055	0.0024	0.0039	0.0037	0.0061	ML
		Mean	0.3131	0.3079	0.3126	0.3105	0.3110	0.3069	
	(95,95,95,95,95,95)	MSE	0.0009	0.0042	0.0011	0.0026	0.0019	0.0049	ML
		Mean	0.3167	0.3167	0.3173	0.3174	0.3174	0.3162	

CONCLUSIONS

These conclusions according to the simulation study results:

1. We concluded from the Table 1.
 - I. With increasing value of parameter β , reliability is decreasing.
 - II. With the increasing value of parameter μ , reliability is increased.
 - III. With the decreasing value of K/M , reliability is increasing.
 - IV. With the increasing value of parameter β and parameter μ , reliability is increased.
 - V. With the decreasing value of parameter β , the decreasing value of parameter μ and the increasing value of K/M , reliability is increased.
2. We concluded from the Table 2 the best estimator for R is ML for 6 experiments and different sample sizes.

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