Performance Parameters Evaluation of Surface Plasmon Resonance Based Fiber Optic Sensor with Different Bilayer Metals: Theoretical Study

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Abstract
Surface plasmon resonance (SPR) based fiber optic sensor with three types of bilayer configurations (silver/gold, copper/gold, and aluminum/gold) is theoretically analyzed. Performance parameters like sensitivity, signal to noise ratio, figure of merit, and resolution are evaluated for each configuration. Signal to noise ratio (SNR), and figure of merit (FOM) are enhanced very well for the selected bilayer configurations as the outer gold layer thickness increased, the sensitivity enhanced also but with small frictions while the resolution has decreased slightly.

Keywords: surface plasmon resonance, optical fiber sensor, bilayer metal, performance parameters.

Introduction
The very rapid evolution of technologies generates important contributions in various fields of science such as chemistry, biochemistry, biology, environment, materials, mechanics, medicine, and physics. The development of very advanced instruments in the field of sensing has become a significant challenge to fulfill needs and requirements that are increasingly important and necessary: Time control, extremely fast response, miniature components, monitoring in hard-to-reach environments, real-time analysis, etc. Among a large number of existing sensing devices, also called "sensors", one of them is based on a physical phenomenon of surface plasmon resonance (SPR). This type of sensors is particularly used today because of its ability to provide a fast response in real time, and without a use of labels [1][2][3]. This comes from the singular property of surface plasmons, to guide the light along a metal/dielectric interface, and this precisely according to the properties of the dielectric. More specifically, these surface waves whose characteristics are very sensitive to the dielectric medium which is in contact with the surface of the metal. A small variation in the dielectric constant of this medium results in a change in the so-called "resonance" conditions. By measuring this change, it is then possible to sense, in real time, the presence of chemical and / or biochemical species and also the change of some physical parameters in the environ surround the metal surface and thus to produce very sensitive sensors[4][5] . Numerous articles on sensors based on the phenomenon of (SPR) have been published since 1982 [6][7][8]. Indeed, several
configurations have been proposed for these sensors. Among these configurations, the most widespread is that proposed by Kretschmann and Raether, which is based on the use of a massive prism. Currently, SPR systems based on this configuration are widely used for its high sensitivity and high detection limit despite the complexity of the assembly and the high cost. It should be noted that the use of this configuration is restricted to non-miniature systems[7]. With the evolution of the industry, there is an increasing demand for miniature and automatic devices, especially for physical, chemical, biochemical and biological analysis systems, which have generated a new interest in optical fibers as a new SPR sensor configuration. The originality of this configuration consists in replacing the massive prism with an optical fiber. The interest of this approach comes from the meeting of two technologies. Firstly, it enables the known advantages of the optical fibers in telecommunication to be realized for new industrial applications. Indeed, the design of such optical fiber SPR sensors makes it possible to visualize miniature, flexible and relatively inexpensive devices making it possible to reach many sectors. On the other hand, the resonance detection of surface plasmons based on a resonant effect does not require a labeling of the target molecules and allows quantifying in real time the presence of molecules in very confined media. At present time, fiber optic sensors based on the phenomenon of surface plasmon resonance have proven their ability to measure[9][10]. Nevertheless, in order to obtain an efficient and competitive fiber-optic SPR sensor, it is necessary to improve its performance parameters like sensitivity, signal to noise ratio, etc. These performance characteristics depend mainly on the geometrical configurations of the sensor as well as on certain physical parameters such as the nature of the surface treatments and the dielectric constants of the materials involved. It can say that the problem is to find the ideal geometry as well as the key parameters influencing the response of the sensor. So far, experimental parametric studies and numerical simulations of (SPR) response seem to be two particularly interesting approaches to this problem. Indeed, several studies have carried out, with the help of numerical simulations, on the optimization of the performance of the sensor as well as on parametric studies. The development and characterization of a fiber-optic sensor with a layer of silver is the subject of several research projects. This type of sensor having good sensitivity and accuracy but it requires a protective layer because the silver oxidizes rapidly in aqueous media. For this reason, a sensor with a layer of silver is generally considered inappropriate in some areas. Other theoretical and experimental researches have been carried out on SPR fiber optic sensors with a layer of gold that is more stable over time than silver [11].

The performance of these sensors is not yet well established. It is always difficult to compare the performance of these sensors because this notion differs according to the published works and the characteristics of the sensor[12]. From these results, it was difficult to take into account all the possible combinations between the different parameters and conditions (e.g., type of molecules to be detected, method of interrogation, equipment used, etc.).

Our main objective is to evaluate the performance parameters of a fiber optic SPR sensor with a bilayer configuration. To achieve that, it will be necessary to develop a valid numerical model which allows us to determine the optimal operating conditions of the sensor. The numerical simulations performed according to certain parameters, calculating from multilayer system after writing it in matrix formalism in order to program it in the Matlab.

Materials and Methodology
According to the configuration of Kretschmann, surface plasmon resonance (SPR) sensing is based on the principle of attenuated total reflection. The system of sensing in SPR-based fiber optic sensor mainly consists of three media, the first is the core of
the fiber and the second is the metal layer while the third is the sensing medium as they shown in Figure 1. Small portion (about 10mm) of step index multimode optical fiber is unclad firstly in order to deposit the second layer of metal directly over the core of fiber. Normally the metals used in this type of sensors are noble metals like gold, silver, cooper, and aluminum. Light from polychromatic source is launched into one end of the optical fiber using a proper collimated lens and the transmitted light is recorded using optical spectrum analyzer (OSA).

Figure 1: Representation of three media considered in our simulations: the silica core of fiber, the sensitive metal layer and the solution to be analyzed.

The media that make up the SPR-based fiber optic sensor are:

1st Medium: the silica core of optical fiber: The refractive index will be indicated by $n_c$. The calculation of $n_c$ gotten from the dispersion relation of fused silica given by the equation (1)[13]:

$$n_c(\lambda) = C_0 + C_1\lambda^2 + C_2\lambda^4 + \frac{C_3}{(\lambda^2 - a)} + \frac{C_4}{(\lambda^2 - a)^2} + \frac{C_5}{(\lambda^2 - a)^3}$$  \hspace{1cm} (1)

Where the coefficients $C_0$, $C_1$, $C_2$, $C_3$, $C_4$, $C_5$ and $a$ are numerical values given as [14]:

- $C_0 = 1.4508554$
- $C_1 = -0.0031268$
- $C_2 = -38.1 \times 10^{-6}$
- $C_3 = 0.0030270$
- $C_4 = -77.9 \times 10^{-6}$
- $C_5 = 1.8 \times 10^{-6}$
- $a = 0.035$.

2nd Medium: The metallic layer: This layer is of thickness $d$; from Drude model, the dielectric function of the metallic layer given as [15]:

$$\varepsilon_m = 1 - \frac{\lambda^2 \lambda_c}{\lambda^2 (\lambda_c + il)}$$  \hspace{1cm} (2)

Where $\lambda_c$ and $\lambda_p$ are the collision wavelength and the plasma wavelength of metallic layer respectively. Table 1 shows both of plasma and collision wavelengths of gold and silver.

Table 1: The plasma and collision wavelengths of gold and silver [16][17].

<table>
<thead>
<tr>
<th>Metal</th>
<th>Plasma wavelength $\lambda_p$ (m)</th>
<th>Collision wavelength $\lambda_c$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold (Au)</td>
<td>$1.6826 \times 10^{-7}$</td>
<td>$8.9342 \times 10^{-6}$</td>
</tr>
<tr>
<td>Silver (Ag)</td>
<td>$1.4541 \times 10^{-7}$</td>
<td>$17.614 \times 10^{-6}$</td>
</tr>
<tr>
<td>Copper (Cu)</td>
<td>$1.3617 \times 10^{-7}$</td>
<td>$40.852 \times 10^{-6}$</td>
</tr>
<tr>
<td>Aluminum (Al)</td>
<td>$1.0657 \times 10^{-7}$</td>
<td>$2.4511 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

3rd Medium: sensing medium: Its dielectric constant is indicated by $\varepsilon_s$. It will most often be taken as a real constant. If $n_s$ is a refractive index of sensing medium then $\varepsilon_s = n_s^2$.

The number of these media can be expand to more than three as in our study. The gold layer is the additional layer which can protect the metal layer from oxidization and enhance some of the performance parameters of the sensor. In this system when the rays of polychromatic light guided through the optical fiber incident on the metal layer coated over the core, an excitation occur in the surface plasmon wave. If the wave vector of the incident wave equal to that of the plasmon wave, a resonance occur. Hence the resonance condition can written as:

$$\frac{2\pi}{\lambda} n_1 \sin \theta = Re[k_{sp}]$$  \hspace{1cm} (3)

Where $n_1$ is core refractive index, $\theta$ is the incident angle on the metal-core interface, $Re[k_{sp}]$ is a real part of the surface plasmon wave vector that can written as:

$$k_{sp} = \omega \frac{c}{\varepsilon_m \varepsilon_s} = \frac{2\pi}{\lambda} \sqrt{\frac{\varepsilon_m n_s^2}{\varepsilon_m + \varepsilon_s}}$$  \hspace{1cm} (4)

Where $\varepsilon_m$ and $\varepsilon_s$ are the dielectric constants of metal and sensing medium respectively and $\omega$ is the frequency of the incident wave while $c$ is a speed of light.
Matrix formulation of the reflectance in a multilayer system

For the calculation of the reflectance $R_p$ by finding an expression for amplitude reflection coefficient of a p-polarized beam incident on a medium consisting of non-magnetic isotropic layers, of number $N$ (the number of interfaces $N-1$) stacked along the direction of propagation of beam (as shown in Figure 2) it is important to use matrix formalism because it well adapted with Matlab software. This formalism, based on the calculation of the Fresnel coefficients, is described precisely by Yeh Pochi and X. Abeles [18][19]. The aim is to determining the electric field $E_k$ and magnetic field $H_k$ in the $N$ layers, taking into account a transition matrix at each interface, between $(k-1)$ and $k$, and the propagation matrix in each layer $k$. The medium of arbitrary layer ($k^{th}$ layer) in Figure (2a) is described by thickness $d_k$, dielectric constant $\varepsilon_k$, permeability $\mu_k$ and refractive index $n_k$. The minimum number of the layers in a surface plasmon resonance based sensors is three. Hence the first consideration of our modeling is three layers system in which the dielectric layer of dielectric constant $\varepsilon_2$, (the dielectric constant = (refractive index)$^2$), sandwiched between two semi-transparent layers of refractive indices $n_1$ and $n_3$ as shown in figure 2b.

Suppose a linearly polarized beam incident on the dielectric layer and both the electric ($E$) and the magnetic ($H$) fields are continuous through the boundaries or the interfaces. At the first interface between layer 1 and layer 2 (Figure 2b) the electric field is given by [21]:

$$E_1 = E_{i1} + E_{r1} = E_{t1} + E'_{r2}$$ (5)

Where $E_{i1}$, $E_{r1}$ and $E_{t1}$ are respectively the electric field amplitudes of incident, reflected and transmitted beam at the boundary 1, whereas $E'_{r2}$ is the electric field amplitude of the reflected beam from the boundary 2 and incident on the boundary 1.

By applying the boundary condition for the magnetic field at the boundary 1, one can get:

$$H_1 = \sqrt{\frac{\varepsilon_1}{\mu_1}} \frac{(E_{i1} - E_{r1})n_1 \cos \theta_{i1}}{E_{r1}'}n_2 \cos \theta_{i2}$$ (6)

$E$ and $H$ are related as:

$$H = \sqrt{\frac{\varepsilon_1}{\mu_1}} n_1 \hat{k} \times E$$ (7)

Where $\hat{k}$ the propagation vector and $\theta_{i1}$, $\theta_{i2}$ are the incidence angles of the beams at the boundaries 1 and 2 respectively.

For the second boundary between 2$^{nd}$ and 3$^{rd}$ layers, it can write the electric field as:

$$E_2 = E_{i2} + E_{r2} = E_{t2}$$ (8)

And the magnetic field as:
where $E_{t2}$, $E_{r2}$ and $E_{z2}$ are the electric field amplitudes of incident, reflected and transmitted beams respectively at the boundary 2 and $\theta_2$ is the refraction angle of the beam at the boundary 2.

When the beam go across a layer of thickness $d_2$, the phase shift occurred can be written as [16]:

$$\beta_2 = \frac{2\pi d_2}{\lambda}(\epsilon_2 - n_2^2\sin^2\theta_2)^{1/2}$$ \hspace{1cm} (10)

The electric field equations at the boundary 2 are then:

$$E_{t2} = E_{t1}e^{-i\beta_2},$$ \hspace{1cm} (11) $$E_{r2} = E_{r1}e^{i\beta_2}$$ \hspace{1cm} (12)

Hence, equations (8) and (9) written as:

$$E_z = E_{t1}e^{-i\beta_1} + E_{r1}e^{i\beta_1}$$ \hspace{1cm} (13)

$$H_z = \sqrt{\frac{\mu_1}{\epsilon_1}}(E_{t1}e^{-i\beta_1} - E_{r1}e^{i\beta_1})n_z\cos\theta_{t1}$$ \hspace{1cm} (14)

By solving equations (13) and (14) for $E_z$ and $E_{r2}$ and substituting their values in equations (5) and (6) we can obtain:

$$E_z = E_z\cos\beta - H_z(\sin\beta_2)/q_2$$ \hspace{1cm} (15) $$H_z = E_zq_2(\sin\beta_2) + H_z(\cos\beta_2)$$ \hspace{1cm} (16)

Where:

$$q_2 = \frac{(\epsilon_2 - n_2^2\sin^2\theta_2)^{1/2}}{\epsilon_2}$$

In matrix notation equations (15) and (16) can be written as:

$$\begin{bmatrix} \cos\beta_2 & m_2 \cos\beta_2 \\ -i q_2\sin\beta_2 & -i q_2\sin\beta_2 \end{bmatrix} \begin{bmatrix} E_z \\ H_z \end{bmatrix} = \begin{bmatrix} E_z \\ H_z \end{bmatrix}$$ \hspace{1cm} (17)

Or

$$\begin{bmatrix} E_z \\ H_z \end{bmatrix} = m_2 \begin{bmatrix} E_z \\ H_z \end{bmatrix}$$ \hspace{1cm} (18)

In generalized form, electric and magnetic fields at the first boundary $z=z_1$ are related to those at the final boundary $z=z_{N-1}$ by:

$$\begin{bmatrix} E_1 \\ H_1 \end{bmatrix} = M \begin{bmatrix} E_{N-1} \\ H_{N-1} \end{bmatrix}$$ \hspace{1cm} (20)

Where $E_1$ and $H_1$, respectively tangential components of electric and magnetic fields at the boundary of 1st and 2nd layers, while $E_{N-1}$ and $H_{N-1}$ are corresponding fields at the boundary of $(N-1)^{th}$ and $N^{th}$.

The characteristic matrix $M$ is given by:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$ \hspace{1cm} (21)

Where $k=2, 3, ..., N-1$ is number of layers.

$$m_k = \begin{bmatrix} \cos\beta_k & (-i\sin\beta_k)/q_k \\ -i q_k\sin\beta_k & \cos\beta_k \end{bmatrix}$$ \hspace{1cm} (22)

By simplification and rearrangement, the amplitude reflection coefficient for p-polarized wave is [20]:

$$r_p = \frac{(M_{11}^p + M_{12}^p q_1^p)q_1^p - (M_{21}^p + M_{22}^p q_1^p)}{(M_{11}^p + M_{12}^p q_1^p)q_1^p + (M_{21}^p + M_{22}^p q_1^p)}$$ \hspace{1cm} (25)

And for s-polarized wave:

$$r_s = \frac{(M_{11}^s + M_{12}^s q_1^s)q_1^s - (M_{21}^s + M_{22}^s q_1^s)}{(M_{11}^s + M_{12}^s q_1^s)q_1^s + (M_{21}^s + M_{22}^s q_1^s)}$$ \hspace{1cm} (26)

Finally, the general expression of the reflectivity $R$ as a function of the reflection coefficients of Fresnel and of the terms of the resulting matrix $M$ is obtained by:

$$R_p = |r_p|^2$$ \hspace{1cm} (27)

for p-polarized, and by:

$$R_s = |r_s|^2$$ \hspace{1cm} (28)

for s-polarized.

**Transmitted Power Calculation**

From equations (27) and (28), it can calculate the power of the light transmitted in the optical fiber. By taking into account of the two types of polarization, -P and -S, the transmitted light power will be calculated separately with the
reflection coefficients for the -s and -p polarizations. The total transmitted power will be deducted from the average of the contributions of the two polarizations:

\[ P_{\text{trans}} = \frac{P_s + P_p}{2} \]

Then the total transmitted power is given by the equation [22]:

\[ P_{\text{trans}} = \frac{1}{2} \left[ \int_{\theta_c}^{\pi/2} R_p \left( n_s^2 \frac{\sin \theta \cos \theta}{1 - n_s^2 \cos^2 \theta} \right) d\theta + 1 \right] - \sum_{p} \int_{\theta_c}^{\pi/2} R_s \left( n_s^2 \frac{\sin \theta \cos \theta}{1 - n_s^2 \cos^2 \theta} \right) d\theta \]

(30)

Where \( R_p \) and \( R_s \) are the reflectivity of the light of polarization -p and -s respectively.

Since that the SPR phenomenon does not occur when the incident ray has an s-polarization, the light power transmitted by the fiber can be, under the best resonance conditions, only half of the power injected at the input face of the fiber. So that the final expression of \( P_{\text{trans}} \) can be written in the following form:

\[ P_{\text{trans}} = \frac{1}{2} \left[ \int_{\theta_c}^{\pi/2} R_p \left( n_s^2 \frac{\sin \theta \cos \theta}{1 - n_s^2 \cos^2 \theta} \right) d\theta + 1 \right] \]

(31)

Where \( N_{\text{ref}} (\theta) \) is the total number of reflections performed by a beam making an angle \( \theta \) with the normal to the core-metal layer interface in the sensing region and it is given by [23]:

\[ N_{\text{ref}} (\theta) = \frac{L}{D \tan \theta} \]

(32)

\( L \) and \( D \) are the length of the sensing region and the core diameter respectively.

The critical angle of the core-clad interface is given by [23]:

\[ \theta_c = \sin^{-1} \left( \frac{n_{\text{cl}}}{n_1} \right) \]

(33)

Where \( n_{\text{cl}}, n_1 \) are the refractive index of the clad and core of the optical fiber respectively.

**Performance parameters**

Sensitivity, signal to noise ratio, figure of merit, and resolution are a performance parameters that taking into account of our study. In the case of wavelength interrogation, sensitivity can be defined as the change in resonance wavelength per unit change in refractive index of the sensing medium and it can be written as:

\[ S_n = \frac{\delta \lambda_{\text{res}}}{\delta n_g} \]

(34)

While the signal to noise ratio (SNR), figure of merit (FOM), are inversely proportional to the width of SPR spectral curve and can written as:

\[ \text{SNR} = \frac{\delta \lambda_{\text{res}}}{\delta \lambda_{1/2}} \]

(35)

\[ \text{FOM} = \frac{S_n}{\delta \lambda_{1/2}} \]

(36)

Finally, the resolution of the sensor can be defined as the minimum of change in refractive index detectable by the sensor and given as:

\[ R = \frac{\delta n_s}{\delta \lambda_{\text{res}}} \delta \lambda_{\text{DR}} \]

(37)

Where \( \delta \lambda_{\text{DR}} \) is the spectral resolution of the optical spectrometer analyzer used to measure the resonance wavelength and it toke to be equal to 0.01 in our study.

**Result and Discussion**

The values of the parameters used in the numerical calculations of this study were:

\( \text{numerical aperture (NA)} = 0.22, \text{fiber core diameter (D)} = 600\mu m, \text{sensing length (L)} = 10\text{mm}, \text{and the thickness of metal layer (d)} =50nm. \) In order to evaluate the effects of an additional gold layer on the performance parameters of the SPR-based fiber optic sensor, different thicknesses i.e. 0, 5, 10, 15, and 20nm of gold layer are used.

Figures (1a-f) show the SPR response curves for all studied bilayer configurations (Ag/Au, Cu/Au, and Al/Au) with inner metal layer of thickness 50nm and different thicknesses of Au layer (0, 5, 10, 15, and 20nm) at different refractive index of the sensing medium i.e. 1.33, 1.34, 1.35, 1.36, and 1.37. The resonance wavelength increase as refractive index of the sensing medium increase for all configurations and with all thicknesses of Au layer. This behavior can be justified by the resonance condition of surface plasmon wave. If the refractive index of the sensing medium is large, then the real part of the propagation constant will be large and hence the resonance condition will satisfied at large value of wavelength. Also if the refractive index of the sensing medium is

\[ \frac{\omega}{c} \]
small, then the real part of the propagation constant will be small and hence the resonance condition will satisfy at small value of wavelength. It is clear that the width of each SPR response curve at all thicknesses of Au layer is different from configuration to another. Wider SPR response curve with Ag/Au configuration while narrower SPR response curve with Al/Au configuration. This difference in the width and also in the magnitude of the resonance wavelength shifting for each configuration leads to difference in the performance parameters which are depend mainly on these two parameters. Tables 2 and 3 show the performance parameters of the sensor with different metals of thickness 50nm and different thicknesses of Au over layer. In Table 2 it is clear that the sensitivity increase slightly as the thickness of Au layer increase for all configurations. This occurs due to the change of refractive index of Au as Au includes high value of real part of its dielectric function and hence its refractive index. The highest sensitivity was with Ag/Au configuration $3.0749 \, \mu m/RIU$. In the other hand, another performance parameters enhanced as Au layer thickness increase for all configuration. The highest SNR and FOM was with Al/Au configuration and the lowest was with Ag/Au configuration for all thicknesses of Au layer. The resolution of the sensor decreased slightly for all configurations Al/Au was the greatest and Cu/Au was lowest.

Figure: 1 the SPR response curves of bilayer configuration: (core/Ag/Au/Analyte) with Au thickness of (a) 0nm (b) 20nm, (core/Cu/Au/Analyte) with Au thickness of (c) 0nm and (d) 20nm, and (core/Al/Au/Analyte) with Au thickness (e) 0nm and (f) 20nm

Table 2: Sensitivity and Signal to noise ratio of SPR fiber optic sensor with bilayer metals at different thicknesses of gold outer layer.

<table>
<thead>
<tr>
<th>Au (nm)</th>
<th>Sensitivity (µm/RIU)</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.002 2.933 2.3129 0.594 1.324 1.944</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3.031 2.952 2.3179 0.727 1.655 2.283</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>3.051 2.966 2.3209 0.866 2.070 2.499</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3.064 2.972 2.3249 0.981 2.507 2.607</td>
<td></td>
</tr>
</tbody>
</table>
Conclusion

Surface plasmon resonance (SPR) based fiber optic sensor has been studied numerically. The performance parameters of the sensor has been evaluated with different bilayer configurations (Ag/Au, Cu/Au, and Al/Au) with different thicknesses of Au additional layer. Increase in Au layer thickness increases the sensitivity, SNR, and FOM for all studied metals and decrease the resolution with small fractions. Sensor with (Ag/Au) configuration shows the largest sensitivity. SPR-based fiber optic sensor with Al/Au configuration shows the highest SNR, FOM, and resolution. The N-layer model with matrix formalism showed good method to simulate SPR fiber optic sensor and also evaluate its performance parameters.

References:


