Research Article

Hybrid Lossless Image Compression Using Wavelet Transform and Hierarchical non Linear Prediction

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Abstract
This paper introduces a promising hybrid lossless image compression method by combining the wavelet transform along with a hierarchal non-linear polynomial approximation model to compress natural and medical images. The test results showed good performance in which the compression ratio is improved about three times or more on average in compered with the results of a non-linear coding system that does not adopt the techniques used in this research.

Keywords: Wavelet Transform, Non-linear Polynomial..

Introduction
Image compression is very important in the present world for efficient archiving and transmission. Lossless image compression is characterized by preserving image quality; where the image can be reconstructed exactly as the original image with error free [1]. Unfortunately, there is a limitation in the compression performance (i.e., small compression ratio from 2 to 10) because of exploiting the statistical redundancy only (i.e., exploits the coding redundancy and/or inter pixel redundancy) [2] [3] [4].

The performance of a lossless compression system can be improved either by combining different techniques such as wavelet and prediction or by exploiting a technique that selects significant blocks and exclude others [5] [6] [7]. Recently, many researchers, such as [8] [9] [10] [11] [12], focused on using the Discrete Wavelet Transforms (DWT) in image compression. In contrast to the discrete cosine transform (DCT); the advantage of DWT is that; it does not require the image to be divided into blocks, but it analyses the image as whole.

In one-dimensional wavelet transform (1D) the image is decomposed into high and low sub-images, more details about 1D transform can be listed in [13], while in two dimensions (2D) DWT, the decomposition is achieved by applying (1D) transform in horizontal and vertical directions; so this will result into four sub bands images; low sub band image (LL), high sub band image (HL), low sub band image (LH), and high sub band image (HH). This process can be repeated with the (LL) image several times. Generally, the approximation sub band (LL) considered the most significantly important part since it contains all image information, while other sub bands considered to be less significant, since they contain very small image information and they can be set to zero without significantly changing the image [13].

In this paper, an efficient, simple and fast hybrid lossless method was suggested to compress images; based on exploiting a two dimensional wavelet transform along with polynomial representation of non-linear base which utilized hierarchically in order to maximize the compression ratio.
Materials and Methodologies
The main taken concerns in the suggested hybrid system are:

First, the polynomial coding of non-linear approximation model is exploited to compress image efficiently using six coefficients \(a_0, a_1, a_2, a_3, a_4, a_5\) [14].

Second, the hierarchal scheme was adopted to improve the compression ratio and preserve image quality [15]. The Hierarchical technique worked reversely from subsequent layers to construct up layers, this means, the coefficients \((a_00, a_{01}, a_{02}, a_{03}, a_{04}, a_{05})\) of layer2 are used to construct layer1 coefficient \((a^{01})\); then layer1 coefficients \((a^{01}, a_1, a_2, a_3, a_4, a_5)\) are used to reconstruct the approximated image \(LL\).

The following steps illustrate the system implantation in more details. Figure (1) shows the basic steps clearly:

The following steps were adopted in this study:

1. Input grayscale image \((I)\) of size \(N\times N\).
2. Apply the wavelet transform which is characterized by simplicity and high compression ratio. The transform based on decomposing image \((I)\) into four quadrants sub band namely \((LL\) and detail sub bands \(LH, HL\) and \(HH)\) each of size \((N/2 \times N/2)\).
3. For the approximation sub band \((LL)\), the polynomial prediction of non-linear based model is utilized hierarchically to remove the redundancy embedded within image pixel values, using the following steps:
   a. Construct layer1 of hierarchal representation, first partition the approximation sub band \((LL)\), \((LL\) considered here as the original image), into non overlapped blocks of fixed size \(n \times n\). Then, the polynomial coefficients \(a_0, a_1, a_2, a_3, a_4, a_5\) was calculated using the following equations [14]:
      \[
      a_1 = \frac{n^{-1} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I(i, j) \times (j - x_c)}{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (j - x_c)^2}
      \]
      \[
      a_2 = \frac{n^{-1} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I(i, j) \times (i - y_c)}{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (i - y_c)^2}
      \]

Where the \(a_1\) and \(a_2\) coefficients corresponds to the ratio of sum pixel multiplied by the distance from the center divided by the squared distance in \(i, j\).

\[
xc = yc = \frac{n - 1}{2}
\]

Where \((j-x_c)\) and \((i-y_c)\) measure the distance from a pixel coordinates to the block center \((x_c, y_c)\).

Other coefficients, namely the \(a_0, a_3\) and \(a_4\) can be founded by applying the Crammers rule, where:

\[
V_1 \quad W_2 \quad W_2 \\
V_2 \quad W_3 \quad W_4 \\
V_3 \quad W_4 \quad W_3 \\
W_1 \quad V_1 \quad W_2 \\
W_2 \quad V_2 \quad W_4 \\
W_2 \quad V_3 \quad W_3 \\
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W_2 \quad W_3 \quad W_4 \\
W_2 \quad W_4 \quad W_3 \
\]

Where:

\[
V_1 = a_0 W_1 + a_3 W_2 + a_4 W_2 \\
V_2 = a_0 W_2 + a_3 W_3 + a_4 W_4 \\
V_3 = a_0 W_2 + a_3 W_4 + a_4 W_3 \\
W_1 = n \times n \\
W_2 = \sum_{j=0}^{n-1} (j - x_c)^2 = \sum_{i=0}^{n-1} (i - y_c)^2 \\
W_3 = \sum_{j=0}^{n-1} (j - x_c)^4 = \sum_{i=0}^{n-1} (i - y_c)^4
\]
\[
W_4 = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (j-x_c)^2 (i-y_c)^2
\]  
(14)

4- Construct layer 2 of the hierarchal representation from layer1 \(a_0\) coefficient. The non-linear polynomial coding technique will be utilized again in this layer using Equations (1-14) to construct coefficients \(a_{00}, a_{01}, a_{02}, a_{03}, a_{04}\) and \(a_{05}\) (in this layer \(a_0\) coefficient from layer1 will be considered here as original image).

5- For Layer 2:
   a- Determine the deterministic part (function formula) \(a\bar{0}\).

\[
a\bar{0} = a_{00}W_1 + a_{01}(j-x_c) + a_{02}(i-y_c) + a_{03}(j-x_c)^2 + a_{04}(i-y_c)^2 + a_{05}(j-x_c)(i-y_c)
\]

b- Find residual image using the following equation [16]:

\[
a0 \text{ Re } sd = a0 - a\bar{0}
\]

c- Build the modeled approximated \(a\hat{0}\)

\[
a\hat{0} = a\bar{0} + a0 \text{ Re } sd
\]

6- Reconstruct layer1 from layer2 hierarchically as follows:
   a- Determine the deterministic part \(L\bar{L}\).

\[
L\bar{L} = \hat{a}0W1 + a1(j-x_c) + a2(i-y_c) + a3(j-x_c)^2 + a4(i-y_c)^2 + a5(j-x_c)(i-y_c)
\]

b- Find the error (residual)

\[
LL \text{ Re } sd = L - L\bar{L}
\]

7- Use Run Length and LZW and Huffman coding techniques to encode:
   a- Layer 2 information of coefficients \(a_{00}, a_{01}, a_{02}, a_{03}, a_{04}\) and \(a_{05}\) and the error \((a0\text{Resd})\) along with the layer1 information of coefficients \(a1, a2,a3,a4,a5\) and the error \((LL\text{Resd})\).
   b- The sub bands LH, HL and HH.

8- Reconstruct the compressed image (that identical to the original one I) using the following steps:
   a- For the approximation sub band LL, the residual along with the coefficients used to rebuild the LL quadrant

\[
LL = L\bar{L} + LL \text{ Re } sd
\]

b- Apply the inverse wavelet transform to reconstruct image I.

**Results and Discussion**

To evaluate the performance of the suggested hybrid method; two sets of image natural and medical were tested (as illustrated in Figure 2) all images in size of 256×256. Figure 3 shows the reconstructed image after the compression process.

In this paper, the compression ratio was adopted as a guide to the performance of the suggested system; because in lossless image compression system there is no degradation needed to be evaluated; i.e., the compressed image will be identical to the original one.

Table 1, summarizes the results of the suggested method; it shows the size of the compressed information and the compression ratio against the utilized block sizes for the tested images.

Table 2, illustrates the results obtained from a non-linear compression system without using the suggested techniques in this paper; these results are used to illustrate the effectiveness of the suggested method.

The results show the high compression ratio is achieved for a lossless compression system characterizes this technique compared to other technique, in which the compression ratio is improved about three times or more on average.
Figure 1: The suggested Compression System Structure.

Figure 2: The Tested Grayscale Images.
Figure 3: The Reconstructed Images using block size of 8*8.

Table 1: Performance of the Suggested Method.

<table>
<thead>
<tr>
<th>Test image</th>
<th>Size of original image (in bytes)</th>
<th>Size of compressed image (in bytes)</th>
<th>Compression Ratio</th>
<th>Size of compressed image (in bytes)</th>
<th>Compression Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camera</td>
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<td>1400</td>
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<td>1388</td>
<td>47.2161</td>
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<td>53.8947</td>
<td>1204</td>
<td>54.4319</td>
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<tr>
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<td>65536</td>
<td>932</td>
<td>70.3176</td>
<td>920</td>
<td>71.2348</td>
</tr>
<tr>
<td>Mr</td>
<td>65536</td>
<td>988</td>
<td>66.3320</td>
<td>976</td>
<td>67.1475</td>
</tr>
<tr>
<td>Brain</td>
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<td>1126</td>
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<td>1084</td>
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Table 2: The Performance of non linear prediction compression system.

<table>
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<tr>
<th>Test image</th>
<th>original image size (in bytes)</th>
<th>Size of compressed image (in bytes)</th>
<th>Compression Ratio</th>
<th>Size of compressed image (in bytes)</th>
<th>Compression Ratio</th>
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</table>
Conclusions
The results in this paper are promising in terms of the higher compression gain achieved compared to the current standard technique. The compression ratio is affected by two factors; the first one is the image nature, natural images contain more details than the medical one, which implicitly means; decreasing in the compression rate compared to the medical. The block size of the approximation sub band LL was the second factor; whereas the block size gets bigger, less coefficient are needed (i.e., 6 coefficients for larger block sizes); and this will implicitly improves the compression ratio. On the other hand; exploiting the wavelet transform along with a hierarchical polynomial approximation of non-linear base effectively improved the compression ratio about three times or more on average.

References