Behavior ofIon Beams For Triplet Quadrupole Lenses

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ABSTRACT

This research includes a study to understand the behavior of charged particles beam through consideration of systems of quadrupole triplet reaching to the optimum design of beam transport through a system of triplet quadrupole triplet lenses. In this work, tracing the path of charged particles beam has been within the free field space and quadrupole electrostatic lens system which include the triplet quadrupole lens by using matrices to described particle trajectories throughout the system. Matrix representation deals with ion beam as bunched and representing phase ellipse for both horizontal and vertical planes. The present work investigated the effect of the main parameters of triplet quadrupole such as length of quadrupole, the distance between the lenses, voltage applying on lenses, second field free region length. Matlab program built to study these parameters, the results indicated that good focusing properties for both horizontal and vertical plane.

Keywords: plasma, ion beam transport, triplet quadrupole lens.

INTRODUCTION

Electron lens is one of important element for focusing an electron (charged particle) beam by using either a magnetic field or an electrostatic field in a way that is analogous to the focusing of light beam by an optical lens[1]. Generally, electron lenses play an important part in the formation and control of the beam of electron or ions, and have important applications in several different fields such as electron microscopy, cathode ray tubes, ion accelerators and electron impact studies[2]. According to the type of field that could be used, the electron lenses may be classified into magnetic lenses and electrostatic lenses. The triplet quadrupole lens designs are prominent among those frequently employed. Such lenses are simple to build and to control, and require lower operating voltages and have less spherical aberration than do axis symmetric electrostatic lenses. This study investigated the focusing properties of charged particle beam passing through a system consist form number of triplet quadrupole lenses.

Theoretical Considerations

The ideal electrostatic-quadrupole lens system consists of four parallel electrodes with hyperbolic cross sections as shown in Figure (1) it has four planes of symmetry intersecting along the z-axis with an angle (a) between them. The lens centered at Z = 0, extends in the
z-direction the aperture of the lens (a) is defined by the diameter of the hypothetical circle tangential to the four electrodes [3, 4]. Apart from round lenses, quadrupole affect the paraxial path of rays. Since their electromagnetic field is transversal inside the quadrupole, its refraction power is proportional to the field strength. This is the reason why these “strong focusing” elements are exclusively employed in high energy accelerators and storage rings[5].

A quadrupole lens focuses in one coordinate direction and defocuses in the other. A single lens cannot be used to focus a beam to a point or to produce a two-dimensional image. Two-dimensional focusing can be accomplished with combinations of quadrupole lenses. Quadrupole lens combinations form the basis for most high-energy particle transport systems. They occur as extended arrays or as discrete lenses for final focus to a target.

![Figure 1: Principle of an ideal electrostatic quadrupole lens system consisting of four hyperbolic](image)

Quadrupole lens combinations are convenient to describe transverse motions are separable in x and y if the poles (electrodes) are aligned with the axes as shown in Figure (2)[7].

There are many configurations for electrostatic lenses, based on shape of their electrodes, such as cylinders, planes containing circular holes, and grid[8]. The total effect of electrostatic lens depends on the strength extent of electrostatic field and on the speed of the ions of the pass beam.

In electrostatic lenses the trajectories of particles, which have passed the same accelerating voltage, are independent of the particle's mass. In magnetic fields, however, under the same conditions the particle trajectories depend on the particle's mass that is heavy particles are less focused than light ones. That is based on the fact that the electrostatic lenses are favored to focus heavy particles such as ions. So that it can be used to control beam of low-energy electrons or ions[2]. Type of electron lenses, electrostatic or magnetic, is limited by the number of poles in the lens. In present study quadrupole lens has been used and the focusing field is obtained by means of four poles. The forces on a positively charged particle in electrostatic quadrupole field are in the same direction as the fields[9, 10]. Quadrupole lenses have convergent action in one direction and divergent action in the other direction, which is perpendicular to the first. Quadrupole lenses are found in many forms (singlet, doublet and triplet).

**Quadrupole Triplet Lenses**

The quadrupole triplet consists from three singlet lens is separated by drift space with distance (S). Most of triplet lenses consist of two doublet lenses. The quadrupole triplet is preferred in many cases, although it has a more complex structure than doublet, for its optical properties to correct the aberration of the lens[2, 11]. The type of triplet depends on the arrangement of the singlet or doublet lenses forming it, for example in the case of using two (CD) doublet lens the action of triplet on transport beam envelope could behave as shown in Figures (2), focusing-defocusing-focusing (FDF) in one plane and defocusing-focusing-defocusing (DFD) in other plane.
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Figure 2: Improved stigmatic properties of a quadrupole triplet lens in the x and y planes [2, 7].

Transfer Matrix of the quadrupole triplet lens

Transfer matrices describe changes in the transverse position and angle of particles relative to the main beam axis. If x and y are the coordinates normal to z, then a particle orbit at some axis position can be represented by the four dimensional vector \((x, x', y, y')\), the quantities \(x'\) and \(y'\) are angles with respect to the axis and in matrix form [12]:

\[
X = \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}
\]  

(1)

An optical element operates on an entrance orbit vector to generate an output orbit vector. The transfer matrix \((R)\) represents this operation. That is, the result of passing particle through systems element is to make new coordinates and a linear combination of the old [2].

\[
X_{j}(out) = \sum_{j=1}^{4} R_{j} X_{j}(in)
\]  

(2)

In charged particles systems, the behavior of an individual particle is often of less concern than in the behavior of a bundle of particles. One can bound the beam with an appropriately sized and shaped surface in phase space, and then transport this surface using the fact that the hyper volume of this closed surface in phase space must remain constant. The standard surface to use for this purpose is an ellipsoid. Phase ellipses represent the properties of a beam. The transverse phase ellipses are represented by the symmetric matrices \((\sigma_x)\) and \((\sigma_y)\) [13].

\[
\sigma(out) = R \sigma(in) R^T
\]

(3)

The \(R\)-matrix in quadrupole triplet given by [11,13]:

\[
R = \begin{bmatrix}
1 & 2 \left(2L_q + S \left(1 \pm \frac{S}{L_q}\right) \right) \\
-2 \left(\frac{\theta}{L_q}\right) \left(2 + \frac{S}{L_q}\right) & 1
\end{bmatrix}
\]

Where: \(\theta = \frac{L_q}{a} \sqrt{\frac{Va}{U}}\); lens length, \(S\): the distance between lenses, \(Va\): pole voltage, \(a\): distance between the pole tip and lens axis, \(U\): particle potential.

The upper sign is for Focusing-defocusing-focusing plane and lower sign is for Defocusing-focusing-defocusing plane.

Sigma matrix for the quadrupole triplet lens in \(x\)-plan:

\[
\sigma_{11}(out) = \sigma_{11}(in) + 4 \sigma_{12}(in) [2L_q + S \left(1 + \theta^2 \left(1 + \frac{S}{L_q}\right)\right)] \\
-4 \sigma_{12}(in) [2L_q + S \left(1 + \theta^2 \left(1 + \frac{S}{L_q}\right)\right)^2]
\]

\[
\sigma_{22}(out) = -2 \sigma_{22}(in) \left(\frac{\theta}{L_q}\right)^2 \\
+ \sigma_{22}(in) \left(1 + \theta^2 \left(1 + \frac{S}{L_q}\right)\right) [2L_q + S \left(1 + \theta^2 \left(1 + \frac{S}{L_q}\right)\right)]
\]

Sigma matrix for the quadrupole triplet lens in \(y\)-plane:
RESULTS AND DISCUSSION

To get good understanding of the behavior of charged particle beam passing through different system designs of quadrupole triplet lens, it can be taken a system of quadruple triplet lens consists of several quadrupole triplet lenses and then studies the action of the system as focusing or defocusing lens. To get the best understanding for the factors affecting the beam.

**Lens Length (L_q)**

We take some different designs for double triplet quadrupole lens system by change the lens length (L_q) with values (L_q=100,150mm) with fixed all other factors value (L_d=50mm, S2=75mm, V_a=750V). Figure (3) illustrates the beam envelope along a channel of quadrupole triplet lens in horizontal plan. In general the beam envelope curve has more than peak which represented the minimum and the maximum values of envelope, and indicating the focus and defocus region along this system. From the same Figure, one could note that the large (L_q), causes additional peak to appear, also in this case high value for beam envelope reaches to (1400mm), the last behavior is not preferable.

**Distance Lenses (L_d)**

To give a clear perspective of distance lenses effect on the charged particle beam passing through the double triplet quadrupole lens. Will be change (L_d) with values (20,50mm) and fixed all factors that effect to ion beam behavior (L_q=100mm, S2=75mm, V_a=750V). Figure (5) illustrates the beam envelope along a channel of the quadrupole triplet lenses in horizontal plane. There is focusing and defocusing action appear after the charged particle beam input the system also there is more than one peak represented the region of focus and defocus of the ion beam, the focus and defocus peak position is not the same for different value of (L_d), there is shift in their position depending on (L_d) value, here one could note that the L_d with value (50mm) gives best behavior than that with value (20mm).
Figure (6) indicates the same behavior of beam envelope for vertical plane which appears the same as in a horizontal plane but the beam envelope has small value than that for horizontal that means the focusing properties of ion beam is better than that in horizontal plane.

Second Free Region (S2)
The effect of the length of the second free region located after the double triplet quadrupole lens and is convergence ion beam referring to the action of the double triplet quadrupole as convergence lens. The effect study be changing length of second free region ($S_2=50, 75, 100\, \text{mm}$) with fixed all other factors ($L_q=100\, \text{mm}$, $L_d=50\, \text{mm}$, $V_a=750\, \text{V}$).

Figure (7) indicated the envelope of ion beam in horizontal plane, there is no clear effect of changing the second free region length that appears at this range of ($S_2$) values. That means the distance between the quadrupole triplet lenses does not have strong effect on the nature of beam envelope.

Lens Voltage ($V_a$)
Here was take different values ($V_a=750, 1250\, \text{V}$) with fixed other parameter that effect in channel triplet quadrupole lens design ($L_q=100\, \text{mm}$, $L_d=50\, \text{mm}$, $S_2=75\, \text{mm}$), a total voltage is applied between pole of lenses effect on the charged particle beam to be parallel to the axis of lens that is, converge and forced this desirable action.

Figure (9) illustrate the beam envelope of charged particle beam in horizontal plane. There is converge action at the end of this system, that means the beam exit from first lens has good focus action when entering the second lens containing with converge action, that means decreasing the value voltage leads to increasing focus beam, this converge seems nearly as straight line at voltage (1250V).
Figure (10) shown beam envelope of charged particle beam in vertical plane. The behavior of beam envelope is same for the different value of voltage as converge, also this converge increase by increasing the voltage.

Figure 10: Beam envelope versus distance for different values of $V_a$ for vertical plane.

From overall the research ion beam passing through number of triplet quadrupole lens , when compare this system that of single triplet lens or double triplet lens, found that the converge action increase by adding the more than one triplet quadrupole lens along the system, the extraction region was up right phase space ellipse, in the first free region is already be diverge in the phase space ellipse. When the ion beam passing through the system the phase space ellipse is changed so that there is converge action appears in both horizontal and vertical plane.

**CONCLUSIONS**

From the results of this research, it can be concluded the following:

1) The ion beam envelope in field free regions strongly depends on the length of these regions, in addition to the behavior of the pervious optical element as converge or diverge lens. So, ion beam envelope in free field region may converge or diverge, that means the field free regions depend on the properties of the previous optical elements in the system.

2) The beam envelope curve for beam passing through a quadrupole triplet lens has more than peak which represented the minimum and the maximum envelope, in other word representing the focus and defocus region along the system.

3) When Increasing of $(L_q, L_d)$ cause additional peak to beam envelope curve.

4) The distance between the quadrupole triplet lenses in the system has not strong effect on the nature of beam envelope.

5) When the voltage at high value (1250V) there is high diverge of ion beam at the end of system, reaching to this, has not practical value.

6) In general the focusing properties in vertical plane are better than that for horizontal plane.

7) Good focusing properties for both horizontal and vertical plane obtained when $(L_q=100\text{mm},\ L_d=50\text{mm},\ S_2=100\text{mm},\ V_a=750\text{V})$

**REFERENCES**


